THE SMITH-BURDGE ARITHMETICS

ADVANCED BOOK

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1937

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THE SMITH-BURDGE ARITHMETICS

ADVANCED BOOK

BY

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PREFACE

This series of arithmetics has been prepared not only to meet the requirements of leading modern courses of study and to keep pace with the present progressive movement in teaching arithmetic, but also with the purpose of setting a new standard in the arrangement of material and in the development of the subject to meet the needs and interests of children. To this end the authors have taken into account all the recent suggestions of any importance that have been advanced by educational psychologists, practical teachers, and business men.

This book provides material for classes in Grades VII and VIII. It is recognized that the work of the first six years has been devoted largely to the development of skill in the fundamental operations, and that the point of view is now somewhat changed. The pupils are beginning to appreciate that they are part of a great social organization in which they must prepare to take their place, and the problems must therefore have a more significant bearing on life. The school fails in its mission if it does not at this time seek to present the principles of fair business dealings and of good citizenship. All this imposes a new responsibility upon the school besides the one of teaching children to compute, which, after all, must be the chief aim of arithmetic. It should be remembered that the schools are not training experts for special technical positions, but that they should give both boys and girls as complete a view as possible of modern business procedure and civic responsibility, such as the average citizen needs in order to take his place as an intelligent member of a community.

Following the usual practice in the leading courses of study, the work of Grade VII relates chiefly to those applications of percentage which the pupil will be most likely to need later, — profit and loss, commission, commercial discount, insurance, taxes, and simple problems of banking. It also includes further work in measurement and in drawing simple plans and maps.

In Grade VIII the pupil completes the customary work in measurements, including the mensuration of the common solids, takes up the practical uses of ratio and proportion, and considers further the applications of arithmetic in relation to thrift, savings, investments, and business organization. There is also included some work in the important field of intuitive geometry and in using simple algebraic expressions. The pupil is thus given an insight into the nature and uses of mathematics in the sciences and in industry. All this work is so arranged, however, that teachers may easily change the order of the topics to meet local requirements, and may, if desired, omit entirely the work in literal arithmetic (pages 351–386). There has been provided also a special chapter of problems which presents a complete survey of the types already studied.

Along with this work there will be found in the review and drill sections at the end of each chapter and in the drill charts a systematic plan of review of the fundamental operations. While the drill charts (pages 3–8 and 186–202) are placed at the beginning of each year's work for the benefit of those who need a rapid review of computation, it is not expected that any class will need all the material, but that these charts will be reviewed at intervals whenever drill of this sort is needed.

In the Teachers' Manual accompanying this book there will be found detailed suggestions that will assist the teacher in still further vitalizing the work of Grades VII and VIII.

DAVID EUGENE SMITH
HOWARD GRIFFITH BURDGE

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THE SMITH-BURDGE ARITHMETICS

ADVANCED BOOK

CHAPTER I

I. REVIEW OF THE FUNDAMENTALS

A SILENT READING LESSON

In our earlier work in arithmetic we studied all the common operations. We learned how to read and write numbers as large as we shall probably need to use, and we learned how to add, subtract, multiply, and divide *whole numbers*, *fractions*, and *decimals*. We also learned that decimals are only fractions written in another way, and that *per cents* are only decimals written in another way; that is,

$$75\% = 0.75 = \tfrac{3}{4}$$

and

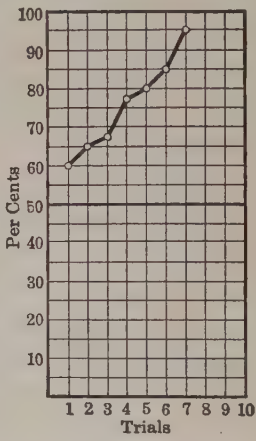
$$250\% = 2.50 = 2\tfrac{1}{2}.$$

We do not yet know how all this is used in everyday business; for example, we do not know about investing money, about insurance, about taxation, and about many other ways in which arithmetic is used. These we shall learn in the work of the next two years.

After our vacation we are a little "rusty" in handling numbers rapidly. We shall therefore take first a short review of the kind of work that we shall need when we begin to use arithmetic in daily life.

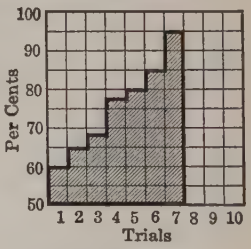
A SILENT READING LESSON (CONTINUED)

Our written drill work will mean much more to us if we keep a record of our standings. This may be done by means of a *curve-line graph* like the one here shown. To make such a graph, we first rule a page of our note-book with quarter-inch squares so that it looks like this picture.



Suppose that Tom gets 60% on the first trial of a test like the one on page 3. In recording his score, he places a dot at the point where the vertical line marked 1 meets the horizontal line marked 60 (per cent). When the same test is taken a few days later, he records his standing in the same way. Thus, if his marks are 60%, 65%, 68%, 77%, 80%, 85%, and 95% for seven trials, his graph will look like this one.

Instead of recording his progress by means of a graph like the one described above, Tom might draw a *bar graph* like the one shown in this picture. In this case the lengths of the seven bars show the different scores that he makes, and the "steps up" show that Tom continually improved his previous scores.



In all our drill work we should try to improve previous scores, so that our graphs will show continued progress.

As shown in the second figure, we need not draw the squares below 50% if we are sure that we shall have no mark below 50%. The story told by the graph will be just as clear, and it will not take so much space.

REVIEW DRILL CHART IN ADDITION

Do as many of the following additions as you can in 5 min., counting each correct answer as 4% in the two top rows and as 6% in the two bottom rows:

1. 247	5. 726	9. 135	13. 239	17. 712
46	835	246	75	384
356	104	790	145	509
918	53	879	687	678
705	689	68	529	89
329	745	568	380	808
2. 142	6. 476	10. 892	14. 359	18. 176
768	72	374	88	829
59	989	607	207	193
905	302	123	848	903
827	165	458	796	87
356	768	36	165	465
3. 6782	7. 8012	11. 5791	15. 9465	19. 4328
148	3466	2648	93	2697
3954	5098	3097	7830	5401
1066	273	8268	6972	2376
5092	7312	609	1586	749
7768	6076	4017	5070	8080
4. 6843	8. 2013	12. 7707	16. 2576	20. 3942
87	5746	8088	3985	8076
6955	475	9909	4072	5979
4432	9789	6660	680	8213
380	6054	820	1879	8019
1202	3071	1555	6048	3091

REVIEW DRILL CHART IN SUBTRACTION

Do as many of these subtractions as you can in 5 min., counting each correct answer as 2%:

1. 9072 − 46	11. 8129 − 649	21. 3842 − 987	31. 5017 − 3926	41. 7230 − 47
2. 5079 − 298	12. 7311 − 5876	22. 8124 − 5248	32. 9007 − 2647	42. 8272 − 5727
3. 6341 − 2870	13. 5700 − 3880	23. 2232 − 1987	33. 6249 − 3189	43. 5722 − 936
4. 8021 − 7231	14. 6802 − 4902	24. 7485 − 4626	34. 5125 − 3642	44. 8082 − 5081
5. 6643 − 4873	15. 6294 − 1919	25. 3340 − 1896	35. 5237 − 2969	45. 8333 − 4777
6. 6232 − 4252	16. 8721 − 7812	26. 4083 − 804	36. 5270 − 4139	46. 6445 − 4207
7. 9872 − 1999	17. 3704 − 1924	27. 7352 − 6199	37. 8012 − 7775	47. 9217 − 3747
8. 8000 − 175	18. 4127 − 1396	28. 8020 − 990	38. 7283 − 696	48. 8020 − 975
9. 6010 − 96	19. 3070 − 480	29. 6421 − 5009	39. 3121 − 1900	49. 4000 − 21
10. 8320 − 407	20. 6121 − 5980	30. 8008 − 6009	40. 5300 − 769	50. 3006 − 1598

REVIEW DRILL CHART IN MULTIPLICATION

Do as many of these multiplications as you can in 7 min., counting each correct answer as 2%:

1. 236 × 14	11. 3041 × 92	21. 8837 × 36	31. 4979 × 58	41. 7281 × 37
2. 364 × 34	12. 8756 × 98	22. 9867 × 46	32. 7854 × 39	42. 1342 × 71
3. 519 × 60	13. 7182 × 38	23. 4907 × 65	33. 5279 × 31	43. 8876 × 94
4. 2736 × 28	14. 7685 × 56	24. 8071 × 48	34. 7096 × 92	44. 3280 × 97
5. 5686 × 49	15. 8129 × 95	25. 3080 × 67	35. 5129 × 47	45. 6277 × 53
6. 6080 × 29	16. 6237 × 370	26. 8843 × 660	36. 7484 × 590	46. 3080 × 57
7. 3929 × 48	17. 4842 × 90	27. 6969 × 540	37. 8300 × 570	47. 6600 × 66
8. 3077 × 77	18. 6908 × 320	28. 8383 × 860	38. 8366 × 670	48. 7474 × 74
9. 7083 × 240	19. 7400 × 490	29. 4707 × 230	39. 7389 × 530	49. 8299 × 99
10. 4033 × 60	20. 8000 × 570	30. 6782 × 490	40. 2929 × 340	50. 5802 × 36

REVIEW DRILL CHART IN SHORT DIVISION

Do as many of these divisions as you can in 6 min., writing fractions in the results instead of remainders, and counting each correct answer as $1\frac{2}{3}\%$:

1. $4\overline{)148}$	16. $8\overline{)328}$	31. $2\overline{)118}$	46. $5\overline{)225}$
2. $6\overline{)396}$	17. $7\overline{)504}$	32. $9\overline{)333}$	47. $3\overline{)222}$
3. $2\overline{)431}$	18. $3\overline{)347}$	33. $4\overline{)125}$	48. $5\overline{)126}$
4. $6\overline{)789}$	19. $7\overline{)123}$	34. $8\overline{)345}$	49. $9\overline{)678}$
5. $7\overline{)247}$	20. $6\overline{)325}$	35. $5\overline{)428}$	50. $4\overline{)979}$
6. $3\overline{)776}$	21. $3\overline{)677}$	36. $3\overline{)708}$	51. $3\overline{)808}$
7. $2\overline{)908}$	22. $2\overline{)809}$	37. $9\overline{)809}$	52. $9\overline{)918}$
8. $7\overline{)644}$	23. $7\overline{)446}$	38. $8\overline{)426}$	53. $8\overline{)904}$
9. $9\overline{)777}$	24. $3\overline{)709}$	39. $6\overline{)888}$	54. $5\overline{)375}$
10. $5\overline{)357}$	25. $5\overline{)735}$	40. $4\overline{)848}$	55. $4\overline{)938}$
11. $3\overline{)777}$	26. $3\overline{)707}$	41. $4\overline{)550}$	56. $4\overline{)560}$
12. $8\overline{)736}$	27. $8\overline{)763}$	42. $8\overline{)376}$	57. $8\overline{)367}$
13. $7\overline{)245}$	28. $7\overline{)542}$	43. $7\overline{)524}$	58. $7\overline{)452}$
14. $8\overline{)999}$	29. $8\overline{)992}$	44. $8\overline{)299}$	59. $8\overline{)792}$
15. $5\overline{)900}$	30. $5\overline{)905}$	45. $4\overline{)905}$	60. $3\overline{)905}$

REVIEW DRILL CHART IN LONG DIVISION

Do as many of the following divisions as you can in 30 min., writing fractions in the results instead of remainders, and counting each correct answer as $1\frac{2}{3}\%$:

1. $25\overline{)6250}$	16. $60\overline{)7350}$	31. $75\overline{)1575}$	46. $77\overline{)7788}$
2. $20\overline{)6520}$	17. $42\overline{)4284}$	32. $75\overline{)1600}$	47. $25\overline{)1505}$
3. $25\overline{)6025}$	18. $42\overline{)8484}$	33. $81\overline{)9999}$	48. $35\overline{)1505}$
4. $20\overline{)6025}$	19. $42\overline{)8421}$	34. $81\overline{)8910}$	49. $22\overline{)8866}$
5. $30\overline{)6250}$	20. $45\overline{)9945}$	35. $81\overline{)8919}$	50. $23\overline{)4830}$
6. $21\overline{)6250}$	21. $16\overline{)2727}$	36. $52\overline{)6262}$	51. $23\overline{)4853}$
7. $21\overline{)7028}$	22. $16\overline{)3760}$	37. $96\overline{)1920}$	52. $69\overline{)2369}$
8. $22\overline{)4742}$	23. $16\overline{)5368}$	38. $96\overline{)2016}$	53. $69\overline{)6923}$
9. $22\overline{)3742}$	24. $18\overline{)7380}$	39. $96\overline{)2064}$	54. $55\overline{)1111}$
10. $31\overline{)6910}$	25. $18\overline{)7308}$	40. $99\overline{)8732}$	55. $55\overline{)2750}$
11. $32\overline{)6728}$	26. $18\overline{)7317}$	41. $46\overline{)5060}$	56. $82\overline{)4141}$
12. $32\overline{)6736}$	27. $29\overline{)5829}$	42. $46\overline{)5083}$	57. $28\overline{)5656}$
13. $50\overline{)7850}$	28. $38\overline{)2936}$	43. $83\overline{)9130}$	58. $28\overline{)5887}$
14. $50\overline{)3925}$	29. $48\overline{)7286}$	44. $83\overline{)9213}$	59. $90\overline{)8190}$
15. $60\overline{)7320}$	30. $48\overline{)7296}$	45. $77\overline{)8877}$	60. $36\overline{)1836}$

REVIEW DRILL CHART IN FRACTIONS

Do as many of these operations as you can in 10 min., counting each correct answer as $1\frac{1}{2}\%$, and adding 1% to your score if you do them all:

1. $\frac{3}{4} + \frac{1}{2}$.
2. $\frac{3}{4} - \frac{1}{2}$.
3. $\frac{3}{4} \times \frac{1}{2}$.
4. $\frac{3}{4} \div \frac{1}{2}$.
5. $\frac{1}{2} \div \frac{3}{4}$.
6. $1\frac{5}{8} + \frac{1}{2}$.
7. $1\frac{5}{8} - \frac{1}{2}$.
8. $1\frac{5}{8} \times \frac{1}{2}$.
9. $1\frac{5}{8} \div \frac{1}{2}$.
10. $\frac{1}{2} \div 1\frac{5}{8}$.
11. $2\frac{7}{8} + 1\frac{3}{4}$.
12. $2\frac{7}{8} - 1\frac{3}{4}$.
13. $2\frac{7}{8} \times 1\frac{3}{4}$.
14. $2\frac{7}{8} \div 1\frac{3}{4}$.
15. $1\frac{3}{4} \div 2\frac{7}{8}$.
16. $5\frac{1}{8} + 2\frac{3}{4}$.
17. $5\frac{1}{8} - 2\frac{3}{4}$.
18. $5\frac{1}{8} \times 2\frac{3}{4}$.
19. $5\frac{1}{8} \div 2\frac{3}{4}$.
20. $2\frac{3}{4} \div 5\frac{1}{8}$.
21. $15 \times 2\frac{1}{2}$.
22. $15 \div 2\frac{1}{2}$.
23. $\frac{1}{2} + \frac{1}{8}$.
24. $\frac{1}{2} - \frac{1}{8}$.
25. $\frac{1}{2} \times \frac{1}{8}$.
26. $\frac{1}{2} \div \frac{1}{8}$.
27. $\frac{1}{8} \div \frac{1}{2}$.
28. $2\frac{3}{8} + \frac{1}{2}$.
29. $2\frac{3}{8} - \frac{1}{2}$.
30. $2\frac{3}{8} \times \frac{1}{2}$.
31. $2\frac{3}{8} \div \frac{1}{2}$.
32. $\frac{1}{2} \div 2\frac{3}{8}$.
33. $3\frac{5}{8} + 1\frac{3}{4}$.
34. $3\frac{5}{8} - 1\frac{3}{4}$.
35. $3\frac{5}{8} \times 1\frac{3}{4}$.
36. $3\frac{5}{8} \div 1\frac{3}{4}$.
37. $1\frac{3}{4} \div 3\frac{5}{8}$.
38. $6\frac{3}{8} + 3\frac{3}{4}$.
39. $6\frac{3}{8} - 3\frac{3}{4}$.
40. $6\frac{3}{8} \times 3\frac{3}{4}$.
41. $6\frac{3}{8} \div 3\frac{3}{4}$.
42. $3\frac{3}{4} \div 6\frac{3}{8}$.
43. $7\frac{1}{2} \times 3\frac{3}{4}$.
44. $7\frac{1}{2} \div 3\frac{3}{4}$.
45. $\frac{1}{2} + \frac{1}{3}$.
46. $\frac{1}{2} - \frac{1}{3}$.
47. $\frac{1}{2} \times \frac{1}{3}$.
48. $\frac{1}{2} \div \frac{1}{3}$.
49. $\frac{1}{3} \div \frac{1}{2}$.
50. $3\frac{2}{3} + \frac{1}{2}$.
51. $3\frac{2}{3} - \frac{1}{2}$.
52. $3\frac{2}{3} \times \frac{1}{2}$.
53. $3\frac{2}{3} \div \frac{1}{2}$.
54. $\frac{1}{2} \div 3\frac{2}{3}$.
55. $5\frac{5}{6} + 1\frac{1}{2}$.
56. $5\frac{5}{6} - 1\frac{1}{2}$.
57. $5\frac{5}{6} \times 1\frac{1}{2}$.
58. $5\frac{5}{6} \div 1\frac{1}{2}$.
59. $1\frac{1}{2} \div 5\frac{5}{6}$.
60. $6\frac{2}{3} + 2\frac{5}{6}$.
61. $6\frac{2}{3} - 2\frac{5}{6}$.
62. $6\frac{2}{3} \times 2\frac{5}{6}$.
63. $6\frac{2}{3} \div 2\frac{5}{6}$.
64. $2\frac{5}{6} \div 6\frac{2}{3}$.
65. $17\frac{5}{8} \times 5\frac{7}{8}$.
66. $17\frac{5}{8} \div 5\frac{7}{8}$.

DIFFERENT EXPRESSIONS FOR FRACTIONS

Copy the following tables and fill the blank spaces:

	Fraction	Decimal	Per Cent
1.	$\frac{1}{2}$	0.50	50%
2.	$\frac{1}{4}$		
3.		0.75	
4.			10%
5.	$\frac{1}{8}$		
6.		0.375	
7.			$62\frac{1}{2}$%
8.	$\frac{7}{8}$		
9.		$0.33\frac{1}{3}$	
10.			$66\frac{2}{3}$%
11.	$\frac{1}{6}$		
12.			$83\frac{1}{3}$%
13.	$\frac{2}{5}$		
14.		0.60	
15.			80%
16.	$\frac{1}{20}$		
17.			6%
18.		0.08	
19.			12%
20.		0.15	
21.	$\frac{1}{12}$		
22.			$6\frac{1}{4}$%

	Fraction	Decimal	Per Cent
23.	$1\frac{1}{4}$	1.25	125%
24.		1.00	
25.			300%
26.	$6\frac{1}{4}$		
27.	$6\frac{3}{4}$		
28.		2.125	
29.			225%
30.		$2.33\frac{1}{3}$	
31.			$216\frac{2}{3}$%
32.	$3\frac{2}{3}$		
33.		$4.66\frac{2}{3}$	
34.			200%
35.		3.375	
36.	$2\frac{5}{8}$		
37.		6.875	
38.			$283\frac{1}{3}$%
39.	$1\frac{1}{6}$		
40.		$2.08\frac{1}{3}$	
41.	5		
42.		7.00	
43.	$6\frac{5}{6}$		
44.			750%

MISCELLANEOUS PROBLEMS

1. Mr. Wheeler works 6 da. each week and earned $31.50 per week. Last month, on account of illness, he lost 9 da. How much did he lose in wages?

2. A grocer charges 8¢ a pound for sugar and 50¢ a doz. for eggs. At a special sale one of his customers bought 6 lb. of sugar for 45¢ and $2\frac{1}{2}$ doz. eggs for $1. How much did the customer save in buying at the special sale?

3. At the rate of three for $1, find the cost of 1 doz. collars; of $\frac{1}{2}$ doz. collars; of 15 collars.

4. A family requires $3\frac{1}{2}$ qt. of milk and 1 pt. of cream daily. If they pay 12¢ a quart for the milk and 18¢ a half pint for the cream, how much do they pay for milk and cream during the month of July?

5. A family of four go to the "movies" on an average of two nights a week. If the tickets are 20¢ each, how much do they cost in a year? If the house rent of this family is $38 a month, for how many months would the money spent for "movies" pay the rent?

6. Mr. Howard, who lives 147 mi. from Toledo, decides to make a trip there and back in his automobile. The price of gasoline is 17¢ a gallon, and his car will make 14 mi. on a gallon. On the trip he pays also 54¢ for oil. Find the cost of gasoline and oil for the trip to Toledo and back.

7. A farmer who planted 40 A. (acres) of corn found that, by carefully selecting the seed corn, he could have raised 45 bu. instead of 30 bu. per acre. If he had done this and had sold the extra corn at 82¢ a bushel, he could have bought with the money some more stock for his farm. If the stock cost $60 a head, how many head could he have bought, and how much money would he have had left over?

II. PERCENTAGE

A SILENT READING LESSON

We have already studied *percentage*, which is the part of arithmetic that uses per cents. We found that per cents are simply another way of writing decimals, because per cent is only another name for hundredths. We learned that all the expressions 6%, $\frac{6}{100}$, 0.06, and $\frac{3}{50}$ have the same value, and that 6% is only another way of writing 0.06. Similarly,

100% means $\frac{100}{100}$, or 1;
125% means $\frac{125}{100}$, or 1.25;
400% means $\frac{400}{100}$, or 4.

On page 9 we reviewed the relation of fractions to decimals and the relation of each to per cents. That is, relations like the following are already familiar to us:

$10\% = 0.10 = \frac{1}{10}$;
$25\% = 0.25 = \frac{1}{4}$;
$50\% = 0.50 = \frac{1}{2}$;
$62\frac{1}{2}\% = 0.62\frac{1}{2} = \frac{5}{8}$.

We also know that other commonly used per cents may be expressed as fractions.

If we wish to find one of the above per cents of a number, it is much easier to use the fractional form. For example, if we wish to find 50% of \$10, we simply take half of \$10. Similarly, if we wish to find $37\frac{1}{2}\%$ of a given number, we take $\frac{3}{8}$ of it; if we wish to find $66\frac{2}{3}\%$ of a number, we take $\frac{2}{3}$ of it; if we wish to take 125% of a number, we may multiply it by $1\frac{1}{4}$, or we may add to the number $\frac{1}{4}$ of itself; and so on.

Sometimes, however, it is best to use decimals. For example, the fraction that is equal to 18% is $\frac{9}{50}$, and it is evidently easier to multiply by 0.18 than by $\frac{9}{50}$.

PER CENTS AS FRACTIONS

All work oral

1. By using these columns show that $20\% = \frac{1}{5}$, $40\% = \frac{2}{5}$, $60\% = \frac{3}{5}$, $80\% = \frac{4}{5}$, and $100\% = 1$.

				20%
			20%	20%
		20%	20%	20%
	20%	20%	20%	20%
20%	20%	20%	20%	20%
20%	40%	60%	80%	100%

2. Show that 20% is $\frac{1}{2}$ of 40%, $\frac{1}{4}$ of 80%, and $\frac{1}{5}$ of 100%.

3. Show that 40% is $\frac{2}{3}$ of 60%, $\frac{1}{2}$ of 80%, and $\frac{2}{5}$ of 100%.

4. Show that 60% is 20% + 40%, $1\frac{1}{2}$ times 40%, 3 times 20%, $\frac{3}{4}$ of 80%, and $\frac{3}{5}$ of 100%.

5. A foot is what fraction of a yard? A foot is what per cent of a yard?

6. A pint is what fraction of a gallon? A pint is what per cent of a gallon?

7. A nickel is what fraction of a dollar? A nickel is what per cent of a dollar?

8. If you wish to find $33\frac{1}{3}\%$ of a number, what fractional part of the number may you find?

9. If a man spends one fifth of his income for rent, what per cent does he spend for this purpose?

10. What per cent of the distance around a square is one side? two sides? three sides?

11. If there are 30 pupils in a class, and if 50% are boys, what fraction are boys? How many boys are there? How many girls are there?

12. How many dollars is $\frac{1}{5}$ of \$40? 20% of \$40? $\frac{1}{2}$ of \$40? 50% of \$40? $\frac{1}{4}$ of \$40? 25% of \$40?

USING PER CENTS

Numbers 1 to 16, oral

1. A man said that he gained 100% on some property which he sold. What does this mean?

2. A dealer said that he would sell some goods at 40% less than the marked price. Express this as a fraction.

3. A boy beginning to play tennis won only 25% of the games played the first week. Express this as a fraction.

Read as fractions, whole numbers, or mixed numbers:

4. 20%.	**7.** 60%.	**10.** 25%.	**13.** 200%.
5. 30%.	**8.** 70%.	**11.** 125%.	**14.** 300%.
6. 50%.	**9.** 90%.	**12.** 225%.	**15.** 325%.

16. One dealer sells an article for $33\frac{1}{3}\%$ more than it costs him, and another sells the same thing for $\frac{1}{3}$ more than it costs him. Is there any difference?

17. How much is $\frac{1}{2}\frac{1}{5}$ of 50? 4% of 50? 4% of 60?

18. How much is $\frac{1}{4}$ of 440? 25% of 440? 25% of 3724?

19. How much is $\frac{1}{2}$ of 3628? 50% of 3628? 50% of 9376?

20. How much is $\frac{3}{4}$ of 6480? 75% of 6480? 75% of 3872?

21. How much is $1\frac{1}{4} \times 964$? 125% of 964? 125% of 896?

22. Instead of multiplying 72 by $12\frac{1}{2}\%$, what fractional part of 72 may we take? Do this, and thus find $12\frac{1}{2}\%$ of 72.

23. Instead of multiplying 640 by $37\frac{1}{2}\%$, what fractional part of 640 may we take? Do this, and thus find $37\frac{1}{2}\%$ of 640.

24. Find $87\frac{1}{2}\%$ of 160 and of 3200 in the shortest way.

25. Find $33\frac{1}{3}\%$ of 96 and of 9672 in the shortest way.

26. Find $3\frac{1}{4} \times 488$; 325% of 488; 325% of 872.

Per Cents as Decimals. If a baseball team has lost 32% of its games, how many hundredths of its games has it lost? How do you write 32% as hundredths?

Since 32%, 0.32, and $\frac{32}{100}$ all mean the same thing, we see that we can easily write per cents as decimals.

To express a per cent as a decimal, omit the per-cent sign and insert or move the decimal point two places to the left.

That is, when we omit the per-cent sign, we must indicate the hundredths in some other way, which we do by the position of the decimal point.

Thus, $1\frac{1}{2}\% = 0.01\frac{1}{2}$, or 0.015; $125\% = 1.25$; $0.6\% = 0.006$.

PER CENTS AS DECIMALS

Numbers 1 to 9, oral

1. If 2% of the people in your town are in the seventh grade of school, how many out of every 100 people are in the seventh grade? How many hundredths of the people in the town are in this grade?

2. If 2% of the pupils in your school are absent today, how many pupils are absent out of every 100? out of every 50?

3. How much is $\frac{1}{100}$ of \$200? 1% of \$200? 1% of \$1700?

4. How much is 0.06 of \$100? 6% of \$100? 6% of \$300?

Read the following per cents as decimals:

5. 35%. **6.** 19%. **7.** 75%. **8.** 120%. **9.** 325%.

10. Write $\frac{1}{10}$ as a decimal; as per cent.

11. Write $\frac{1}{10}\%$ as a decimal; as a fraction.

Write as decimals or as whole numbers:

12. 800%. **14.** 150%. **16.** $37\frac{1}{2}\%$. **18.** 1.67%.

13. 0.5%. **15.** 0.005%. **17.** $66\frac{2}{3}\%$. **19.** 3.75%.

Decimals as Per Cents. If a basketball team has won 0.6 of the games it has played, how many hundredths of the games has it won? This is what per cent?

That is, to express a decimal as per cent we ask how many hundredths it represents, as in the following examples:

1. Express 0.375 as per cent.

Here $0.375 = 37.5$ hundredths, or $37\frac{1}{2}$ hundredths. Therefore 0.375 is the same as 37.5%, or $37\frac{1}{2}\%$.

2. Express 0.00125 as per cent.

Here $0.00125 = \frac{125}{100{,}000} = \frac{0.125}{100} = 0.125\%$. Also, since $0.125 = \frac{125}{1000} = \frac{1}{8}$, we have $0.00125 = \frac{1}{8}\%$.

3. Express 6.55 as per cent.

Here $6.55 = \frac{655}{100}$, and therefore $6.55 = 655\%$.

To express a decimal as per cent, move the decimal point two places to the right and place the per-cent sign after the number.

Since 655. and 655 are the same, in cases like Ex. 3, above, we simply omit the decimal point.

DECIMALS AS PER CENTS

Numbers 1 to 5, oral

1. A pint, or 0.5 qt., is what per cent of a quart?

2. A peck, or 0.25 bu., is what per cent of a bushel?

3. A quart, or 0.125 pk., is what per cent of a peck?

4. What per cent of a dollar is 49¢? 49 dimes? $1?

5. Express 1 qt. as per cent of a pint; of 2 bu.; of 4 bu.

Write the following decimals as per cents:

6. 0.9.	**8.** 0.3.	**10.** 0.59.	**12.** 0.325.	**14.** $6.666\frac{2}{3}$.
7. 6.2.	**9.** 3.0.	**11.** 0.72.	**13.** 0.650.	**15.** 0.00375.

Fractions as Per Cents. If we wish to know the amount of a reduction of 20% on a bill of goods, which is easier, to take $\frac{1}{5}$ of the amount of the bill or to take 20% of it? Is it easier to divide by 5 or to multiply by 2?

Sometimes it is easier to use per cents than to use fractions. Since per cent means hundredths, to express fractions as per cents we have only to reduce them to hundredths.

For example, $\frac{4}{5} = \frac{80}{100}$, which is 0.80, or 80%. Similarly, $\frac{2}{25} = \frac{8}{100} = 8\%$, and $2\frac{1}{2} = \frac{5}{2} = \frac{250}{100} = 250\%$.

To express a fraction as per cent, first reduce it to hundredths, omit the denominator, and then write the numerator followed by the per-cent sign.

FRACTIONS AS PER CENTS

Numbers 1 to 9, oral

1. 25¢ is what fraction of 50¢? what per cent of 50¢?
2. $33\frac{1}{3}$¢ is what fraction of \$1? what per cent of \$1?

Express the following fractions as per cents:

3. $\frac{1}{4}$. 4. $\frac{1}{5}$. 5. $\frac{1}{10}$. 6. $\frac{3}{4}$. 7. $\frac{2}{5}$. 8. $\frac{7}{10}$.

9. $\frac{5}{8}$ of some games played is what per cent of the games?
10. An inch is what per cent of a foot? of a yard?
11. A pint is what per cent of a peck? of a bushel?
12. What per cent of a gallon is 2 qt.? $2\frac{1}{2}$ qt.? 4 qt.?
13. What per cent of a pound is 1 oz.? $3\frac{4}{5}$ oz.? 16 oz.?
14. What per cent of a rod is 1 ft.? 1 yd.? $5\frac{1}{2}$ yd.? 11 yd.?
15. If you have increased $\frac{1}{9}$ in weight during the past year, what per cent have you increased in weight?
16. If you have increased $12\frac{1}{2}\%$ in weight during the past year, what fractional part have you increased?

Reading Per Cents. We already know that 0.75 may be read "seventy-five hundredths" or "seventy-five per cent." In business it is more common to write 75% than 0.75.

READING PER CENTS

Numbers 1 to 3, oral

1. If your school has twice as many pupils as it had 10 yr. ago, it has now what per cent as many as then?

2. Read the following numbers and tell which of them are equal: 2; $\frac{200}{100}$; 200%; $\frac{100}{50}$; 7; 700%.

3. Since $3.75 = \frac{375}{100}$, 3.75 is what per cent?

Write the following numbers in words:

4. 475%. 6. 3.9%. 8. 0.39%. 10. 5.25. 12. 0.6%.
5. 4.75%. 7. 39%. 9. 390%. 11. 7.35. 13. 0.06%.

Express the following per cents as decimals:

14. 225%. 15. $\frac{1}{4}$%. 16. 300%. 17. $\frac{3}{8}$%. 18. 0.4%.

Express as fractions or mixed numbers:

19. 78%. 20. $6\frac{1}{4}$%. 21. 325%. 22. 475%. 23. $33.3\frac{1}{3}$%.

Express the following numbers as per cents:

24. $\frac{3}{5}$. 25. $\frac{1}{3}$. 26. $\frac{3}{8}$. 27. $\frac{4}{5}$. 28. $2\frac{1}{2}$. 29. $3\frac{3}{4}$.

Express as decimals or whole numbers:

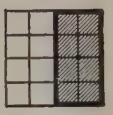
30. 28%. 33. $66\frac{2}{3}$%. 36. 250%. 39. 5000%.
31. 34%. 34. 600%. 37. 275%. 40. 0.006%.
32. 0.6%. 35. 355%. 38. 325%. 41. 4550%.

42. Arrange in order of size, beginning with the smallest: 64%, $\frac{18}{25}$, $\frac{3}{5}$, and $0.6\frac{3}{5}$.

PER CENTS AND FRACTIONS

1. Read 0.50, using the words "per cent"; using the word "hundredths." How many halves are there in 1? Then how many times is 50% contained in 1? in 100%?

2. If this square is called 1, how many sixteenths of the square are shaded? What part of the square is shaded? How many hundredths of it are shaded? What per cent is shaded?



3. Read 0.25, using the words "per cent"; using the word "hundredths." How many fourths are there in 1? Then how many times is 25% contained in 1? in 100%?

4. If this square is called 1, how much is the shaded part? If the square is called 100, how much is the shaded part? What per cent of the square is shaded?

5. How many eighths are there in 1? Then how many times is $12\frac{1}{2}\%$ contained in 1?

6. Draw a square on the board and shade 10% of it. How many tenths of the square is 10% of it?

7. Draw another square on the board and shade 75% of it. How many fourths of the square is 75% of it?

8. How many times is $16\frac{2}{3}\%$ contained in 1? in 2?

9. How many times is $6\frac{1}{4}\%$ contained in 1? in 3?

10. How many times is 20% contained in 1? in 5?

11. How many times is 50% contained in 7? in 12?

12. How many times is 25% contained in 9? in 20?

13. Express $\frac{15}{100}$, 15%, and 0.15 as twentieths.

14. Express $\frac{3}{4}\%$ as a decimal; as a fraction.

15. Express $666\frac{2}{3}\%$ as a decimal; as an improper fraction.

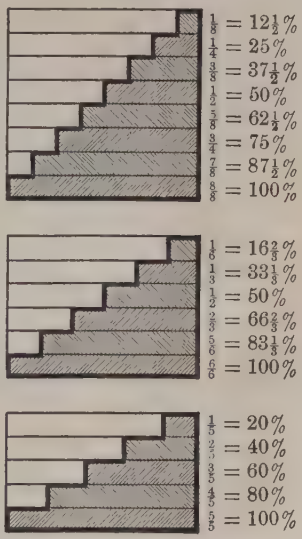
A SILENT READING LESSON

Which is easier, to find $\frac{3}{8}$ of 72 games played or to find $37\frac{1}{2}\%$ of 72 games played?

Sometimes it is easier to use the fraction and sometimes it is easier to use the per cent. It is easier in this case, of course, to use $\frac{3}{8}$ than to use $37\frac{1}{2}\%$.

We have already learned the fractions that are equivalent to certain important per cents. The most important ones are

$50\% = \frac{1}{2}$	$33\frac{1}{3}\% = \frac{1}{3}$
$25\% = \frac{1}{4}$	$66\frac{2}{3}\% = \frac{2}{3}$
$75\% = \frac{3}{4}$	$16\frac{2}{3}\% = \frac{1}{6}$
$12\frac{1}{2}\% = \frac{1}{8}$	$83\frac{1}{3}\% = \frac{5}{6}$
$37\frac{1}{2}\% = \frac{3}{8}$	$20\% = \frac{1}{5}$
$62\frac{1}{2}\% = \frac{5}{8}$	$40\% = \frac{2}{5}$
$87\frac{1}{2}\% = \frac{7}{8}$	$60\% = \frac{3}{5}$
$6\frac{1}{4}\% = \frac{1}{16}$	$80\% = \frac{4}{5}$



We should become so familiar with these equivalents that we can change at once from one to the other. We should never multiply by 25%, $12\frac{1}{2}\%$, $33\frac{1}{3}\%$, or any other per cent, when the fraction is easier for the purpose. That is, in finding $62\frac{1}{2}\%$ of \$1640 and $83\frac{1}{3}\%$ of \$720, we have

$$62\tfrac{1}{2}\% \text{ of } \$1640 = \tfrac{5}{8} \text{ of } \$1640 = 5 \times \$205 = \$1025,$$

and $$83\tfrac{1}{3}\% \text{ of } \$720 = \tfrac{5}{6} \text{ of } \$720 = 5 \times \$120 = \$600.$$

In finding 40% of \$725, however, there is nothing to be gained by using $\frac{2}{5}$ instead of 40%.

IMPORTANT PER CENTS

Numbers 1 to 9, oral

1. Recite the per-cent equivalents of the eighths.

2. Using the equivalent fractions, find the result of dividing $87\frac{1}{2}\%$ by $12\frac{1}{2}\%$.

3. Find the result of dividing 75% by $62\frac{1}{2}\%$.

4. Find the result of dividing $62\frac{1}{2}\%$ by 50%.

5. Recite the per-cent equivalents of the sixths.

6. Divide $83\frac{1}{3}\%$ by $33\frac{1}{3}\%$; $33\frac{1}{3}\%$ by $66\frac{2}{3}\%$.

7. Divide 50% by $16\frac{2}{3}\%$; 50% by 25%; 25% by 50%.

8. Recite the per-cent equivalents of the fifths.

9. Divide 1 by 80%; 80% by 1; 80% by 100%.

Draw figures as on page 19 and write the per-cent equivalents of

10. The sevenths.
11. The ninths.
12. The twelfths.
13. The sixteenths.
14. The tenths.
15. The twentieths.

Perform the following operations:

16. $50\% \div 40\%$.
17. $80\% \div 50\%$.
18. $25\% \div 60\%$.
19. $12\frac{1}{2}\%$ of 80%.
20. $83\frac{1}{3}\%$ of 60%.
21. 60% of $33\frac{1}{3}\%$.
22. $50\% \div 12\frac{1}{2}\%$.
23. $33\frac{1}{3}\%$ of 75%.
24. $66\frac{2}{3}\% \div 40\%$.

Find the value in each of the following cases:

25. 50% of \$1273.80.
26. 25% of \$3724.64.
27. $12\frac{1}{2}\%$ of \$5128.72.
28. $6\frac{1}{4}\%$ of \$23,376.16.
29. $37\frac{1}{2}\%$ of \$17,768.32.
30. $33\frac{1}{3}\%$ of \$77,163.69.
31. $66\frac{2}{3}\%$ of \$80,070.12.
32. $83\frac{1}{3}\%$ of \$71,108.10.
33. $112\frac{1}{2}\%$ of \$87,264.48.
34. $137\frac{1}{2}\%$ of \$96,328.32.

A SILENT READING LESSON

There are three important kinds of problems in percentage. The most important of these, and the one that we shall consider first, is the problem of finding some per cent of a number.

For example, suppose that a man who has a contract to remove 9600 cu. yd. of earth in building a canal reports that $23\frac{1}{4}\%$ of the earth has been removed.

In finding how many cubic yards this represents, we see that $23\frac{1}{4}\% = 0.23\frac{1}{4}$, and so we multiply 9600 by $0.23\frac{1}{4}$. The result, as shown at the right, is 2232; that is, 2232 cu. yd. of earth have been removed.

9600
$0.23\frac{1}{4}$
2400
28800
19200
2232.00

To find a per cent of a number, multiply the number by the given per cent expressed as a decimal.

In the above case a still easier way would be simply to multiply 96 by $23\frac{1}{4}$. We should always look for such short cuts, which save time and labor.

Just as we sometimes use special names like "multiplier" and "product" in multiplication, so in percentage we sometimes use special names for the different numbers. The case at the right, in which we find 40% of \$300, shows how the names "base," "rate," and "percentage" are used.

Base,	\$300
Rate,	0.40
Percentage,	\$120.00

To find the percentage, multiply the base by the rate.

Sometimes the 40 in the above case is called the *rate per cent*, but generally we speak simply of the rate. All these terms, however, are seldom used outside the school, and even in school we ordinarily use the word "percentage" to mean that part of arithmetic which treats of per cents.

FINDING A PER CENT OF A NUMBER

Numbers 1 to 7, oral

1. In a school of 400 pupils 9% were in the seventh grade. How many pupils were in that grade?

2. In a school of 500 pupils 11% were in the third grade. How many pupils were in that grade?

3. In a school of 200 pupils 5% were tardy or absent last month. How many pupils were not tardy or absent?

4. If there should be an increase of $3\frac{1}{3}$% next year in a school of 300 pupils, how many more pupils would there be then? How many would there be in all?

5. In a school of 250 pupils there were 10% fewer pupils four years ago. How many pupils were there then?

6. If $16\frac{2}{3}$% of the pupils in a school of 240 pupils are to graduate this year, how many will graduate?

7. A man borrowed \$200 and paid 6% of that sum for the use of it for a year. How much was this?

8. A taxicab company that uses an average of 1750 gal. of gasoline per week plans to equip its cabs with devices which are guaranteed to save 15% on gasoline. How many gallons must be saved to make this guaranty good?

9. In Ex. 8, how many gallons of gasoline will the taxicab company use per week after the cabs are equipped?

10. On a job requiring great haste, a contractor charged 12% of the cost for his services. If the cost of the job was \$168,000, how much did the contractor receive?

11. An architect who furnished the plans and supervised the erection of a new school building costing \$550,000 was paid 5% of the cost for his services. How much money did the architect receive on this job?

12. An overcoat that cost a dealer \$40 was sold at 40% above cost. Find the selling price.

13. Mollie had a test in spelling. There were 50 words in the test, and she missed 6% of them. How many words did she miss? How many did she spell correctly?

14. Mr. Russell has just received \$125 for some work which he did on a house. He puts 20% of this money in the bank and buys a Treasury Savings Certificate for \$84.60. How much money has he left?

15. Mr. Baker saved coal during the winter by using a little less each day and made up for it by taking better care of his furnace. If he used $14\frac{1}{2}$ T. last year and saved 9% this winter, how many tons did he save?

16. A manufacturer sells shoes so as to realize a gain of 11% on the manufacturing cost. If the manufacturing cost is \$3.70 a pair, how much is his gain on 1000 pairs?

17. Of 276 locomotives manufactured in a large plant, 75% were sold for \$74,125 each and the rest for \$72,825 each. How much was received for all?

18. A man can have a wall built for \$1.20 a foot, but he can have a better job done for 15% more. What will be the cost of building 54 ft. of the better kind of wall?

19. A farmer owned 1424 sheep and sold $12\frac{1}{2}$% of them. How many sheep did he sell? How many were left?

20. A man was paid $\frac{1}{2}$% of the cost for checking the plans of a new building costing \$475,000. How much did he receive?

21. In Ex. 20, how much more would the man have received if he had been paid $\frac{5}{8}$%?

22. A suit of clothes was marked \$40, but after the holidays the price was reduced 15%. How much was it reduced, and what was the price after the reduction?

MISCELLANEOUS PROBLEMS

Numbers 1 to 6, oral

1. If 20% of the pupils in a school of 425 pupils are in the seventh and eighth grades, how many are in those grades? How many are not in those grades?

2. If 25% of the pupils of a school of 420 pupils are in the first and second grades, how many are in those grades?

3. If 54% of the pupils in a school of 300 pupils are girls, how many girls are there in the school?

4. How much must be added to 250 ft. to increase it 50%? to increase it 100%?

5. How much must be added to 240 lb. to make it 125% of the present weight?

6. Water is standing in a tank to a depth of 6 in. If the depth is increased $16\frac{2}{3}\%$, how deep is the water then?

7. How much butter fat is there in 425 lb. of milk in which the creamery test shows that 3.9% of the weight of the milk is butter fat?

8. A farmer takes 420 lb. of milk to a creamery and is paid 26¢ a pound for the butter fat. The tests show that the butter fat is 3.8% of the weight of the milk. Find the weight of the butter fat and the amount received by the farmer.

9. One girl who works in a store is paid $25 a week, and another is paid $24 a week. The first girl saves 6.2% of her salary, and the second girl saves 6.5%. Which girl saves more in a year, and how much more does she save?

10. A farmer who owns 90 hogs, which average 225 lb. each in weight, decides to sell 60% of them. If he sells the hogs at a price of $12 per hundredweight, how much money does he receive for them?

Another Problem in Percentage. The problem in percentage which is second in importance to finding a per cent is that of finding what per cent one number is of another.

We already know how to find what fractional part one number is of another. For example, we know that 1 is $\frac{1}{2}$ of 2; 2 is $\frac{2}{4}$ of 4, or $\frac{1}{2}$ of 4; 7 is $\frac{7}{10}$ of 10; and so on. That is,

To find what fractional part one number is of another, divide the first by the second.

We should see that a problem which asks what per cent one number is of another is only asking in another way what part one number is of another number. The result, however, is to be expressed with the per-cent sign.

FRACTIONAL PARTS

All work oral

1. A team lost 7 games out of 28 games played. The games lost were what part of the games played?

2. Mary's baby sister weighs 14 lb., and Mary herself weighs 70 lb. The baby's weight is what part of Mary's weight? Give the result as a fraction and also as a decimal.

In the following cases state what part the first number is of the second, giving the answer as a fraction and also as a decimal:

3. 6, 36.	10. 20, 200.	17. $\frac{1}{4}$, 5.	24. 1, 20.
4. 12, 36.	11. 3, 6.	18. $\frac{1}{2}$, 4.	25. 5, 20.
5. 18, 72.	12. $\frac{3}{10}$, 6.	19. 0.5, 4.	26. 10, 20.
6. 10, 60.	13. 0.3, 0.6.	20. 3, 9.	27. 20, 20.
7. 8, 80.	14. 0.30, 0.60.	21. 2, 8.	28. 5, 25.
8. 10, 80.	15. 0.60, 0.30.	22. 0.2, 8.	29. 25, 50.
9. 10, 20.	16. 300, 600.	23. 2, 80.	30. 50, 25.

Finding What Per Cent One Number is of Another. If Fred made 6 base hits last June and was at bat 20 times, the base hits were what per cent of the number of times at bat?

If we think of this problem as asking what part the base hits were of the times at bat, we see that the result is $\frac{6}{20}$. If we follow the general rule of dividing one number by the other and expressing the result as a decimal, we have

$$6 \div 20 = 0.30.$$

Since 0.30 is the same as 30%, we thus see that the base hits were 30% of the times at bat; that is, we find the result as a decimal and then write it with the per-cent sign.

RELATIONS OF NUMBERS AS PER CENTS

All work oral

1. If Jack made 12 base hits and was at bat 24 times, what per cent of base hits did he make?

2. If a baseball team wins 16 games out of 20, what per cent of games does it win?

3. If 3 of the 30 pupils in a class are absent today, what per cent are absent?

4. If you answer correctly 15 questions out of 20 on an arithmetic test, what per cent do you answer correctly?

Find what per cent the first number is of the second:

5. 1, 2.	11. 20, 40.	17. 22, 44.	23. 5, 25.
6. 1, 3.	12. 30, 40.	18. 44, 44.	24. 2.5, 25.
7. 1, 4.	13. 45, 90.	19. 40, 60.	25. $12\frac{1}{2}$, 25.
8. 2, 4.	14. 15, 60.	20. 72, 72.	26. 25, 25.
9. 3, 4.	15. 16, 32.	21. 27, 36.	27. 50, 75.
10. 4, 4.	16. 32, 16.	22. 27, 270.	28. 50, 250.

Using Per Cents. A man sold at a profit of \$120 an automobile for which he had paid \$600. What per cent of the cost did he gain?

In finding what part \$120 is of \$600, we have

$$\$120 \div \$600 = 0.20 = 20\%.$$

To find what per cent one number is of another, divide the first by the second, finding the result as a decimal and then expressing it with the per-cent sign.

USING PER CENTS

1. \$130 is what per cent of \$2600? of \$3900?
2. \$11.25 is what per cent of \$375? of \$56.25?
3. \$10.12 is what per cent of \$253? of \$202.40?
4. \$29.10 is what per cent of \$465.60? of \$174.60?
5. A quart is what per cent of a gallon? of 25 gal.?
6. A rod is what per cent of a foot? of a yard?
7. $\frac{2}{3}$ is what per cent of $\frac{1}{4}$? $\frac{3}{8}$ is what per cent of $\frac{1}{4}$?
8. 25% is what per cent of 75%? of 125%? of 250%?
9. $33\frac{1}{3}\%$ is what per cent of $66\frac{2}{3}\%$? of 1? of $133\frac{1}{3}\%$?
10. $16\frac{2}{3}\%$ is what per cent of $33\frac{1}{3}\%$? of $66\frac{2}{3}\%$? of 1?

Find what per cent the first number is of the second:

11. 387, 2709. 12. 52.8, 31.68. 13. 56.7, 453.6.

14. If a man rents for \$360 a year a house that costs \$4000, the rent is what per cent of the cost?

15. If $12\frac{1}{2}$ T. of iron are obtained from 235 T. of ore, what per cent of the ore is iron?

16. Last year the population of a certain city increased from 26,275 to 28,530. What was the per cent of increase?

17. If the seventh grade devotes 45 min. a day to arithmetic and the sixth grade 42 min., what per cent more time does the seventh grade devote to arithmetic?

18. If a class had 16 examples to solve on Monday and 20 on Tuesday, what was the per cent of increase?

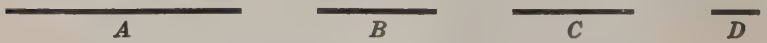
19. If you have read 69 pages in a book of 300 pages, what per cent of the pages have you read?

20. If the last chapter of a book is Chapter XXXII, and if you have read to the end of Chapter XXIV, what per cent of the chapters have you read?

21. A man's income is $2300 a year, and he spends $1886. What per cent of his income does he save?

22. In a town, 576 out of the 1200 pupils in school are boys. What per cent are boys?

23. Measure these lines, and then state what per cent B is of A; B is of C; D is of C; D is of A; A is of B; A is of D.



24. The purity of gold is measured in carats, or twenty-fourths, 18 carats meaning $\frac{18}{24}$ pure gold. What is the per cent of pure gold in an 18-carat ring? in a 14-carat ring?

25. What is the per cent of pure gold in a watchcase that is 16 carats fine? in a chain that is 12 carats fine?

26. If 2 members of a class of 32 have been neither tardy nor absent during the year, and 4 others have not been tardy, what per cent have been neither tardy nor absent? What per cent have not been tardy?

27. A man's income is $6350 a year, and he saves $1087.50. What per cent does he save? What per cent does he spend? What is the sum of these two per cents? How does the answer to the last question check the first two results?

The Next Problem in Percentage. The third problem in percentage is that of finding the number of which a given number is a given per cent.

For example, if a ball team lost 18 games, which was 40% of the total games played, how many games were played?

We have $\frac{40}{100}$ of the total = 18.

Therefore $\frac{1}{100}$ of the total = $\frac{1}{40}$ of 18,

and the total = $\frac{100}{40}$ of 18;

or, expressed in another way, we have

the total = $18 \div \frac{40}{100}$.

That is, the total number of games played by the team is

$18 \div 0.40$, or 45.

Check. 40% of 45 = 18.

To find the number of which a given number is a certain per cent, divide the given number by the per cent.

FINDING THE NUMBER

1. If 20% of the inhabitants of a certain city are school children, and there are 12,400 school children in the city, what is the total population of the city?

2. If 36% of the inhabitants of a certain city voted at a certain election, and there were 7200 ballots cast, what was the total population of the city?

3. A player on the seventh-grade baseball team reached first base 29 times, which was $33\frac{1}{3}$% of the number of times he was at bat. How many times was he at bat?

4. A player scored 24 times, which was $8\frac{1}{3}$% of the number of times he was at bat. How many times was he at bat?

5. If, in paying $16\frac{2}{3}$% of the expenses of a camping trip, you pay $12.75, what are the total expenses of the trip?

Find the numbers of which the following are the given per cents:

6. 44, 11%.	11. 37, $12\frac{1}{2}$%.	16. 32.45, $16\frac{2}{3}$%.
7. 75, 5%.	12. 49, $3\frac{1}{3}$%.	17. 57.8, 17%.
8. 24, 6%.	13. 53, $33\frac{1}{3}$%.	18. 22.22, 110%.
9. 98, 7%.	14. 186, $37\frac{1}{2}$%.	19. 65.72, 106%.
10. 44, 4%.	15. 325, $62\frac{1}{2}$%.	20. 622.6, 5.5%.

21. \$4.65 is 15% of what amount? 20% of what amount?

22. \$7.84 is 16% of what amount? 10% of what amount?

23. 12 ft. 5.6 in. is 11% of what length? 5% of what length?

24. 126 lb. 14 oz. is 35% of what weight?

25. If a farmer sold a horse for 80% of what it cost him and received \$160, how much did the horse cost him? How much did he lose on the sale?

26. A poultry raiser set some eggs, and 15% of them failed to hatch. If the number that failed to hatch was 66, how many eggs did he set?

27. A man sells a building lot for \$360, thereby gaining 20% on the cost. What per cent of the cost is the selling price? How much is the cost?

28. A man saves \$1283.50 a year, which is 17% of his income. How much is his income?

29. A school has 20% of its pupils in the seventh grade, which has 24 pupils. How many pupils are there in the school? How many are not in the seventh grade?

30. In an eastern city a school had 112 pupils in the seventh grade. This number was $16\frac{2}{3}$% of the total number in the school. How many pupils were there in the school?

31. Solve Ex. 30, supposing that there were 119 pupils in the seventh grade, and that this was 17% of the total number.

Per Cents Greater or Less. Mr. Lane, who has been earning \$30 a week, has his wages raised to \$36 a week. His wages are now what per cent greater than before?

The increase is \$36 − \$30, or \$6. The question then reduces to this: \$6 is what per cent of \$30? This is like the case on page 27, and so we have

$$\$6 \div \$30 = 0.20 = 20\%.$$

That is, Mr. Lane's wages were raised 20%.

Problems which ask us to find what per cent one number is less than another are solved in a similar way.

PER CENTS GREATER OR LESS

Numbers 1 to 12, oral

1. 101 is what per cent greater than 100?
2. 99 is what per cent less than 100?
3. \$1010 is what per cent greater than \$1000?
4. \$990 is what per cent less than \$1000?
5. \$106 is what per cent greater than \$100?
6. \$95 is what per cent less than \$100?
7. A man who is now earning \$6 a day formerly earned \$5 a day. What per cent have his daily wages increased?
8. A man who earned \$150 one month worked only part of the time the next month and earned \$100. His earnings were then what per cent less than before?
9. \$125 is what per cent greater than \$100?
10. \$250 is what per cent greater than \$200?
11. \$750 is what per cent less than \$1000?
12. 150 ft. is what per cent greater than 100 ft. and what per cent less than 200 ft.?

13. \$150 is what per cent greater than \$100?

14. \$100 is what per cent less than \$150?

15. A half mile is what per cent less than $\frac{3}{4}$ mi.?

16. \$4080 is what per cent greater than \$3400?

17. \$21,000 is what per cent greater than \$17,500?

18. \$3500 is what per cent greater than \$3000 and what per cent less than \$4000?

19. \$256.15 is what per cent greater than \$235?

20. \$216.20 is what per cent less than \$235?

21. 40 in. is what per cent longer than 1 yd.?

22. A yard is what per cent longer than 1 ft.?

23. A foot is what per cent shorter than 1 yd.?

24. A mile is what per cent greater than 2640 ft.?

25. On a boiler which supplies steam for an engine there is a gauge which shows the pressure in pounds that the steam exerts against every square inch of the boiler surface. If the steam pressure increases from 120 lb. to 150 lb. per square inch, it is then what per cent greater than before?

26. A man bought a horse for \$150, and sold it so as to gain \$30. Find the per cent of gain on the cost.

Find the per cent of gain or loss on the cost in each case:

	Cost	Gain
27.	\$425	\$29.75
28.	\$235	\$16.45
29.	\$320	\$44.80
30.	\$225	\$11.25
31.	\$575	\$74.75

	Cost	Loss
32.	\$725.50	\$145.10
33.	\$2750	\$247.50
34.	\$12,375	\$1361.25
35.	\$16,750	\$3015
36.	\$875.25	\$140.04

Using What You Have Learned. We shall now take up some miscellaneous problems where it will be necessary for you to decide for yourself the method of solution. You will find it best first to ask yourself exactly what is given. Then consider carefully whether you are to find (1) a required per cent of a given number, (2) what per cent one of the given numbers is of the other, or (3) of what number the given number is the given per cent.

USING WHAT YOU HAVE LEARNED

1. A cubic foot of water weighs $62\frac{1}{2}$ lb. From a tank containing 16 cu. ft. there is drawn off an amount of water that weighs 125 lb. What per cent of the water is drawn off?

2. To a class of 32 pupils 8 more are admitted. The number admitted is what per cent of the original number? of the present number? If 10 pupils now leave the class, what per cent leave it?

3. In an arithmetic class it is planned to solve 550 problems in a certain number of days. All but 44 problems have been solved. What per cent of the problems remain to be solved? What per cent of them have been solved?

4. If 45% of the 240 pupils that attend a school are boys, and the rest are girls, how many girls are there?

5. A man purchased 530 head of cattle and sold 30% of them. How many head of cattle had he left?

6. If an automobile going at the rate of 14 mi. an hour increases its speed 37%, at what rate will it then be going?

7. If a man who earns $2400 a year has his salary increased 5%, how much does he then receive?

8. If a farmer sold 15 cows, which was $33\frac{1}{3}$% of the number he owned, how many cows did he own?

9. A cubic foot of platinum weighs 1220 lb., and a cubic foot of pressed gold weighs 10 lb. less. Platinum is what per cent heavier than pressed gold?

10. Of a salary of $24 a week a clerk spends 60% for board and room, 10% for clothes, and 15% for other expenses. If he saves the rest, how much does he save in 50 wk.?

11. If a baker uses 1278 lb. of flour in making bread and adds to the flour 426 lb. of liquid, the weight of the liquid is what per cent of the weight of the mixture?

12. There are 3 teaspoonfuls of sugar to a tablespoonful of sugar. A teaspoonful is what per cent of a tablespoonful?

13. There are 16 tablespoonfuls to a cup. A tablespoonful is what per cent of a cup?

14. A teaspoonful of butter is $\frac{1}{3}$ of a tablespoonful of butter. A tablespoonful is what per cent of a teaspoonful?

15. The record of the leading team in a baseball league in a recent year was 52 games won and 14 lost. What per cent of the games did the leading team win that year?

16. One of the best records in baseball of recent years was made when Brooklyn won the pennant with a total of 82 games won and 54 lost. What was the per cent of games won?

17. Two of the best averages for runs in baseball in a recent year were as follows: 597 times at bat, 139 runs; 543 times at bat, 145 runs. The runs in each case were what per cent of the number of times at bat?

18. If the expenses of a village were $66,500 last year and $66,005 this year, what is the per cent of decrease?

19. A man bought a horse for $225 and sold it at a loss of 8% on the cost. How much did he lose on the deal? How much did he receive for the horse?

III. HOW PER CENTS ARE USED

A SILENT READING LESSON

A merchant generally plans to sell his goods for enough more than their cost to pay for the goods, to cover the expenses of running his store, and to leave him a *profit* besides. If he fails to do this, he suffers a *loss*.

A merchant has many expenses, such as rent, heat, light, clerks' salaries, insurance, and so on. We shall call all these the *expenses*, or the *cost of doing business*, but the expression *overhead charges*, or simply *overhead*, is often used. In large stores the buying, selling, and operating expenses are sometimes considered separately. In city retail stores the total expenses often amount to about 30% of the total sales, and are sometimes as high as 45%. A merchant must allow for these expenses before he can say that he has made a profit.

If a merchant sells goods rapidly, so that he is continually buying new goods and selling them, we say that his *turnover* is rapid. In such cases, he can sell at a smaller profit on each article, because he sells so many of them. He can even afford to have *mark-down sales* to dispose of old stock. In obtaining a rapid turnover, good salesmanship and effective advertising are exceedingly valuable.

The merchant should have a larger profit than the man who simply puts his money in a savings bank and gets interest on it, for the merchant not only runs a much greater risk of losing the money invested, but is called upon to exercise keen business judgment in buying and selling goods.

Merchants usually speak of their profits or losses as per cents. The per cent is reckoned sometimes on the cost of goods and sometimes on the selling price. We shall first consider the plan of basing the per cent on the cost, beginning with simple transactions such as buying and selling a house.

Selling at a Profit. **1.** Suppose that I bought a house for \$8000 and sold it so as to gain 20% on what it cost me. How much did I gain?

The gain was 20% of \$8000, or $\frac{1}{5}$ of \$8000, and so, as shown at the right,

$\frac{1}{5}$ of \$8000 = \$1600, the amount of gain.

\$1600
5)\$8000

2. What was the selling price in Ex. 1?

The selling price was the cost plus the gain; that is, I received \$8000 (cost) plus \$1600 (gain), and so, as here shown,

\$8000 + \$1600 = \$9600, the selling price.

\$8000
1600
\$9600

3. In Ex. 2, find the selling price directly.

Since I gained 20%, it is evident that

100% of the cost = the cost
20% of the cost = the gain
120% of the cost = the selling price

Hence, as shown at the right,

1.20 × \$8000 = \$9600, the selling price.

\$8000
1.20
160000
8000
\$9600.00

SELLING AT A PROFIT

Given the cost and the per cent of gain, find the amount of gain and the selling price as in Exs. 1 and 2, above:

	Cost	Gain
1.	\$700	10%
2.	\$800	14%
3.	\$950	$12\frac{1}{2}$%
4.	\$725	13%

	Cost	Gain
5.	\$2000	15%
6.	\$2500	16%
7.	\$3000	$17\frac{1}{2}$%
8.	\$4000	18%

	Cost	Gain
9.	\$4200	8%
10.	\$3600	$8\frac{1}{4}$%
11.	\$4800	$9\frac{3}{4}$%
12.	\$7500	15%

13. Solve Exs. 4, 8, and 12 by the method given in the third illustrative example, above.

Selling at a Loss. **1.** If I bought a house for \$8000 and sold it at a loss of 20% of the cost, what was the selling price?

The selling price was the cost less 20% of the cost; that is,

100% of the cost = the cost
20% of the cost = the loss
80% of the cost = the selling price

\$8000
0.80
\$6400.00

That is, I sold it for 80% of \$8000, and so

$0.80 \times \$8000 = \$6400$, the selling price.

2. If I sold a house and lost \$1600, which was 20% of the cost, how much did the house cost me?

Since 20% of the cost is \$1600,
1% of the cost is $\frac{1}{20}$ of \$1600,
and 100% of the cost is $\frac{100}{20}$ of \$1600;
that is, $5 \times \$1600 = \$8000$, the cost.

\$1600
5
\$8000

3. If I sold a house that cost me \$8000 and lost \$1600, what per cent did I lose?

I lost $\frac{1600}{8000}$ of \$8000, or $\frac{1}{5}$ of \$8000. That is, since $\frac{1}{5} = 20\%$, I lost 20% on the cost of the house.

SELLING AT A LOSS

Copy the following tables and fill all the blank spaces:

	Cost	Loss	Amount of Loss
1.	\$500	8%	
2.	\$500		\$30
3.		12%	\$24
4.	\$800	$12\frac{1}{2}\%$	
5.	\$600		\$48

	Cost	Loss	Amount of Loss
6.	\$2000	$16\frac{2}{3}\%$	
7.		$16\frac{2}{3}\%$	\$200
8.	\$3000		\$600
9.	\$5000	15%	
10.	\$8000		\$520

Profit or Loss on the Cost. **1.** A merchant's expenses average 30% of the cost of goods. If he buys an article for \$30 and wishes to make a profit of 10% on the cost, at what price should he sell the article?

Since the cost is 100% of itself, the selling price must be

100% + 30% + 10%, or 140%, of the cost,

and so $1.40 \times \$30 = \$42$, the selling price.

100% of cost = cost	\$30
30% of cost = expenses	1.40
10% of cost = profit	1200
140% of cost = selling price	30
	\$42.00 (selling price)

2. If the merchant in Ex. 1 sells the article for \$33, what per cent does he lose on the cost?

The cost plus expenses is 130% of the cost, and so

$1.30 \times \$30 = \$39$, the cost + expenses.

Then $\$39 - \$33 = \$6$, the amount of loss,

and $\$6 \div \$30 = 0.20 = 20\%$, the loss.

100% of cost = cost	\$30	\$39	0.20
30% of cost = expenses	1.30	33	\$30)\$6.00
130% of cost = cost + expenses	900	\$6 (loss)	
	30		
	\$39.00		20%. *Ans.*

Do not try to understand all of each solution at once. Begin with the first step and see if you understand why that was taken. Then take the next step, and so on. Do not ask your teacher for help until you have done some hard thinking.

PROFIT OR LOSS ON THE COST

1. Disregarding expenses, if you buy an article for \$1 and sell it for \$1.30, what is your per cent of gain? What would it be if you bought it for \$1.25 and sold it for \$1.75?

2. A dealer buys some goods for \$750 and sells them so as to make a profit of \$87.50. Find, to the nearest 0.1%, the per cent of profit.

3. If a grocer buys some canned goods for \$2475 and sells them at a profit of \$221.50, what is the per cent of profit?

4. A dry-goods merchant knows that his expenses average 25% of the cost of goods. If he buys some silk for \$4800 and has to sell it for \$5280, what is the per cent of loss?

5. A merchant's cost of doing business is 22% of the cost of goods. If he buys an article for \$22.50 and wishes to make a profit of 12%, at what price should he sell the article?

6. Mr. Robinson bought a house for \$5600 and sold it at a profit of 15%. Find the selling price.

7. If Mr. Robinson had sold the house mentioned in Ex. 6 at a loss of 15%, what would have been the selling price?

8. A merchant bought some goods for \$5700, but had to sell them at a loss of \$285. Find the per cent of loss.

9. A man bought a city lot for \$24,000 and sold it at a profit of $16\frac{2}{3}\%$. Find the amount of his profit and the price at which he sold the lot.

10. A man who bought an automobile for \$2400 estimated that the use he got out of it was worth \$360. He sold the car for $12\frac{1}{2}\%$ less than he paid for it. Find the profit or the loss.

11. A man bought some horses and sold them at a profit of \$1960. If he made 20% on the deal, how much did he pay for the horses and for how much did he sell them?

Profit or Loss on the Selling Price. Formerly the per cent of profit or loss was generally computed on the cost of goods. Now, however, it is often the custom to figure the cost of the goods, the expenses, and the profit as per cents of the selling price. This method has been adopted almost universally by large retail stores and some other types of business concerns. In many establishments, however, the method of figuring profit on the cost is still used.

By keeping careful records of sales and expenses, a department store, for example, is able to know the selling price to be placed upon an article in order to cover expenses and to yield the desired profit. From a bookkeeping standpoint, there are many advantages in this method of figuring profit, but they are in general too technical to be considered in the schools.

PROFIT OR LOSS ON THE SELLING PRICE

All work oral

State the profit at 10% on each of the following selling prices:

1. $1.	4. $2.	7. $15.	10. $100.	13. $2000.
2. $5.	5. $4.	8. $25.	11. $300.	14. $2500.
3. $8.	6. $9.	9. $30.	12. $575.	15. $7500.

State the loss at 6% on each of the following selling prices:

16. $3.	19. $1.	22. $10.	25. $100.	28. $3000.
17. $5.	20. $2.	23. $30.	26. $200.	29. $4000.
18. $8.	21. $6.	24. $70.	27. $500.	30. $8000.

State the profit at 15% on each of the following selling prices:

31. $1.	34. $5.	37. $10.	40. $100.	43. $1000.
32. $2.	35. $7.	38. $20.	41. $200.	44. $2000.
33. $3.	36. $9.	39. $30.	42. $400.	45. $3000.

Finding the Selling Price. In a store where the expenses average 24% of sales, and the desired profit is 12% of the selling price, what should be the selling price of an article costing $9.60?

The selling price (100%) includes the cost, the expenses, and the profit, and so, as shown below, the cost is

$$100\% - 36\%, \text{ or } 64\%, \text{ of the selling price.}$$

Then $\$9.60 \div 0.64 = \$15$, the selling price.

24% = expenses	$0.64\overline{)\$9.60}$
12% = profit	$\$15.$
36% = expenses + profit	$64\overline{)\$960.}$
100% = selling price	64
36% = expenses + profit	320
64% = cost	320

Check. $0.36 \times \$15 = \$5.40$; $\$15 - \$5.40 = \$9.60$.

FINDING THE SELLING PRICE

Copy the following table and fill the blank spaces:

	Cost	Expenses	Profit	Selling Price
1.	$240	30%	10%	
2.	$480	40%	12%	
3.	$750	35%	15%	
4.	$840	32%	8%	
5.	$1200	33%	7%	
6.	$1160	30%	12%	
7.	$5400	36%	10%	

PROFIT OR LOSS ON THE SELLING PRICE

1. If you sell an article for \$9, and your profit is 20% of the selling price, what is the profit?

2. A store's profit was \$3.96 on an article that sold for \$26.40. Find the per cent of profit on the selling price.

3. A merchant, whose expenses average 20% of the selling price, sells for \$13 an article that costs \$9. How much are the expenses in connection with the sale of this article? How much profit does the merchant make? What is the per cent of profit on the selling price?

4. If the expenses average 24% of sales, at what price, to the nearest 5¢, should a department store sell an article that cost \$4.50 so as to make a profit of 10% on the selling price?

5. A store with expenses averaging 28% of the selling price sells for \$17.50 an article that cost \$10.50. Find the per cent of profit on the selling price.

6. A dealer's expenses are 25% of sales, and he desires a profit of 12% of sales. At what price, to the nearest 10¢, should he sell an article costing \$12?

7. At what price should the dealer in Ex. 6 sell an article which costs \$4.50? \$6.75? \$12.25? \$17.60?

8. A dealer's expenses average 22% of sales, and he desires a profit of 14% of sales. If he buys silk at \$3.30 per yard, at what price, to the nearest 5¢, should he sell it?

9. A department store made a profit of 10% on an article which cost \$1.95 and sold for \$3. Find the per cent of the selling price allowed for expenses.

10. A merchant, whose expenses average 25% of the selling price, sells an article that cost \$10 at the cost price. Find the amount of loss.

A SILENT READING LESSON

We learned last year that, when one person acts as an *agent* for another, one of the ways by which the agent is paid for his services is by being given a *commission*; that is, by being paid a certain per cent of the money involved in the transaction, or a certain amount for each article bought or sold.

Thus, if an agent sells a house for a man for \$6000 and is paid a commission of 5%, he receives 5% of \$6000, or \$300, for his services in connection with the sale.

We may speak of the 5% as the *rate* of commission and of the \$300 as the *amount* of the commission.

In some kinds of business, agents are sent out to buy instead of sell goods. For example, suppose that a wool dealer sends an agent to the sheep-raising country to buy wool. If the agent buys \$5000 worth of wool, and if his commission is 5%, he receives 5% of \$5000, or \$250, for his work.

We therefore see that an agent may be employed to sell on a commission, or to buy on a commission. In either case, we have this relation :

Amount of commission = *rate* × *amount of transaction.*

If an agent who is paid a commission of 5% sells a house for Mr. Brown for \$6000, Mr. Brown receives \$6000 less the amount of the agent's commission; that is,

In selling, the sum (or net amount) received is the amount of the sale less the amount of the commission.

Similarly, if an agent who is paid a commission of 5% buys \$5000 worth of wool for Jones & Co., the total cost of the wool to the company is \$5000 plus the amount of the agent's commission; that is,

In buying, the total cost is the amount of the purchase plus the amount of the commission.

Finding the Commission. If an agent sells a house for Mr. Henderson for \$8000 and receives a commission of $2\frac{1}{2}\%$, what is the amount of his commission?

The amount is $2\frac{1}{2}\%$ of \$8000, but instead of multiplying \$8000 by $0.02\frac{1}{2}$, where we might make a mistake in the decimal point, it is easier to see that 1% of \$8000 is \$80, and that $2\frac{1}{2}\%$ of it is $2\frac{1}{2}$ times \$80, or \$200.

That is, the agent receives \$200.

\$8000	\$80
$0.02\frac{1}{2}$	$2\frac{1}{2}$
	40
	160
	\$200

We should always watch for such short cuts. Can you see the other short cut of simply dividing \$800 by 4? Can you see why it will give the same answer?

FINDING THE COMMISSION

All work oral

State the value of the following:

1. 2% of \$100.
2. 2% of \$200.
3. 4% of \$100.
4. 4% of \$300.
5. 6% of \$100.
6. 6% of \$500.

7. An agent sells a house for \$9000 and receives a commission of $2\frac{1}{2}\%$. Find the amount of the commission.

8. On a sale amounting to \$50,000 an agent receives a commission of 4%. Find the amount of the commission.

9. An agent who sells electric-light fixtures is paid a commission of 2%. Find the amount of his commission on a sale amounting to \$12,000.

State the amount of the commission on a sale amounting to

10. \$300 at 6%.
11. \$200 at 3%.
12. \$750 at 8%.
13. \$4000 at 5%.
14. \$3000 at 3%.
15. \$2500 at 9%.
16. \$3000 at 2%.
17. \$5000 at 4%.
18. \$15,750 at 3%.

Finding the Net Amount. A used-car dealer sells an automobile for Mr. Martin and charges a commission of 4%. If the dealer sells the car for \$850, what is the amount of his commission, and what is the net amount received by Mr. Martin?

\$850	\$8.50	\$850
0.04	4	34
	\$34.00 (commission)	\$816 (net amount)

Could we have multiplied \$850 by 0.04 as easily as we multiplied \$8.50 by 4? Either method may be used.

Why do we subtract \$34 from \$850?

COMMISSIONS AND NET AMOUNTS

1. Mr. Haskins has a car that he has used for some time. He pays a garage man $4\frac{1}{2}\%$ for selling it for him at a price of \$1100. How much commission does the garage man receive? What is the net amount that Mr. Haskins receives?

In each of the following, which show the amounts of sales at different rates of commission, find the amount of the commission and the net amount received:

2. \$50, 2%.
3. \$75, 2%.
4. \$75, 3%.
5. \$85, 4%.
6. \$60, $4\frac{1}{2}\%$.
7. \$80, $3\frac{1}{4}\%$.
8. \$80, $4\frac{3}{4}\%$.
9. \$200, 2%.
10. \$250, $2\frac{1}{2}\%$.
11. \$320, $3\frac{3}{4}\%$.
12. \$5000, 4%.
13. \$4800, 5%.
14. \$6200, $5\frac{1}{2}\%$.
15. \$7200, $4\frac{3}{4}\%$.
16. \$1250, 3%.
17. \$1750, $2\frac{1}{4}\%$.
18. \$1875, 4%.
19. \$2250, $3\frac{1}{2}\%$.
20. \$3375, 4%.
21. \$3270, $4\frac{1}{2}\%$.
22. \$15,000, 3%.
23. \$20.000, 2%.
24. \$25,000, 3%.
25. \$30,000, $2\frac{1}{2}\%$.

A SILENT READING LESSON

Farmers and market gardeners often sell their vegetables and other farm produce through *commission merchants* or *brokers*. Both the commission merchant and the broker are agents, and each receives a commission for his work.

The commission merchant actually receives the produce and sells it; the broker does not receive it, but finds a purchaser to whom the farmer makes delivery. Both cases, however, involve practically the same arithmetic operations.

The commission merchant *deducts* (subtracts) the amount of his commission from the amount that he receives from selling the produce, and *remits* (sends) the net amount, called the *proceeds*, to his *principal* (the man for whom he acts). That is,

Amount of sale	
Proceeds	Commission

Proceeds = amount of sale — amount of commission.

Suppose that a market gardener ships to a commission merchant a quantity of vegetables, and that the transportation charges are \$18. If the merchant sells the vegetables for \$500 and charges a commission of 8%, what are the proceeds?

To solve this problem, we first take 8% of \$500, and have

$$0.08 \times \$500 = \$40, \text{ amount of commission.}$$

Then, deducting the amount of the commission, we have

$$\$500 - \$40 = \$460, \text{ the proceeds.}$$

There still remains the question of the transportation charges of \$18. If they are paid by the commission merchant, he deducts \$18 more and remits the balance. If paid by the market gardener, he charges them up as expenses on this lot of vegetables; that is, in either case the net amount received for the vegetables is \$460 — \$18, or \$442.

PROBLEMS IN COMMISSION

1. At $2\frac{1}{2}\%$, find the amount of the commission on the sale of some furniture for $2400.

2. At $5\frac{1}{2}\%$, how much is the commission on the sale of some wagons for $4800? How much are the proceeds?

3. A commission merchant sells for a factory 4800 lb. of cheese at 32¢ a pound and deducts a commission of 5%. What sum does he remit to the factory?

4. An auctioneer sells property for Mr. Sinclair for $750 and receives a commission of 8%. How much is the commission? How much is left for Mr. Sinclair?

5. A city dealer sends an agent to a fruit region to buy for him and pays him a commission of 2%. The agent's purchases amount to $2000 at one grove, $3200 at another, and $1600 at a third. What is his total commission? What is the total cost to the dealer, including the commission?

6. A real-estate dealer sells 30 A. of land at $400 an acre. How much is his commission at $3\frac{1}{2}\%$?

7. An agent sells $90 worth of maps to our school, and his firm pays him a commission of $8\frac{1}{2}\%$. How much is his commission on the sale?

8. Some butter and eggs which a farmer shipped to the city are sold for $116.50. The commission merchant charges the farmer 8% for selling. What are the proceeds?

9. At 8%, how much is the auctioneer's commission on a sale of furniture amounting to $1425?

10. At 6%, how much is the architect's commission on a house which costs $12,500 to build?

11. A commission merchant receives 8% for selling some produce for $175. How much is his commission?

Further Work in Commission. **1.** A salesman who received a commission of 4% was paid $25.60 for selling a second-hand touring car. At what price did he sell the car?

Since $25.60 is the result of multiplying the selling price by 4%, we find the selling price by dividing $25.60 by 0.04.

The answer is $640.

$$0.04\overline{)\$25.60} \qquad \begin{array}{r}\$640.\\ 4\overline{)\$2560.}\end{array}$$

Amount of sale = amount of commission ÷ rate.

2. A salesman received $25.60 for selling a second-hand touring car for $640. What was the rate of commission paid him?

Since $25.60 is the result of multiplying $640 by the rate, we find the rate by dividing $25.60 by $640.

The answer is 0.04, or 4%.

$$\begin{array}{r}0.04\\ \$640\overline{)\$25.60}\\ \underline{2560}\end{array}$$

Rate = amount of commission ÷ amount of sale.

FURTHER WORK IN COMMISSION

All work oral

Given the amount of a commission and the rate paid, as follows, find the amount of the sale:

1. $2, 1%.	3. $8, 1%.	5. $4, 2%.	7. $5, 5%.
2. $2, 2%.	4. $3, 3%.	6. $10, 10%.	8. $50, 5%.

Given the amount of a sale and the amount of the commission, as follows, find the rate paid:

9. $1000, $10.	12. $2000, $20.	15. $3000, $60.
10. $1000, $30.	13. $2000, $40.	16. $3000, $120.
11. $1000, $80.	14. $2000, $100.	17. $3000, $180.

PROBLEMS IN COMMISSION

1. If you buy a car for \$600, and the salesman's commission is 5%, how much commission does he receive?

2. If you buy a car for \$500, and the salesman's commission is \$30, what rate of commission is he paid?

3. If you buy a car from a salesman who is paid a commission of 6% by his employer, and if the salesman receives \$42 on the sale, how much do you pay for the car?

4. If you buy some wool from an agent to whom you pay a commission of 6%, and if you pay him \$42, how much in all do you pay for the wool?

5. A farmer ships some vegetables to a city dealer, who sells them for \$380 and charges a commission of 10%, plus \$6.80 for express. How much does the farmer receive?

6. Helen's father gave her a commission of 25% on all the raspberries that she picked and sold from his bushes. From this record of her sales, find how much Helen made:

July 10	25 qt. @ 25¢	July 15	12 qt. @ 16¢
11	15 qt. @ 20¢	17	15 qt. @ 12¢
12	20 qt. @ 20¢	18	17 qt. @ 15¢
13	18 qt. @ 18¢	19	20 qt. @ 20¢

7. If the amount of a sale of farm produce is \$1280, and the rate of commission is 6%, how much is the commission?

8. If the amount of a sale is \$1725, and the commission amounts to \$138, what is the rate of commission paid?

9. Which receives the greater amount, a salesman who sells \$10,250 worth of goods at a commission of $1\frac{1}{2}$%, or one who makes a sale amounting to \$1375 at a commission of 15%?

REVIEW OF COMMISSION

Copy the following tables and fill the blank spaces:

	Amount of Sale	Rate of Commission	Amount of Commission
1.	$350	4%	
2.	$480		$24
3.		5%	$17
4.	$300		$12
5.	$750	6%	
6.		8%	$2.40
7.	$750		$15
8.	$500	$4\frac{1}{2}$%	
9.		12%	$240
10.	$800		$64

	Amount of Sale	Rate of Commission	Amount of Commission
11.	$825	4%	
12.		6%	$14.40
13.	$2500		$500
14.	$1200	$3\frac{1}{2}$%	
15.	$3600		$196
16.		8%	$33.60
17.	$1600	$3\frac{3}{4}$%	
18.	$275		$4.50
19.		10%	$14.75
20.	$2500	$4\frac{1}{2}$%	

In each of the following cases, which show the amounts of sales and the net amounts received, find the amount and rate of commission paid:

21. $200, $188.
22. $300, $285.
23. $500, $480.
24. $600, $564.
25. $650, $624.
26. $750, $735.
27. $750, $727.50.
28. $825, $800.25.
29. $1275, $1224.

In the following cases, each of which shows the cost of a purchase and the cost plus the commission, find the amounts and the rates of commission paid:

30. $200, $212.
31. $200, $216.
32. $300, $315.
33. $500, $550.
34. $700, $756.
35. $900, $927.
36. $750, $772.50.
37. $950, $959.50.
38. $1650, $1674.25.

A SILENT READING LESSON

After the baseball season a boy may be glad to sell his catcher's mask for less than the original price, so that he can use the money toward buying a football. If he sells the mask for 25% less than the original price, he sells it at a *discount* of 25%; that is, the price at which he sells it is found by deducting 25%, or $\frac{1}{4}$, of the original price.

In the above case, the 25% is sometimes called the *rate* of discount. Such rates are often given as fractions instead of as per cents. For example, merchants frequently advertise special sales at which they are to sell goods at "$\frac{1}{4}$ off," at "$\frac{1}{3}$ off," or at other reductions in price.

By selling at a discount, dealers often dispose of goods that they do not wish to carry over until another season, or goods which are slightly soiled or damaged.

The price that we pay for goods at a store is usually marked on the goods. This is why it is called the *marked price*. If we buy goods at a discount, the amount which we pay after taking off the discount is called the *net price*.

Thus, if we buy a tricycle which was marked \$7, but has been reduced 10%, it costs us \$7 less 10% of \$7; that is, it costs us \$7 − 70¢, or \$6.30. In this case the discount is 10%, the amount of the discount is 70¢, and the net price is \$6.30.

Amount of discount = rate × marked price.

Net price = marked price − amount of discount.

If a discount is allowed on a bill, the amount paid in settling the debt is called the *net amount* of the bill.

In a printed list or catalogue such as manufacturers or wholesale dealers often issue, the prices are generally given without discounts. Such prices are called *list prices*, and in selling to retailers discounts from these prices are allowed.

Amount of discount = rate × list price.

FINDING DISCOUNTS

Numbers 1 to 18, oral

1. If a certain article is marked \$60, and a discount of 10% is allowed, what is the amount of the discount?

2. If on a book marked \$4 a dealer offers a discount of $\frac{1}{4}$, the net price is how much below the marked price?

3. If on an order for \$300 worth of goods a merchant is allowed a discount of 30%, what is the net amount the merchant must pay?

Find the amount of a discount of:

4. 10% on \$70.
5. 10% on \$45.
6. 20% on \$80.
7. 20% on \$65.
8. $\frac{1}{4}$ on \$36.
9. 25% on \$48.
10. $\frac{1}{2}$ on \$50.
11. 50% on \$80.
12. 10% on \$200.
13. 20% on \$400.
14. 25% on \$640.
15. 50% on \$260.

16. If an article listed at \$15 is sold to a dealer at a discount of 20%, how much does it cost him?

17. If a merchant marks a lot of suits at \$45 each and sells them at $\frac{1}{3}$ off, what is the net price of each suit?

18. Watch advertisements in the newspapers to see if discounts are offered. Be prepared to state to the class tomorrow some possible reasons for such discounts.

Find the net price of an article which is marked and sold at a discount, as follows:

19. \$29.50, 20%.
20. \$38.75, 20%.
21. \$46.40, $\frac{1}{4}$.
22. \$86.60, 25%.
23. \$84.40, 25%.
24. \$78.90, $\frac{1}{3}$.
25. \$71.70, $33\frac{1}{3}$%.
26. \$49.50, $16\frac{2}{3}$%.
27. \$81.60, $\frac{1}{8}$.
28. \$86.40, $12\frac{1}{2}$%.
29. \$31.50, 5%.
30. \$58.50, 10%.
31. \$44.75, 20%.
32. \$65.25, 20%.
33. \$38.60, 10%.

A SILENT READING LESSON

When we purchase goods at a store and do not pay cash for them, the dealer sends us a *bill* for what we owe, usually on the first of the next month.

The following model shows such a bill, and on the bill is the dealer's receipt stamp to show that payment has been made:

HOYT AND WHITE

480–486 Main Street

Topeka, Kansas

Mr. Fred B. Coleman
426 Summit Avenue
Topeka, Kansas

Oct. 1, 1928

Account No. 2426

Sept.	4	2 shirts	@ 2.65	5	70		
		3 neckties	@ 1.25	3	75		
	21	1 pr. flannel trousers		6	75		
	28	1 leather jacket		10	50		
						26	70

RECEIVED PAYMENT
OCT. 9, 1928
HOYT AND WHITE
PER C. F. L.

Filling out a bill by inserting the amounts of the various purchases in one of the columns at the right is called *making the extensions* on the bill. The sum of the extensions ($26.70 in the above case) is the total amount owed.

We sometimes hear the terms *debtor* and *creditor* used in speaking of bills. A debtor (Dr.) is one who owes money to another, and the creditor (Cr.) is the one to whom it is owed; that is, in the above case the debtor is Mr. Coleman and the creditor is Hoyt and White.

A SILENT READING LESSON (CONTINUED)

The bill that a manufacturer or a wholesaler sends to a retailer at the time that goods are shipped is often called an *invoice*. If the retail dealer is allowed a discount from the manufacturer's list prices, the amount of the discount generally appears on the invoice, as in the following model:

CHICAGO, ILL., Nov. 17, 1928

DUFFY, WEST, AND BROWN

Manufacturing Jewelers

768 Webster Street

Sold to

R. E. Long Co.
124 First St.,
Oklahoma City, Okla.

Quantity	Stock No.	Description	Price					
3 doz.	406 A	Dessert forks	32	00	96	00		
1/6 doz.	327 P	Salad forks	33	00	5	50		
					101	50		
		Discount, 20%			20	30		
							81	20

Business men usually pay their bills by check. As we shall find, later on, a paid check is itself a receipt, and for this reason invoices and other bills paid in this way are not often receipted. A receipt may, however, be written separately, and in such cases would have a form something like this:

$81.20 CHICAGO, ILL., Nov. 20, 1928

Received of R. E. Long Co., Oklahoma City, Okla._____

Eighty-one and 20/100_____ *Dollars*

in payment of invoice of Nov. 17, 1928

Duffy, West, and Brown

Per C. J. Mayers

Net Price by One Multiplication. We have learned that we can find the net price by first finding the amount of the discount and then subtracting this from the list price. However, when we take off 8% of a price, we have 92% of it left. We can therefore find the net price of an article by a single multiplication, if we wish.

For example, if the list price of an article is \$300, and the discount allowed is 8%, the net price is

$$(100\% - 8\%) \times \$300,$$

or $$0.92 \times \$300;$$

that is, as shown at the right, it is \$276. Notice that we make the subtraction in the parentheses before we multiply.

100%	\$300 (list price)
8%	0.92
92%	\$276 (net price)

$$\textit{Net price} = (100\% - \textit{rate}) \times \textit{list price}.$$

In problems we can now use either method, as required, but the one given on page 51 is sometimes simpler.

NET PRICE BY ONE MULTIPLICATION

Given the list price of an article and the rate of discount as follows, find the net price by one multiplication:

1. \$200, 2%.
2. \$350, 4%.
3. \$725, 4%.
4. \$560, 3%.
5. \$128, 5%.
6. \$2000, 2%.
7. \$2500, 3%.
8. \$3200, 8%.
9. \$4800, 6%.
10. \$7500, 8%.
11. \$25, 20%.
12. \$60, $33\frac{1}{3}$%.
13. \$60, $16\frac{2}{3}$%.
14. \$800, 25%.
15. \$600, $12\frac{1}{2}$%.
16. \$400, 15%.
17. \$450, 14%.
18. \$620, 10%.
19. \$800, 16%.
20. \$900, $16\frac{2}{3}$%.
21. \$1000, 12%.
22. \$1600, $12\frac{1}{2}$%.
23. \$1200, 14%.
24. \$2000, 12%.

A SILENT READING LESSON

In some kinds of wholesale business two or more discounts are sometimes allowed on the same bill. For example, a dealer may buy \$400 worth of hardware with discounts of 20% and 10%, often called "a discount of 20%, 10%." This means that 20% is first deducted from the amount of the bill, and 10% is then deducted from the remainder, as follows:

$$0.20 \times \$400 = \$80, \text{ and } \$400 - \$80 = \$320;$$
$$0.10 \times \$320 = \$32, \text{ and } \$320 - \$32 = \$288.$$

That is, the net amount of the bill for the hardware is \$288.

This explains the general nature of the problem. Practically, however, we should take such short cuts as we can.

The solution is much shorter if we remember that $20\% = \frac{1}{5}$; that $10\% = \frac{1}{10}$; that if we take away $\frac{1}{5}$, we have $\frac{4}{5}$ left; and that if we take away $\frac{1}{10}$ from the result, we have $\frac{9}{10}$ left. That is, the solution becomes simply

$$\frac{9}{10} \text{ of } \frac{4}{5} \text{ of } \$400 = \frac{9 \times 4 \times \overset{80}{\cancel{400}}}{10 \times \cancel{5}} = \$288.$$

Similarly, for a discount of $12\frac{1}{2}\%$, $16\frac{2}{3}\%$, we have

$$\frac{5}{6} \text{ of } \frac{7}{.8} \text{ of } \$400 = \frac{5 \times 7 \times \overset{25}{\cancel{50}}\overset{}{\cancel{400}}}{\cancel{6} \times \cancel{8}_{3}} = \frac{\$875}{3} = \$291.66\tfrac{2}{3}.$$

That is, the net amount of the bill is \$291.67.

In computing the amount of a discount, we disregard fractions less than $\frac{1}{2}¢$ and call $\frac{1}{2}¢$ or more a full cent.

The giving of several discounts, often called *chain discount*, is now becoming less common. The use of the single discount which is equivalent to two or more discounts obviously saves time and labor in bookkeeping.

SEVERAL DISCOUNTS

1. From \$200 take 10%, and 10% from the remainder.

2. From \$800 take 20%, and 10% from the remainder.

Find the net amounts of bills with discounts as follows:

3. \$400, 20%, 20%.
4. \$600, 25%, 5%.
5. \$700, 25%, 10%.
6. \$300, 15%, 2%.

7. What is the difference between a discount of 50% on \$100 and one of 25%, 25%?

8. Is there any difference between a discount of 10%, 5% on \$100 and one of 5%, 10%? How is it on \$200? on \$900?

9. What is the amount of the discount on a bill for \$100 on which a discount of 10%, 8% is allowed? What single discount is equal to a discount of 10%, 8%?

Find the single discount that is equal to a discount of

10. 10%, 10%.
11. 10%, 12%.
12. 5%, 16%.
13. 8%, 12%.
14. 8%, 20%.
15. 10%, 20%.

16. Find the net price of a tractor listed at \$1220 less 10%, 5%, first by taking off the discounts separately, and then by taking a single discount as in Exs. 10–15.

17. An article listed in a catalogue at \$278 is sold at a discount of 12%, with an extra discount of 2% for cash. Find the total amount of discount given a dealer who buys for cash.

18. A contractor received two bids for trim on a building that he was erecting. One was \$3500 less 65%, 10%, and the other was \$3400 less 50%, 16%. Which was the cheaper bid, and how much cheaper was it?

19. Find the net amount of a bill for \$1000 less 10%, 5%, 2%; less 5%, 2%, 10%; less 2%, 10%, 5%.

Discounts on Invoices. The following model shows a form of a wholesaler's invoice for goods sold with two discounts:

J. T. Wells & Co.
Abilene, Tex.

Dallas, Tex. June 13, 1927

Shipped by Express
Date shipped June 13

STEVENS AND SON
JEWELERS

Your order No. 69
Our order No. 1721

Quantity	Pattern	Articles				
8 doz.	Colonial	Teaspoons, sterling @ 24.75	198			
7 doz.	Reed	Dinner forks, plated 7.20	50	40		
			248	40		
		Discount, 20%, 10%	69	55		
					178	85

DISCOUNTS ON INVOICES

Write invoices for the following items, making up headings with dates and names like the above model:

1. 36 doz. files @ $8.30. Discount, 30%, 20%.

2. 480 yd. silk @ $2.50; 840 yd. velvet @ $2.75; 425 yd. satin @ $3.10; 635 yd. lawn @ 50¢. Discount, 25%, 10%.

3. 45 doz. locks with keys @ $7.35; 9 doz. pairs hinges @ $6.75; 36 doz. table forks @ $8.40. Discount, 20%, 10%.

4. 36 doz. locks @ $6.30; 19 doz. mortise locks @ $6.75; 28 doz. pairs hinges @ $8.85. Discount, 25%, 8%.

5. 360 yd. taffeta @ $1.37½; 9 gross pompons @ $185; 6 doz. pieces braid @ $31.80. Discount, 10%, 5%.

6. 780 yd. silk @ $2.65; 440 yd. lawn @ 45¢; 560 yd. taffeta @ $1.42. Discount, 10%, 5%, 5%.

Finding the Marked Price. Suppose that an article which I bought at a bargain sale for \$187.50 was sold to me at a discount of 25%. What was the original marked price?

The net price (\$187.50) is evidently the marked price less 25% of itself; that is, it is 75% of the marked price. Since \$187.50 is the result of multiplying the marked price by 75%, we divide \$187.50 by 0.75 to find the marked price, \$250.

100% = marked price	$0.75\overline{)\$187.50}$	$75\overline{)\$18750.}$
25% = discount		150
75% = net price		375
		375

Marked price = net price ÷ (100% − rate).

In the above case it is easier to use fractions; that is,

$$\$187.50 \div \frac{3}{4} = \overset{62.50}{\$187.50} \times \frac{4}{3} = \$250.$$

Always watch for an opportunity to use such short cuts.

Finding the Rate of Discount. If in selling an article a discount of \$62.50 is allowed on the marked price of \$250, what is the rate of discount?

The amount of the discount is the rate times the marked price; that is, we have

$$\$62.50 = \text{rate} \times \$250.$$

To find the rate, therefore, we divide \$62.50 by \$250 and have

$$\$62.50 \div \$250 = 0.25;$$

that is, the rate of discount is 25%, or $\frac{1}{4}$.

Rate = amount of discount ÷ marked price.

Price List. The following shows the list prices of certain school supplies, with the discounts allowed to dealers:

Composition books,	\$11.00 per gross, less 40%
Drawing compasses,	\$5.40 per dozen, less 20%, 5%
Drawing paper, 9 × 12,	\$3.80 per package, less 40%
Drawing pencils,	\$11.42 per gross, less 30%
Penholders,	\$8.40 per gross, less 30%, 5%
Pens,	\$1.62 per gross, less 25%, 10%
Rulers,	\$0.95 per dozen, less 25%
Thumb tacks,	\$1.20 per 100, less 35%
Tubes of paste,	\$10.10 per gross, less 20%, 10%

DISCOUNTS FROM LIST PRICES

1. A dealer orders 400 thumb tacks and 8 packages of drawing paper at the prices shown in the above list. After deducting the discount, how much does he pay for them?

2. How much does a dealer pay for 5 gross of penholders and 9 gross of pens? for 3 gross of pens and 2 doz. rulers?

3. For a class of 36 pupils a dealer buys drawing compasses and rulers, one for each pupil. How much do they cost him?

4. A dealer buys a gross each of rulers and drawing pencils. He sells the rulers at 15¢ each and the pencils at 12¢ each. His total receipts are how much more than the cost?

5. If a dealer buys a gross of tubes of paste and sells them at 15¢ each, his receipts are how much more than the cost?

6. On an order for 24 doz. rulers of a better kind, a dealer is allowed a discount of 25%. If he pays \$22.50 for the lot, what is the list price per dozen?

7. In buying 72 doz. erasers at 45¢ a dozen, a dealer is allowed a discount amounting to \$9.72 on the lot. What rate of discount is he allowed?

A SILENT READING LESSON

Another subject that we need to know about is *insurance.* We may never be insurance agents; but we all hope sometime to have property enough to insure, or it is probable that we may wish sometime to insure our lives.

If a man's home should be destroyed by fire, the loss would be very great to him and his family. It might take the man a long time to save money enough to buy or build a new home. It would be much better if people would join together, each contributing a small sum annually to a common fund. Then if the house of any one of them took fire, the owner could be paid, from the fund, enough to cover his loss.

This shows us the general nature of insurance. Usually, however, instead of a number of persons joining together, each pays a small annual sum, or *premium,* to an insurance company. The written agreement covering the details of the insurance is called the *policy.* The sum of money specified to be paid in case of loss is called the *face* of the policy.

The necessity for insuring against loss by fire is seen when we consider that the fire losses in this country in recent years have averaged over a million dollars a day. In times of disaster, such insurance is helpful not only to the one who insures but also to the entire community.

We should, if possible, make inquiry about the fire insurance on houses and other property in our vicinity and report to the class. We should also inquire about other kinds of insurance, such as life, accident, and health insurance, burglary insurance, and automobile insurance.

The growth of insurance on people's lives is seen from the fact that in 1850 the total insurance of this kind in our country was less than $70,000,000, whereas, in a recent year, the total amounted to more than $54,000,000,000.

Fire Insurance. The *rate* in fire insurance, unlike the rate in commission or discount, is not stated as a per cent. It is usually given as a certain sum for each \$100 of insurance for a certain period, generally 1 yr., 3 yr., or 5 yr.

For example, in finding the premium, at \$1.20 a year, for insuring a store for \$4000 against loss by fire, we have

$$\$4000 = 40 \times \$100,$$

$$40 \times \$1.20 = \$48, \text{ the premium.}$$

Premium = number of hundreds in face × rate per \$100.

This relation is usually expressed briefly, as follows:

Premium = face × rate.

If possible, bring an old fire-insurance policy to school and show it to the class.

FIRE-INSURANCE PREMIUMS

All work oral

1. At 90¢ a year, what is the annual premium on a policy for \$1000? at 70¢ a year on a policy for \$3000?

2. A man insured his house for \$10,000 at a rate of \$2.50 for 3 yr. What was the premium?

State the premium on a fire-insurance policy for

3. \$1000 at 60¢.	**7.** \$2750 at \$1.	**11.** \$3000 at \$1.20.
4. \$2000 at 70¢.	**8.** \$3000 at \$2.	**12.** \$1000 at \$2.25.
5. \$2250 at \$1.	**9.** \$7000 at \$2.	**13.** \$10,000 at \$0.80.
6. \$4000 at \$2.	**10.** \$8000 at \$2.	**14.** \$10,000 at \$1.50.

15. What is the usual length of time for which fire insurance is written in your city or town? What is the usual premium per \$100 on the most common type of residential property? What conditions determine the rates of insurance?

Finding the Face of the Policy. The face of a policy is always stated in the policy itself. Even though the problem would hardly arise in business, it is interesting to see that we can easily find the face if we know the premium and rate.

For example, at 97¢ per $100, what is the face of a fire-insurance policy when the premium is $24.25?

$$\$0.97\overline{)\$24.25} \qquad \$97\overline{)\$2425}$$

$$\begin{array}{r} 25 \\ \$97\overline{)\$2425} \\ \underline{194} \\ 485 \\ \underline{485} \end{array}$$

Since 97¢ buys $100 of insurance, the face is as many times $100 as 97¢ is contained in $24.25; that is,

$$\$24.25 \div \$0.97 = 25, \text{ the number of hundreds,}$$

and

$$25 \times \$100 = \$2500, \text{ the face.}$$

That is, briefly stated, we have the following relation:

Face = premium ÷ rate.

FACE OF THE POLICY

Find the face of a fire-insurance policy, given the premium paid and the rate per $100 as follows:

1. $17.60, 80¢.
2. $28.80, 90¢.
3. $137.50, $1.
4. $59.50, 85¢.
5. $76.95, 95¢.
6. $69, 75¢.
7. $81.78, 87¢.
8. $69, 92¢.
9. $100.80, $1.12.
10. $85.40, 70¢.
11. $81.78, 94¢.
12. $80.36, 98¢.
13. $80.36, 82¢.
14. $80.36, $1.64.
15. $160.72, $1.64.
16. $160.72, 82¢.
17. $101.50, 70¢.
18. $101.50, $1.45.
19. $34.20, 76¢.
20. $62.70, $1.14.
21. $72.80, $1.12.
22. $145, $1.16.
23. $146.25, $1.17.
24. $169.40, $1.21.
25. $356.70, $1.23.
26. $281.25, $1.25.
27. $233.45, $1.15.

Finding the Rate. If it is necessary, we can easily find the rate when the face of a policy and the premium are known.

For example, if the face of a policy is \$8500 and the premium is \$93.50, what is the rate?

Since $\$8500 = 85 \times \$100$,

we see that each \$100 of insurance costs $\frac{1}{85}$ of \$93.50; that is, as shown in the work at the right, we have

$$\$93.50 \div 85 = \$1.10, \text{ the rate.}$$

$$\begin{array}{r} \$1.10 \\ 85\overline{)\$93.50} \\ \underline{85} \\ 85 \\ \underline{85} \end{array}$$

This shows us that to find the rate, we may divide the premium by the number of hundreds of insurance represented by the face of the policy; that is, stated briefly, we have the following relation:

$$\textit{Rate} = \textit{premium} \div \textit{face}.$$

FINDING THE RATE

Find the rate per \$100, given the face of a fire-insurance policy and the premium as follows:

1. \$2000, \$18.	8. \$7000, \$77.	15. \$4000, \$48.
2. \$2000, \$14.	9. \$7500, \$75.	16. \$4500, \$49.50.
3. \$3000, \$24.	10. \$8000, \$100.	17. \$5700, \$62.70.
4. \$4000, \$34.	11. \$8000, \$120.	18. \$7800, \$93.60.
5. \$5000, \$35.	12. \$9000, \$90.	19. \$4200, \$54.60.
6. \$5000, \$45.	13. \$10,000, \$125.	20. \$8400, \$109.20.
7. \$6000, \$60.	14. \$12,000, \$150.	21. \$16,800, \$218.40.

22. Two neighbors were discussing fire insurance. One, who lived in a brick house, said that he paid a premium of \$21.60 for a policy for \$12,000. The other, who lived in a wooden house, paid \$19.80 for a policy for \$9000. Which paid the higher rate? Why are the rates different?

FIRE INSURANCE

1. At 98¢, what is the premium on a policy for $4000?

2. At $1.20, what is the premium on a policy for $6000?

3. At $1.25, what is the premium on a policy for $8000?

4. A building worth $15,000 is insured for $\frac{2}{3}$ of its value at $1.20. What is the premium?

5. If a man insures the contents of his store for $9000 at $1.25, what is the premium?

6. The premium at $1.50 for insuring a stock of goods is $105. What is the face of the policy?

7. A man insured his factory, valued at $225,000, for $\frac{4}{5}$ of its value. At $1.80, what was the premium?

8. A dealer insured his stock of goods, valued at $24,000, for $\frac{7}{8}$ of its value. At $1.45, what was the premium?

9. If a three-year policy for $3600 costs $59.40, what is the rate for the three years?

10. If the rate for 3 yr. is $2\frac{1}{2}$ times the annual rate, what is the annual rate in Ex. 9?

11. If a 3-year policy for $6000 costs $72, what is the rate for the 3 yr.? As in Ex. 10, what is the rate at which a 1-year policy would be written?

12. The premium paid by a manufacturer who insured his factory for $\frac{5}{8}$ of its value was $420. At a rate of $2, what was the face of the policy? What was the value of the factory?

13. A factory worth $42,000 is insured for 80% of its value, at $2. If fire caused a total loss, how much would the owner lose, including the premium paid?

14. A man insures his house for its full value, $5000, at $1.25. What is the premium? If fire causes a loss of 60% of the value, how much does he receive from the company?

Life Insurance. If a man has a family or others dependent upon him, he may provide protection for them, in case of his death, by insuring his life. Then, if he should die while the policy is in force, the insurance company will pay the face of the policy to the *beneficiary* that he has named.

The earlier a person takes out life insurance, the lower will be the premium, and the sooner will he cease to make payments if the policy calls for a definite number of payments. Every young person should consider taking out a life-insurance policy in some first-class company, because such insurance is valuable not only in providing protection for families but also in making provision for old age when a person's earning power is usually decreased.

Rates in life insurance are stated at so much per year for each $1000 of insurance. They vary with the age of the person insured, reckoned at the time he takes out the policy. The premiums are payable in advance.

LIFE INSURANCE

Find the premium on a life-insurance policy, given the face and the rate per $1000 as follows:

1. $2000, $27.10.
2. $3000, $25.64.
3. $2000, $21.80.
4. $4000, $32.45.
5. $3000, $18.72.
6. $5000, $28.64.
7. $6000, $31.65.
8. $8000, $27.30.
9. $15,000, $28.40.
10. $25,000, $22.20.
11. $35,000, $27.65.
12. $65,000, $24.12.

Find the face for a premium and rate per $1000 of

13. $262, 26.20.
14. $220, $55.
15. $225.40, $32.20.

Find the rate per $1000 for a face and premium of

16. $2000, $60.24.
17. $4000, $96.36.
18. $5000, $158.25.

A SILENT READING LESSON

The chief characteristics of the four leading kinds of life-insurance policies are as follows:

In an *ordinary life* policy the person insured agrees to pay during his lifetime a certain annual premium. Premiums may be paid also semiannually or quarterly, but usually there is a little saving by paying only once a year. The face of such a policy is not payable until the death of the one insured, and the payment is then made to the beneficiary.

In a *limited-payment life* policy the premiums are payable for a fixed number of years, often twenty. At the end of the specified time the policy is said to be *paid up*, but the face is not paid until the death of the person insured.

In an *endowment* policy the premiums are paid for a fixed number of years, often forty, thirty, or twenty. At the end of this period, the face is paid to the person insured, either all at once or in installments. The premiums on limited-payment life and endowment policies are naturally higher than those on ordinary life policies.

In a *term* policy the premiums are paid for a specified length of time, but the face of the policy is payable only if the person insured dies within the specified period. Such policies do not usually run for more than five or ten years.

There are various other forms of policies. Some are arranged so that a certain amount will be paid annually (an *annuity*) during the lifetime of the one for whose benefit the policy is written.

A man may also insure his health for a definite period or he may insure himself against death or injury by accident. The first of these forms of insurance is called *health insurance*; the second, *accident insurance*. The principle of accident and health insurance is substantially like that of life insurance.

LIFE INSURANCE

1. At $31.05 per $1000, how much is the annual premium that a man must pay on a 30-year endowment policy for $8000? How much would he pay in the 30 yr., not considering the question of interest on the several payments?

2. If a man takes out a 20-payment life policy for $10,000 at $27.24, how much, not considering interest, will he have paid when the policy matures; that is, after 20 payments?

3. How much is the annual premium on a 20-payment life policy for $14,000 if taken out by a man at such an age that the rate is $29.95? Find the total amount of the premiums for the 20 yr., not considering interest.

4. A man takes out a policy for $2500 in one company, at $24.12. If the next year he takes out a policy for $3500 in another company, at $24.89, how much does he then pay each year for life insurance?

5. At a rate of $29.53, a young man takes out an endowment policy for $10,000 on the 30-payment plan. How much is the annual premium? How much does he pay in premiums during the 30 yr., not considering interest on the payments? How much more than he has paid does he receive from the company when the policy matures?

6. A man took out a policy for $5000 at $26.40. He died just before the eighth annual payment was due. How much more did the beneficiary receive than the man had paid the company in premiums?

7. Twenty years ago a man took out a 20-payment life policy for $5000 at $25.41. How much has he paid in the 20 yr.? At his present age the rate is $38.51. If he takes out a new policy for $5000, how much more will he have to pay in 20 yr. on this policy than on the first one?

A SILENT READING LESSON

We all owe a great deal to our parents, to our school, to the state in which we live, and to the United States. Each one of our several types of government, national, state, city, and town, spends large sums of money each year for educating us, erecting schoolhouses, protecting lives and property, building roads, and keeping public buildings in repair, — all of which help to make the conditions of our life such as to give us a chance to succeed in the world.

In return we owe it to the state to help support it and to make it strong and safe not merely for ourselves but for those who come after us.

One of the ways in which we can help in supporting the state is through the payment of *taxes*. To illustrate what a tax is, suppose that a class wishes to buy a picture for the schoolroom. There are 30 pupils in the class, and the picture that they wish to buy costs $4.20. Each pupil agrees to pay his share. The committee which is appointed to raise the money finds that the share of each member of the class is 14¢; that is, the total amount to be raised by the class is $4.20, and the tax on each pupil is 14¢. The committee appoints a *collector* to receive the money, and with the money which he collects the committee buys the picture.

Another way to support the state is to see, by taking an interest in politics, that the best men and women are elected to office, and so bring it about that the money raised by taxes is spent wisely. Experience has shown that it is a wise thing for every person, at the beginning of each year, to make out a budget in which he puts down his probable income and how he will spend it. It is now being generally recognized that it is desirable that the state and the nation should also budget their receipts and expenditures.

Taxes in a Village. Suppose that a village needs $75,000 for building a new schoolhouse, and its citizens vote to raise the amount by taxes. To raise so large a sum by taxing each citizen equally would be too heavy a burden upon some members of the community. Each citizen, therefore, is required to pay according to the value of his property; for example, a man who owns $10,000 worth of property pays twice as much as one who owns $5000 worth.

Taxes in a City. It takes a large amount of money to pay all the expenses of a city. The streets have to be cleaned, lighted, and kept in repair, the policemen have to be paid, the fire department has to be kept up, the city buildings have to be cared for, and the schools must be supported. Besides these there are many other expenses, and all of them must be met by money raised by taxes.

Taxes in the County, State, or Nation. In the same way, money is needed to pay the expenses of the county, the state, and the nation. All this money must be raised by taxes of some kind.

Tax Rate. Officials, called *assessors*, are generally elected or appointed to place a value, called the *assessed valuation*, on all property, such as land, buildings, automobiles, and other things, subject to taxation. After the total assessed valuation and the amount to be raised for all expenses is determined, it is a simple matter to determine the *tax rate*.

For example, if a city with a total assessed valuation of $2,500,000,000 must raise $75,000,000 to pay the year's expenses, each citizen should pay an amount equal to $75,000,000 ÷ 2,500,000,000, or $0.03, on each $1 worth of property that he owns. This rate of 3¢ on $1 may be expressed as 30 mills on $1, as $3 on $100, or as $30 on $1000, the last being the most common way.

Finding the Tax Rate. Suppose that a town with a total assessed valuation of \$6,400,000 must raise \$118,400 to meet the running expenses of the schools this year. What will be the school tax rate this year?

Since the tax on \$6,400,000 is \$118,400, the tax on \$1 is found by dividing \$118,400 by 6,400,000, as here shown. That is, to find the tax rate, we divide the amount to be raised by the number of dollars of assessed valuation, or briefly stated,

$$\begin{array}{r} \$0.0185 \\ 6400000\overline{)\$1.18400} \\ \underline{64} \\ 544 \\ \underline{512} \\ 320 \\ \underline{320} \end{array}$$

Rate = total to be raised ÷ assessed valuation.

The above result, \$0.0185, may be expressed as a tax rate of $18\frac{1}{2}$ mills on \$1, as \$1.85 on \$100, or as \$18.50 on \$1000.

Finding the Amount of the Tax Bill. It is now a simple matter to find the tax that each property owner must pay. For example, if the rate is \$18.50 per \$1000, in finding the amount paid on property assessed at \$12,000, we have

$$\$12{,}000 = 12 \times \$1000,$$

and $$12 \times \$18.50 = \$222.00, \text{ amount of the tax.}$$

That is, the amount of the tax is the number of thousands of assessed valuation (or the number of hundreds at a given tax rate per \$100) times the rate per \$1000 (or per \$100), or briefly expressed,

Amount of tax = assessed valuation × rate.

Finding the Assessed Valuation. It is also an easy matter to find the assessed valuation of a piece of property when we know the rate and the amount of taxes paid. That is, from the last relation, given above, we see that

Assessed valuation = amount of tax ÷ rate.

PROPERTY TAXES

1. At \$2.18 on \$100, find the tax on a valuation of \$8000.

Find the amount of the tax bill when the assessed valuation and the rate are as follows:

2. \$8500, \$1.46 on \$100.
3. \$9000, \$2.10 on \$100.
4. \$16,500, \$15.80 on \$1000.
5. \$25,000, \$17.60 on \$1000.

6. The assessed valuation of a city is \$75,000,000, and the total amount to be raised is \$1,845,000. Find the tax rate, expressing it as so many dollars and cents on each \$1000.

7. At a tax rate of 19.6 mills on \$1, what is the tax on property assessed at \$10,000?

8. At a tax rate of \$2.20 on \$100, what is the tax on property assessed at \$25,000?

9. At a tax rate of \$16.30 on \$1000, what is the tax on property assessed at \$12,500?

Find the tax at \$21.60 on \$1000 on each of these valuations:

10. \$4500.
11. \$6500.
12. \$8000.
13. \$4600.
14. \$7000.
15. \$4500.
16. \$8700.
17. \$9700.
18. \$6750.
19. \$8500.
20. \$12,000.
21. \$25,000.
22. \$32,500.
23. \$38,500.
24. \$45,000.
25. \$75,000.

26. A man died, leaving an estate valued at \$23,750, of which \$22,000 was willed to his son and the rest to a nephew. The son did not have to pay any state inheritance tax on the first \$5000, but paid 1% on the rest, and the nephew paid 5% on all he received. Find the total inheritance tax paid.

27. If a man's tax is \$195, and the rate is \$13 on \$1000, what is the assessed valuation of his property? What is the assessed valuation of property which is taxed \$234?

Collectors' Fees. A collector of taxes usually receives a fixed salary for his work. In some places he receives a certain per cent on the taxes collected, which is added to the taxpayer's bill in advance.

For example, if the collector's fee is 1% of the tax, and if a man's property is assessed at $8000 in a place where the tax rate is $18.50 per $1000, the total amount that the taxpayer must pay is found as here shown; that is, he must pay $149.48.

Tax per $1000,	$18.50
Number of thousands,	8
Amount of tax,	$148.00
	0.01
Collector's fee,	$1.48ØØ
Amount of tax,	$148.00
Total amount,	$149.48

COLLECTING TAXES

When the collector is paid a salary, find the amount of the tax bill for a valuation and rate per $100 as follows:

1. $4000, $1.32.
2. $4500, $1.37.
3. $5200, $1.50.
4. $5500, $1.54.
5. $7000, $1.76.
6. $8000, $2.29.
7. $8500, $2.31.
8. $2700, $1.93.
9. $2500, $2.37.
10. $4500, $2.40.
11. $9000, $2.32.
12. $9500, $2.48.
13. $12,000, $1.87.
14. $12,500, $2.90.
15. $15,000, $2.45.

Find the amount of the tax bill, when the valuation, the rate per $1000, and the collector's fee are as follows:

16. $10,000, $12.60, 1%.
17. $15,000, $13.70, 1%.
18. $37,000, $13.20, 1%.
19. $47,000, $14.90, 1%.
20. $16,500, $18.70, $1\frac{1}{4}$%.
21. $17,500, $21.60, $1\frac{1}{2}$%.
22. $25,000, $23.40, $1\frac{1}{2}$%.
23. $27,500, $28.80, $1\frac{1}{2}$%.

Tax Table. To assist in the work of making out the tax bills, after the rate has been fixed, a *tax table* similar to the one below is often prepared. The following table, which is only a small specimen, is arranged for a tax rate of $1.78 on $100 (or of 17.8 mills on a dollar, or of $17.80 on $1000):

	0	1	2	3	4	5	6	7	8	9
0	0.00	0.178	0.356	0.534	0.712	0.890	1.068	1.246	1.424	1.602
1	1.78	1.958	2.136	2.314	2.492	2.670	2.848	3.026	3.204	3.382
2	3.56	3.738	3.916	4.094	4.272	4.450	4.628	4.806	4.984	5.162
3	5.34	5.518	5.696	5.874	6.052	6.230	6.408	6.586	6.764	6.942
4	7.12	7.298	7.476	7.654	7.832	8.010	8.188	8.366	8.544	8.722
5	8.90	9.078	9.256	9.434	9.612	9.790	9.968	10.146	10.324	10.502
6	10.68	10.858	11.036	11.214	11.392	11.570	11.748	11.926	12.104	12.282
7	12.46	12.638	12.816	12.994	13.172	13.350	13.528	13.706	13.884	14.062
8	14.24	14.418	14.596	14.774	14.952	15.130	15.308	15.486	15.664	15.842
9	16.02	16.198	16.376	16.554	16.732	16.910	17.088	17.266	17.444	17.622

In this table the hundreds' figure of the assessed valuation is shown in the column at the left, and the tens' figure at the top. Thus, at $1.78 on $100, the tax on $300 is $5.34; on $140 it is $2.49 (2.492); and on $90 it is $1.60 (1.602).

Using the Table. 1. At $1.78 per $100, how much tax must a man pay on property assessed at $8500?

We proceed as if we wished to find the tax on $850. In the table we look in the first column for **8**, the first figure in $850, and then follow along this line to the column headed **5**, the second figure in $850. We there find the number 15.130, which means that the tax on $850 is $15.13. Therefore, on $8500, which is $10 \times \$850$, the tax is $151.30.

2. At $17.80 on $1000, how much tax must a man pay on property assessed at $17,500?

The tax on $17,000 is $302.60 (that is, it is 100 times the tax on $170, or $100 \times \$3.026$), and the tax on $500 is $8.90. Hence the tax on $17,500 is $\$302.60 + \$8.90$, or $311.50.

USING THE TAX TABLE ON PAGE 76

1. At a tax rate of \$1.78 on \$100, what is the amount of the tax on property assessed at \$6000? at \$7000? at \$7500? at \$8600? at \$9800?

2. At a tax rate of \$17.80 on \$1000, what is the amount of the tax on property assessed at \$10,000? at \$12,000? at \$16,000? at \$16,500? at \$25,600?

3. At a tax rate of 17.8 mills on \$1, what is the amount of the tax on property assessed at \$7800?

4. At a tax rate of \$0.0178 on \$1, what is the amount of the tax on property assessed at \$22,800?

At a tax rate of \$17.80 on \$1000, find the taxes on property assessed at the following amounts:

5. \$9300.	10. \$15,000.	15. \$55,000.	20. \$125,000.
6. \$6700.	11. \$22,500.	16. \$75,000.	21. \$675,000.
7. \$8000.	12. \$25,000.	17. \$75,500.	22. \$275,000.
8. \$9000.	13. \$32,500.	18. \$80,000.	23. \$350,000.
9. \$8500.	14. \$42,500.	19. \$87,500.	24. \$875,000.

25. Taxes in a certain city are payable in semiannual installments on May 1 and Nov. 1. If the whole tax is paid on May 1, a discount of half a year's interest at the rate of 4% a year is allowed on the second payment. If the rate is 17.8 mills on \$1, and if the whole tax is paid on May 1, how much is due on a piece of property assessed at \$3000?

26. A company owns four parcels of land in four different cities. One parcel is assessed at \$35,000, the second at \$8000, a third at \$20,000, and the fourth at \$40,000. If the tax rates in these cities are, respectively, \$21.40, \$17.80, \$22.60, and \$19.90 on \$1000, what is the total of the company's taxes?

A SILENT READING LESSON

In order to make sure that all money collected for the village, city, county, state, or nation goes to the government for which it is intended, tax collectors have to *give bonds* that they will pay to the proper officers all money collected. This generally means that some company will insure the village, county, and so on against loss on the part of the collector. In the same way, the treasurer or any other officer through whose hands the money passes must give bonds for the honest and careful discharge of his duty.

A man may be perfectly honest, but he might lose the government's money through carelessness or through lack of ability to handle large sums. Against such dangers the government must protect itself.

In order to be able to discuss the question of local taxes in class, we should find out, if possible, what the property tax rate is in our community, what the chief expenses are, how our rate differs from some other rates in the state, and what the reasons are for this difference. At the same time we can find out what officials have to give bonds.

Part of the money raised by some states is collected through taxes on people's incomes. The general principle of such taxes is that those who receive large incomes during a year pay more taxes than those who receive less. In a recent year the rate of income tax in one state was 1% on the first $10,000, 2% on the next $40,000, and 3% on larger incomes. A man whose taxable income was $30,000 a year had, at that time, to pay 1% of $10,000, or $100, and 2% of $20,000, or $400, making a total of $500.

Much of the income of the United States government comes from taxes of this kind, the rates for which are higher than those of the state mentioned above.

TAXING FOR GOOD ROADS

1. The highway tax, which was included in the county tax, on a certain farm for four successive years was as follows:

Year	Assessed Valuation		Highway Tax Rate per $1000
	Land	Buildings	
1927	$4600	$1300	$1.56
1928	4600	1300	1.78
1929	5000	1450	2.32
1930	5000	1450	1.96

The increase in the valuation of the land and buildings was due to the construction of good roads. If the roads had not been improved, and the valuation of the farm had remained unchanged, what would the total highway tax have been for the four years at the 1927 rate? How much would the owner of the farm have saved on his highway taxes?

2. At the end of 1930, after paying the year's taxes, the owner sold the farm for its assessed valuation in that year as given in Ex. 1. If the assessed valuation in 1927 represented fairly the value of the farm on that date, and if we consider the increase in value as due wholly to the building of good roads, did the owner gain or lose by their construction? Determine the gain or loss.

3. In the same county in which the farm in Ex. 1 was located, it cost $24,000 to maintain 60 mi. of macadam road and $1000 for the maintenance of 55 mi. of concrete road. Find the cost per mile of maintaining each type of road.

4. In Ex. 3, the cost of maintaining a mile of the macadam road was what per cent greater than the cost of maintaining a mile of the concrete road?

A SILENT READING LESSON

The expenses of the United States government naturally vary from year to year. The enormous cost of the World War made a corresponding increase in the annual expenses, because much of the national income must be devoted to paying for the war. But however large they may be, it is our duty to raise the money to pay the debts and expenses of the government. If the governments of the world would agree not to make war in the future, the cost of government could be greatly reduced throughout the world.

Recently the expenditures of our government averaged about $3,800,000,000 a year, or about $10,000,000 a day, but they are now being reduced from year to year. Some of the national receipts and expenditures were recently as follows:

RECEIPTS

Customs (duties on imported goods)	$546,000,000
Internal revenue (income tax, and so on)	2,800,000,000
Miscellaneous	666,000,000
Total receipts	$4,012,000,000

EXPENDITURES

War Department	$341,000,000
Navy Department	324,000,000
Pensions	230,000,000
Veterans' Bureau	101,000,000
Interest on the public debt	940,000,000
Other money appropriated by Congress	1,892,000,000
Total appropriations	$3,828,000,000

We can find out the receipts and expenditures for recent years by consulting one of the large almanacs, and can then discuss in class the reasons for such enormous expenditures.

Tariff. The United States collects a large part of its income by a tax on goods brought into the country. This income is called *customs revenue, tariff,* or *duty.* Duties are collected at *customhouses,* which are situated at important harbors on the coasts or important places on the Canadian and Mexican borders. Such places are called *ports of entry.*

Classes of Goods. Goods imported may be on the *free list*; that is, not subject to duty, as in the case of raw silk. They may be subject to *ad valorem* duty; that is, to a certain per cent on the value, as in the case of toys on which the duty specified in the present tariff law is 70% ad valorem. They may be subject to *specific* duty; that is, to a certain amount per unit of measure, as in the case of sheep on which the duty is $2 a head. They may be subject also to both ad valorem and specific duty, as in the case of perfumery on which the duty is 40¢ per pound and 75% ad valorem.

DUTIES ON IMPORTS

Numbers 1 to 10, oral

State the duty in each of the following cases:

1. At 1¢ per pound on 800 lb. of lard; on 8000 lb.
2. At 5¢ per pound on 100 lb. of cheese; on 1000 lb.
3. At 30% on $1000 worth of baseballs; on $1200 worth.
4. At 50% on straw hats worth $1000; worth $8000.
5. At 70% on dolls worth $1000; worth $3000.
6. At 50% on fur coats worth $1500; worth $20,000.
7. At $5 per dozen pairs on 450 doz. pairs of gloves.
8. At 40% on a shipment of violins worth $8000.
9. At 20% on a string of pearls worth $1250.
10. At 40% on a shipment of umbrellas worth $4000.

11. How much is the duty at 55% on a shipment of silk handkerchiefs worth $6500? Is this a specific duty or an ad valorem duty?

12. How much is the duty on 3500 bu. of barley at 20¢ per bushel? Is this a specific duty or an ad valorem duty?

13. At 24¢ a pound, find the duty on 2750 lb. of hops; on 12,500 lb.; on $2\frac{1}{2}$ T.

14. At 8¢ a pound, how much is the duty on 350 lb. of butter? on 1250 lb.? on $2\frac{1}{2}$ T.?

15. At 15¢ a bushel, how much is the duty on 8750 bu. of oats? on 9500 bu.? on 12,500 bu.?

16. A dealer imports 1500 lb. of maple sugar from Canada. How much is the duty at 4¢ per pound?

17. A wholesale grocery house imports from Canada 900 lb. of cheese. How much is the duty at 5¢ a pound?

18. A dealer imports 4750 lb. of butter. How much is the duty at 8¢ per pound?

19. The duty on honey is 3¢ a pound. If a dealer imports 1250 lb., how much is the duty?

20. A dealer imported 1800 lb. of raisins, paying duty at the rate of 2¢ per pound. How much duty was paid?

21. How much is the duty at 35% on a shipment of thread which is valued at $8750?

22. How much is the duty at 30% on a shipment of napkins which is valued at $4520?

23. How much is the duty at 40% on 2000 yd. of carpet valued at $2.50 a yard?

24. The duty on cigars is $4.50 per pound and 25% ad valorem. If a shipment of cigars weighs 550 lb. and is valued at $1650, how much is the total duty?

A SILENT READING LESSON

The largest part of the money needed to run the United States government is raised by a tax on incomes. The chief features of this form of taxation are as follows:

Each year every resident of the United States, whether a citizen or an alien, who has an income above a specified amount must pay the government a certain per cent of his income after deducting the exemptions that are allowed.

Persons whose incomes are large have to pay more, the rate of taxation increasing as the income increases. Certain types of business concerns also have to pay income taxes.

For example, suppose that a man's salary is $5000 in a year when the income-tax law specifies that $2500 of this is exempt from taxation on account of his being the head of a family, that the rate of taxation on the balance is 2%, and that he may deduct a credit of 25% because his income is earned instead of being derived from invested funds. His total income tax, $37.50, is then found as shown in the work at the right.

Salary,	$5000
Exempt,	−2500
	$2500
Rate,	0.02
	$50.00
Less $\frac{1}{4}$,	12.50
Total tax,	$37.50

The details of the income tax are so complex, owing to the great number of different rates and to frequent changes in the laws, that we cannot take them up in school. We ought, however, to understand in a general way about such taxes, and if more information is desired we can find out the details for the current year at any bank.

A considerable part of our national revenue comes from taxes on tobacco. These, together with the income tax and certain other taxes, constitute the *internal revenue.*

INTERNAL REVENUE

1. If an unmarried man has an income of \$4475.50, of which \$1000 is exempt from taxation, find his income tax at 2%, deducting 25% for earned income.

2. Solve Ex. 1 for a married man with an income of \$5650.80, of which \$2900 is exempt from taxation.

3. In a year in which the rate was $12\frac{1}{2}$%, a company had to pay a tax on \$374,285. How much was the tax?

4. Mr. Wilson has a salary of \$20,000 and is allowed an exemption of \$2500. If the first \$4000 of the balance is taxable at 2%, the second \$4000 at 4%, and the balance at 6%, what is the amount of the tax before deducting any credit? If he may now deduct a credit of \$55 for earned income, but is obliged to pay an additional tax (called a *surtax*) of \$220 because his income is more than \$10,000, how much is the amount of his income tax?

5. A certain quality of cigar is taxed \$3 per 1000, and an extra tax of \$7 per 1000 is also collected. Find the total tax on an output of 275,000 cigars.

6. A certain quality of cigarette is taxed \$3.60 per 1000, and an extra tax of \$1.20 per 1000 is also collected. Find the total tax on an output of 3,250,000 cigarettes.

7. When tobacco is taxed at the rate of 8¢ per pound, with an extra tax of 5¢ per pound, what is the total tax on an output of 2,758,500 lb. of tobacco?

8. In one year the receipts from a special tax dropped from \$89,000,000 to \$62,000,000. Find the per cent of decrease.

9. When the rates were raised during the World War, the income-tax receipts increased in one year from \$360,000,000 to \$4,000,000,000. Find the per cent of increase.

A SILENT READING LESSON

We deal with the United States government more frequently through the post office than in any other way. Everyone needs to know the following general facts which relate to mailing (1) letters, (2) printed matter, and (3) parcels:

1. The rate for letters to be delivered in the United States, Canada, Great Britain, Ireland, and certain other countries is 2¢ per ounce, all fractions of an ounce being counted as a full ounce.

The rate on letters to countries not having the 2-cent rate is 5¢ for the first ounce and 3¢ for each additional ounce.

For letters sent in the United States by airplane the rate is 8¢, 16¢, or 24¢ an ounce, according to the distance traveled.

On government postal cards the rate to places in this country is 1¢, but on private mailing cards it is 2¢.

2. On newspapers and periodicals sent by individuals the rate to places in this country is 2¢ for each 2 oz. Publishers of newspapers and magazines pay special rates for mailing.

On other printed matter the rate to places in this country or abroad is $1\frac{1}{2}$¢ for each 2 oz., with certain restrictions.

3. To points less than 300 mi. away, parcels weighing not more than 70 lb. may be sent by *parcel post*; for points more than 300 mi. away, the maximum weight allowed is 50 lb. There are various other restrictions, which may be learned on inquiry at any post office. The cost of sending by parcel post varies with the weight and distance, and parcels may be insured against loss by paying a small additional fee.

A piece of mail matter may be *registered* by paying an additional fee, thus insuring greater security, as well as reimbursement, up to a certain amount, in case of loss. *Special delivery* of a letter or parcel can also be obtained by payment of an additional fee.

Parcel Post. In the operation of the parcel post (fourth-class mail matter) the country is divided into zones or rings, the center of which is the place at which the parcel is mailed. Roughly speaking, the first zone includes distances up to 50 mi.; the second, from 50 mi. to 150 mi.; the third, from 150 mi. to 300 mi.; the fourth, from 300 mi. to 600 mi.; the fifth, from 600 mi. to 1000 mi.; the sixth, from 1000 mi. to 1400 mi.; the seventh, from 1400 mi. to 1800 mi.; the eighth, over 1800 mi. The basic rates for small parcels which are to be delivered in the first few zones are at present as follows:

Pounds	Local	Zones 1, 2	Zone 3	Zone 4	Zone 5	Zone 6
1	$0.05	$0.05	$0.06	$0.07	$0.08	$0.09
2	.06	.06	.08	.11	.14	.17
3	.06	.07	.10	.15	.20	.25
4	.07	.08	.12	.19	.26	.33
5	.07	.09	.14	.23	.32	.41
6	.08	.10	.16	.27	.38	.49
7	.08	.11	.18	.31	.44	.57
8	.09	.12	.20	.35	.50	.65
9	.09	.13	.22	.39	.56	.73
10	.10	.14	.24	.43	.62	.81

To the postage on each parcel as determined by the above table, a flat service charge of 2¢ is then added.

Postal Statistics. In a recent year there were about 52,000 post offices in the United States, and the length of the postal routes was over 460,000 mi. The income of the Post Office Department was about $533,000,000, and the expenditures were about $557,000,000, the loss being paid by the government. The number of stamps and postal cards issued was more than 16,000,000,000, which does not include much free and unstamped matter.

THE POST OFFICE

Numbers 1 to 12, oral

State the postage in each of the following cases:

1. A letter to Chicago, weighing $\frac{3}{4}$ oz.; weighing 2 oz.

2. A special-delivery (fee, 10¢) letter to San Francisco, weighing $1\frac{1}{4}$ oz.; weighing 6 oz.

3. A registered (fee, 15¢) letter to Boston, weighing $2\frac{1}{4}$ oz.

4. A registered (fee, 20¢), special-delivery (fee, 10¢) letter to New Orleans, weighing $3\frac{1}{2}$ oz.

5. A registered (fee, 15¢) letter to England, weighing 1 oz.

6. A newspaper to Galveston, weighing 3 oz.

7. A printed pamphlet to Sacramento, weighing 2 oz.

8. A letter weighing $1\frac{3}{4}$ oz., sent to Italy.

9. A letter weighing $2\frac{1}{2}$ oz., sent to Holland.

10. A letter weighing $\frac{3}{4}$ oz., sent to France.

11. A letter weighing $1\frac{1}{2}$ oz., sent by airplane mail to a place such that it takes the cheapest rate; the next higher rate; the highest rate.

12. A special-delivery (fee, 10¢) letter weighing $2\frac{1}{2}$ oz., sent by airplane mail to a place such that it takes the highest rate.

Using the table on page 86, find the postage in each of the following cases, remembering to add the 2-cent service charge:

13. A parcel weighing 6 lb., sent to a place in the fifth zone.

14. A parcel weighing 9 lb., sent to a place in the sixth zone.

15. A parcel weighing 8 lb., sent to a place in the local zone.

16. In a year in which our population was 105,000,000 we used about 15,400,000,000 postage stamps. Find the average number of stamps used per person.

MISCELLANEOUS PROBLEMS

1. A school football team won 7 out of 12 games played during a season. What per cent of the games played did the team win that season?

2. A merchant advertised a discount of 20% for a special sale. How much must you pay at the sale for a table that was originally marked $28?

3. A farmer raised 950 bu. of potatoes one year and 1120 bu. the second year. The second year's crop was what per cent greater than that of the first year?

4. A dealer tells me that my car has decreased 30% in value since it was purchased and is now worth $910. What was the original value of the car?

5. Walter says that 6% of 104 is more than $4\frac{1}{2}$% of 108; Tom says that it is not. Which one is right?

6. A farmer delivered to a creamery 420 lb. of milk, which tested 3.8% butter fat. If he was paid 34¢ a pound for the butter fat, how much money did he receive?

7. A class voted to give to a children's hospital 75% of the amount that was cleared at a school play. If the class cleared $86.40, how much was given to the hospital?

8. If you weighed 86 lb. when school was over last June, and if you gained 5% in weight during the summer vacation, how much did you weigh in the fall?

9. In dressing a calf for market 40% of the live weight of the animal is lost. A calf that weighs 120 lb., live weight, should weigh how much less when dressed for market?

10. A city which was asked to subscribe $35,000 for a certain charity raised $38,225. By what per cent did the amount raised exceed the amount requested?

IV. REVIEW AND DRILL

MINIMUM ESSENTIALS

1. Write in words the numbers 400,075, 40.0075, 4000.75, 0.475, and 0.00475.

Add, without writing in columns:

2. $\$8.20 + \$5.50 + \$7.75 + \$6.73 + \$4.30 + \$25.50$.

3. 7 ft. 3 in. + 8 ft. 6 in. + 9 ft. 8 in. + 2 ft. 3 in. + 3 ft. 5 in.

Subtract as follows:

4. $3\frac{3}{4} - 2\frac{15}{16}$. **5.** $\$890.60 - \$389.96$. **6.** $\$710.30 - \$578.28$.

Multiply or divide as indicated, giving the results of the divisions to the nearest 0.01:

7. $386 \times \$976.50$. **8.** $1150.6 \div 29.4$. **9.** $94.79 \div 92.6$.

Find the discount on a bill for

10. \$275 at 9%. **11.** \$480 at $2\frac{1}{2}$%. **12.** \$800 at $6\frac{3}{4}$%.

Find the value in each of the following cases:

13. 27% of \$625. **14.** $3\frac{1}{4}$% of \$550. **15.** $4\frac{1}{2}$% of \$985.

Find the commission on the sale of property for

16. \$275 at 2%. **18.** \$220 at 3%. **20.** \$8000 at 5%.

17. \$350 at $4\frac{1}{2}$%. **19.** \$380 at $3\frac{1}{2}$%. **21.** \$9500 at $5\frac{1}{2}$%.

22. Find the annual premium on a life-insurance policy for \$3000 at \$24.50 per \$1000.

Find the amount received at a sale of property when the cost and the gain or loss are as follows:

23. \$8500; gain, 8%.

24. \$7500; loss, 12%.

25. \$16,750; gain, $12\frac{1}{2}$%.

26. \$27,600; loss, $16\frac{2}{3}$%.

PROBLEMS WITHOUT NUMBERS

All work oral

1. If you buy a catcher's mitt and sell it at a certain per cent below cost, how do you find the selling price? How do you find the loss?

2. If you buy a dog and sell him at a certain per cent above what you paid for him, and know how much it has cost you to keep him while you owned him, how do you find whether you gained or lost, and the amount?

3. If you know the number of times a baseball player has been at bat during a season, and the number of hits he has made, how do you find the per cent of hits?

4. If you know the number of times a baseball player has been at bat during a season, and the per cent of runs he has made, how do you find the number of runs?

5. If you know the number of pupils in a school and the number in your room, how do you find the per cent that are in your room?

6. If you know a man's income and the amount which he pays for house rent, how do you find the per cent of his income that he pays for this purpose?

7. If you know the list price of an article which you wish to purchase, and if you are allowed certain discounts, what are you then able to find and how do you find it?

8. If you know the assessed valuation of a house and the tax rate, how do you find the amount of the tax bill?

9. If you know the face of a life-insurance policy and the rate per thousand, how do you find the annual premium?

10. Consider Ex. 9 for a fire-insurance policy at a known rate per hundred.

PROBLEMS FOR COMPLETION

1. A man buys a second-hand automobile for $520. After paying $140 for repairs, he is offered $750 for the car. Complete the problem in any reasonable way and solve it.

Complete and solve the following problems:

2. An aviator covered the distance from Cleveland to Bellefonte, Pa., which is 215 mi., in 83 min.

3. In an automobile race one car made 1.4 mi. in 51 sec., and another car made 1.7 mi. in 54 sec.

4. In a state where the legal weight of a bushel of sweet potatoes is 54 lb., a commission merchant sells 4565 lb.

5. Mr. Howie's wages were $32 a week, but have now been raised to $34.56 a week.

6. Carl is multiplying 15 by a certain number and gets 100 for the result.

7. An automobile driver notices that his speedometer has registered 4.8 mi. in 10 min. He is going to a place that is 36 mi. farther on and expects to keep up the same rate.

8. John finds that, on the average, he covers 26 in. at a step. He lives $\frac{3}{4}$ mi. from his school, and goes 5 da. a week.

9. A man has received a bill for goods amounting to $720 and is allowed a discount of 2% for prompt payment.

10. A sales manager tells a salesman that, with a commission of $4\frac{1}{2}\%$, he ought to make $45 a week.

11. In a year when the tax rate is $1.20 per $100, a man pays a tax of $102 on his house.

12. A married man with a salary of $4200 a year is allowed an exemption of $2500. The income-tax rate is 2%, less a deduction of 25% for earned income.

Testing Your Alertness. In arithmetic you must be alert. To find how alert you really are, you will be asked to copy rows of figures like the ones below and to follow the directions that are given you.

The teacher will read three sets of directions that will be something like the following:

1. In the first row write 3 below each figure containing a curve line, write 7 above each one containing a right angle, and write 8 inside each one containing an acute angle.

2. In the second row draw lines above the squares in such a way that the lines will connect every pair of squares containing two numbers whose sum is 9, and then draw lines below the squares in such a way that these lines will connect every pair of squares containing two numbers whose sum is less than 7.

3. In the third row draw a line that will pass above every circle whose number is divisible by 2, and below every circle whose number is not divisible by 2. Put the letter X below every circle whose number is divisible by 3.

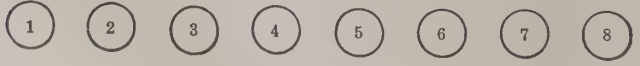
You will have to work quickly, and your scores will be kept to see if you improve in later trials.

ALERTNESS TEST

Copy the following figures and, at a signal given by the teacher, do as you are directed, allowing 30 sec. for each row:

1.

2.

3. 

MONEY IN FERTILIZERS

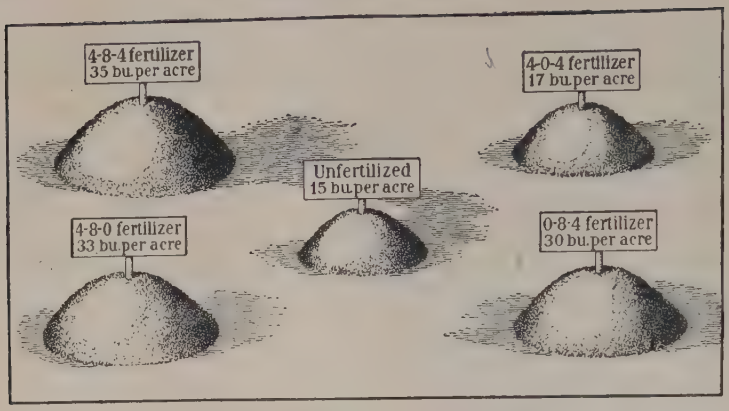
1. George's big brother came home from the agricultural college and decided that on their farm they ought to try more modern methods of feeding the plants. He told George that plants need nitrogen, phosphoric acid, and potash, and that if these are put into the soil by means of fertilizers, the plants grow better. George's brother tried applying fertilizer, costing \$5.16, to an acre of corn, and the increase in the yield was 8.6 bu. At 90¢ a bushel at the farm, what was the value of the increase? The value of the increase was how much more than the cost of the fertilizer?

2. George's brother tried another kind of fertilizer, costing \$7.64, on another acre of corn, and increased the yield by 7.2 bu. At 90¢ a bushel at the farm, was this experiment profitable or not? Find the gain or loss.

3. George's brother knew of another fertilizer, said to be excellent for wheat. They put some of this kind on an acre at a cost of \$11.26. The increase in yield was 8.4 bu. At \$1.05 a bushel, what was the gain or loss in this case?

4. The man who lived on the farm next to theirs used another kind of fertilizer for his wheat. He said that it was better than the kind in Ex. 3. George's brother tried it on an acre at a cost of \$9.75, and the increase in yield was 9.6 bu. At \$1.05 a bushel, how much was the gain or loss in this case? Was this fertilizer better than the kind in Ex. 3? How much more money per acre would this fertilizer return?

5. George's brother explained to him that a complete fertilizer is one containing all three of the plant foods, nitrogen, phosphoric acid, and potash. A 2–8–2 fertilizer is one that contains 2% of nitrogen, 8% of phosphoric acid, and 2% of potash. How many pounds are there of each of these foods in a ton of 2–8–2 fertilizer?



6. George's brother showed him the above picture, which shows the yields of wheat resulting from experiments with different fertilizers, and asked George to find how many pounds of nitrogen, phosphoric acid, and potash there are in 300 lb. (the amount used per acre) of the fertilizer that gave the 35-bushel yield. How many pounds are there of each?

7. In the same picture, what plant food is omitted in the fertilizer which gave the 17-bushel yield? How many pounds of each of the other plant foods are there in 300 lb. of this fertilizer? in a ton of this fertilizer?

8. In the same picture, what plant food is omitted in the fertilizer which gave the 33-bushel yield? in the fertilizer which gave the 30-bushel yield? How many pounds of each of the other plant foods are there in 300 lb. of each fertilizer? in a ton of each fertilizer?

9. Perhaps you know some practical farmer who can tell you something about the subject of fertilizers. If so, make a report to the class about it. If you live in a city, you may not know much about farming, but these problems will give you an idea of how plants have to be fed.

A SCHOOL-GARDEN PROJECT

1. For a school-garden project the teacher of a country school rented a piece of land 20 rd. long and 16 rd. wide, at a rental of $12.50 an acre. The length of the garden ran east and west. She divided the entire piece into ten plots, beginning at the northwest corner, and assigned these plots as follows: to John, a strip 20 rd. long and 3 rd. wide; to William, Clarence, and Roger, each a strip 20 rd. long and 2 rd. wide; to Mary, Susan, Ella, and Ann, each a piece 5 rd. square; to Henry and Frank, each a strip 10 rd. long and 2 rd. wide. Draw a plan of the land with its divisions, using the scale of 1 in. to 4 rd.

2. The teacher paid $4.10 for having the whole piece plowed, and $2.50 a load for 5 loads of stable fertilizer. How much did the rent, plowing, and fertilizer cost?

3. From the result of Ex. 2 find the share that each of the pupils should pay, considering the fractional part of the land that each uses. That is, if John has $\frac{3}{16}$ of the land, he should pay $\frac{3}{16}$ of the expenses. If he does not make $\frac{3}{16}$ of the profits, that is his concern.

4. John planted his strip to early potatoes. He paid $1.80 for seed potatoes, $5.60 for marketing, and sold 18 sacks of potatoes at $4.60 a sack. Deducting his share of the expenses as found in Ex. 3, find his profit; that is, find how much he receives for his labor, since no account is taken in the expenses of the value of the time that he put in.

5. William planted his strip to tomatoes. He paid $16.55 for seed and for a hotbed and raised 108 boxes of tomatoes, which he sold at the garden at an average price of 90¢ a box. Find his profit, as in Ex. 4.

6. Clarence raised onion sets on his strip, paying $10.72 for seed and $29.68 for marketing. He sold 2986 lb. of onion sets at $3\frac{1}{2}$¢ per pound. Find his profit, as in Ex. 4.

7. Roger raised onions on his strip, paying $1.50 for seed. He sold the onions at the garden, selling 98 bags at 50¢ a bag. Find his profit, as in Ex. 4.

8. Henry raised celery on his strip, paying $12.80 for plants and $14.25 for marketing. He sold 262 bunches of celery at 25¢ a bunch. Find his profit, as in Ex. 4.

9. Frank raised lettuce and cabbage on his strip, paying $8.80 for seeds and plants and $13.20 for marketing. He sold 75 boxes of lettuce at 60¢ a box and 270 cabbages at 5¢ a head. Find his profit, as in Ex. 4.

10. From the results found in Exs. 4–9, find the total profit made by the six boys. Mark the plan which you drew in Ex. 1 so that each boy's strip will show what he raised and the profit that he made on it.

11. Considering the results in Exs. 4 and 5, would John probably have done better if on his strip he had planted tomatoes, as William did, instead of potatoes? Give the reasons on which you base your conclusion.

12. Mary raised onions, lettuce, and celery on her piece of land, paying \$1.60 for seed and \$11.58 for marketing. She sold 17 boxes of lettuce at 45¢ a box, 12 bags of onions at 50¢ a bag, and 246 bunches of celery at 20¢ a bunch. Find her profit, as in Ex. 4.

13. Susan raised radishes, beans, and spinach on her piece of land, paying \$1 for seed and \$7.50 for marketing. She sold 18 boxes of beans at 60¢ a box, 2200 bunches of radishes at 1¢ a bunch, and 41 boxes of spinach at 55¢ a box. Find her profit, as in Ex. 4.

14. Ella raised lettuce and cabbage on her piece of land, paying \$6 for seed and plants and \$14 for marketing. She sold 86 boxes of lettuce at 60¢ a box and 330 cabbages at 5¢ a head. Find her profit, as in Ex. 4.

15. Ann raised celery on her piece of land, paying \$2.35 for seed and planting and \$17.80 for marketing. She sold 286 bunches of celery at 25¢ a bunch. Find her profit, as in Ex. 4.

16. From the results found in Exs. 12–15, find the total profit made by the four girls. Complete the plan by marking the remaining strips as suggested in Ex. 10.

17. At the end of the year the question arose as to which boy or girl had made the most money per square rod. Find this profit in each case to the nearest cent and determine who stood highest, who stood next, and so on.

18. The question also arose as to the total profit of the group of the ten children, and what this profit amounted to per acre. Find these two amounts.

19. From Ex. 18 find the average profit made by the children. Find how much more than this average profit was made by the one who had the largest profit, and how much less by the one who had the smallest.

V. LITTLE EXAMINATIONS

I. 1. \$300 is what per cent more than \$200? than \$100?
2. \$200 is what per cent less than \$300? than \$400?
3. How much is $4\frac{1}{2}\%$ of \$350? of \$3500? of \$35,000?
4. How much is 150% of \$3500? $\frac{3}{4}\%$ of \$8000?
5. How much is $87\frac{1}{2}\%$ of \$7200? of \$6496?

II. 1. Find 0.2% of 3000, 2% of 300, and 20% of 30.
2. Express $3\frac{1}{2}$ qt. as per cent of a gallon.
3. Express an inch as per cent of a yard.
4. Express $33\frac{1}{3}\%$, $66\frac{2}{3}\%$, and $83\frac{1}{3}\%$ as fractions.
5. Express $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{16}$, and $\frac{5}{8}$ as per cents.

III. 1. \$60 is what per cent of \$720? of \$7200? of \$72,000?
2. 9 in. is what per cent of 10 in.? of 10 ft.? of 10 yd.?
3. A pint is what per cent of 1 qt.? of 1 gal.? of 6 gal.?
4. 96 ft. is 8% of what distance? 4% of what distance?
5. 9 ft. 8 in. is 10% of what distance?

IV. 1. \$12.35 is 5% less than what amount of money?
2. \$204 is 15% less than what amount of money?
3. Find the discount on a bill for \$380 at 20%.
4. Find the discount on a bill for \$400 at 15%, 2%.
5. At $95¢$ per \$100, find the face of a policy costing \$79.80.

V. 1. Find the commission at $5\frac{3}{4}\%$ on a sale totaling \$1650.
2. Find the tax on \$12,500 at \$22.60 per \$1000.
3. Find the cost of a policy for \$9000 at \$21.80 per \$1000.
4. If the tax on \$8200 is \$205, what is the rate per \$1000?
5. Find the tax at 2% on a taxable income of \$3750.

CHAPTER II

I. BORROWING AND LENDING MONEY

A SILENT READING LESSON

Mr. Moore has worked in a grocery store for a number of years and decides to open a store of his own. He has saved some money, but he finds that it will take more than his savings to pay for the fixtures for the store and to buy the goods which he needs in order to start in business. He must therefore borrow some money. When a man borrows money, he pays for the use of it until such time as he pays it back.

The money paid for the use of money is called *interest.* For example, if Mr. Moore borrows $2000 for a year and pays $120 for the use of it, the $120 is interest.

Interest is very similar to rent. Rent is payment for the use of property, such as a house, and is usually paid at a fixed rate for a certain length of time. Interest is payment for the use of money and is usually paid at a fixed rate for a certain length of time.

The sum of money on which interest is paid is called the *principal.* That is, when Mr. Moore borrows $2000, the $2000 is the principal.

The principal plus the interest at any given time is called the *amount* of the debt at that time. For example, the amount of Mr. Moore's debt at the end of the year for which he borrowed the money is $2000 + $120, or $2120; that is, if he does not pay the interest until he pays back the principal, he will have to pay $2120 at that time.

Rate of Interest. Interest is usually specified as a certain per cent of the principal. The per cent is called the *rate of interest*, and means the rate for borrowing money for 1 yr. unless it is otherwise specified.

Interest for One Year. Finding interest for 1 yr. is therefore simply a case of finding a per cent of a number. For example, suppose that you borrow \$2000 at 6% for 1 yr. The interest for 1 yr. is then 6% of \$2000; that is, as shown by the work at the right, it is \$120.

Principal,	\$2000
Rate,	0.06
Interest for 1 yr.,	\$120.00

Interest for 1 yr. = principal × rate.

In applying this relation, which is the usual form of expressing the rule for interest, we, of course, multiply the principal by the rate, and not the rate by the principal.

INTEREST FOR ONE YEAR

Without using pencil and paper, find the interest for 1 yr. on

1. \$100 at 5%.
2. \$100 at 6%.
3. \$1000 at 6%.
4. \$200 at 4%.
5. \$200 at 6%.
6. \$300 at 4%.
7. \$300 at 6%.
8. \$3000 at 5%.
9. \$4000 at 6%.

10. At 6% find the amount (principal plus interest) at the end of 1 yr. of a debt of \$2000; of \$4000; of \$7500.

Find to the nearest cent the interest for 1 yr. on

11. \$750 at 3%.
12. \$875 at 4%.
13. \$475 at 5%.
14. \$425 at 6%.
15. \$75 at $4\frac{1}{2}$%.
16. \$85 at $3\frac{1}{2}$%.
17. \$95 at $4\frac{1}{2}$%.
18. \$65 at $2\frac{1}{2}$%.
19. \$375.60 at 5%.
20. \$550.25 at 4%.
21. \$835.40 at 6%.
22. \$925.20 at 6%.

Interest for Several Years. If you pay \$400 a year for house rent, you pay twice as much for 2 yr., three times as much for 3 yr., and so on for any number of years. In the same way, if you pay \$60 for the use of \$1000 for 1 yr., you pay twice as much for 2 yr., three times as much for 3 yr., and so on.

For example, suppose that you borrow \$2000 at 6% for 3 yr.

The interest for 1 yr. is 6% of \$2000, or \$120, and for 3 yr. it is three times as much; that is, as shown at the right, it is \$360.

Principal,	\$2000
Rate,	0.06
Interest for 1 yr.,	\$120.00
	3
Interest for 3 yr.,	\$360.00

Interest = principal × rate × number of years.

INTEREST FOR SEVERAL YEARS

Find the interest for 2 yr. on

1. \$250 at 6%.
2. \$375 at 5%.
3. \$480 at 6%.
4. \$150 at 4%.
5. \$280 at 5%.
6. \$850 at 6%.
7. \$1200 at $5\frac{1}{2}$%.
8. \$1600 at 6%.
9. \$2400 at $5\frac{1}{2}$%.

Find the interest for 3 yr. on

10. \$250 at 4%.
11. \$275 at 5%.
12. \$360 at 6%.
13. \$180 at $4\frac{1}{2}$%.
14. \$260 at $5\frac{1}{2}$%.
15. \$950 at $5\frac{1}{2}$%.
16. \$75 at 6%.
17. \$750 at 6%.
18. \$7500 at $5\frac{1}{2}$%.

Find the interest on

19. \$2500 for 2 yr. at 4%.
20. \$3600 for 3 yr. at 5%.
21. \$4800 for 4 yr. at 6%.
22. \$7500 for 2 yr. at $4\frac{1}{2}$%.
23. \$8500 for 3 yr. at $5\frac{1}{2}$%.
24. \$12,500 for 4 yr. at 6%.
25. \$18,000 for 2 yr. at 5%.
26. \$25,000 for 3 yr. at $4\frac{1}{2}$%.

Interest for Part of a Year. If the interest for 1 yr. on a certain sum is \$50, for $\frac{1}{2}$ yr., or 6 mo., it will be $\frac{1}{2}$ of \$50, or \$25. Similarly, if the interest for 1 yr. is \$60, the interest for 3 mo., which is $\frac{1}{4}$ yr., will be $\frac{1}{4}$ of \$60, or \$15, and so on.

For example, to find the interest on \$6000 at 5% for 4 mo., we first find it for 1 yr., which is \$300.

Since 4 mo. $= \frac{1}{3}$ yr., we then take $\frac{1}{3}$ of the \$300; that is, as shown by the work at the right, the interest for 4 mo. is \$100.

Principal,	\$6000
Rate,	0.05
Interest for 1 yr.,	\$300.00
Interest for 4 mo.,	\$100.
	3)\$300.00

Interest = principal × rate × fraction of a year.

In practical work we simply write the 3 before the result of the multiplication, draw the line to represent division, and divide, writing the result below the number divided.

INTEREST FOR PART OF A YEAR

Find the interest for 6 mo. on

1. \$100 at 6%.
2. \$275 at 4%.
3. \$350 at 4%.
4. \$750 at 6%.
5. \$475 at 3%.
6. \$250 at $2\frac{1}{2}$%.
7. \$350 at 6%.
8. \$550 at $4\frac{1}{2}$%.
9. \$695.30 at 3%.
10. \$964.80 at 3%.
11. \$575.75 at 6%.
12. \$950.80 at 5%.

Find the interest on

13. \$250 for 4 mo. at 6%.
14. \$380 for 3 mo. at 5%.
15. \$850 for 2 mo. at 6%.
16. \$750 for 4 mo. at 7%.
17. \$1200 for 9 mo. at 6%.
18. \$1500 for 8 mo. at 5%.
19. \$2400 for 10 mo. at 6%.
20. \$2760 for 8 mo. at 7%.

General Rule for Interest. The plan of finding interest for any length of time is always the same; that is,

Find the interest for one year, and then multiply this by the number representing years or the fractional part of a year.

This rule may be expressed briefly, as follows:

Interest = principal × rate × time.

For example, in finding the interest on \$250 for 10 mo. at 6%, we see that for 1 yr. it is 6% of \$250, and for 10 mo. it is $\frac{10}{12}$ of this. That is, as here shown, the interest for 10 mo. is \$12.50.

$$\frac{\$25\cancel{0} \times \cancel{6} \times 1\cancel{0}}{1\cancel{0}\cancel{0} \times \cancel{12}_{2}} = \frac{\$25}{2} = \$12.50$$

This example may be solved mentally by noticing that $\frac{10}{12}$, or $\frac{5}{6}$, of 6% is 5 × 1%, or 5%. We then have

5% of \$250 = 5 × \$2.50 = \$12.50.

At present, in finding interest, we shall consider 30 da. as a month. The interest for 1 da. is then $\frac{1}{30}$ of the interest for 1 mo., for 5 da. it is $\frac{1}{6}$ of the interest for 1 mo., and so on.

We disregard a fraction less than $\frac{1}{2}$¢ in any interest found, and call $\frac{1}{2}$¢ or greater a full cent.

FINDING INTEREST

1. How much is the interest on \$872, at 6%, for 1 yr.? for 6 mo.? for 4 mo.? for 8 mo.? for 10 mo.?

2. How much is the interest on \$720, at 5%, for 1 yr.? for 6 mo.? for 10 mo.? for 1 mo.? for 9 mo.?

3. How much is the interest on \$1080, at 4%, for 1 yr.? for 1 mo.? for $\frac{1}{2}$ mo.? for 5 da.? for 1 da.?

4. How much is the interest on \$225.50, at 4%, for 1 yr.? for 6 mo.? for 4 mo.? for 1 mo.? for 10 da.?

Interest for Years and Months. Suppose that Mr. Dean, needing \$125 to pay for painting his house, borrows the money at 5%. If he does not pay the interest until 2 yr. 3 mo. later, how much interest is then due?

$$\frac{\overset{5}{\cancel{\$125}} \times 5 \times 9}{\underset{4}{\cancel{100}} \times 4} = \frac{\$225}{16} = \$14.06\tfrac{1}{4}$$

Since 2 yr. 3 mo. is the same as $2\frac{1}{4}$ yr., or $\frac{9}{4}$ yr., we arrange the work as here shown, cancel, and find the required interest to be $\$14.06\frac{1}{4}$, which, disregarding the fraction, we give as \$14.06.

Ordinarily we save time and avoid errors by using cancellation, but in the problem above we might have said

5% of \$125 = \$6.25, interest for 1 yr.,

and $2\frac{1}{4} \times \$6.25 = \$14.06\frac{1}{4}$, or \$14.06.

We do not frequently need to compute interest for years and months. Even if Mr. Dean, in the above case, did not pay back his loan for 3 yr. or more, he would probably have paid the interest every year or every 6 mo.

INTEREST FOR YEARS AND MONTHS

Find the interest on

1. \$350 for $1\frac{1}{4}$ yr. at 6%.
2. \$400 for $1\frac{1}{2}$ yr. at 5%.
3. \$500 for $2\frac{1}{2}$ yr. at 5%.
4. \$725 for $3\frac{1}{2}$ yr. at 4%.
5. \$850 for $1\frac{3}{4}$ yr. at 6%.
6. \$9600 for 2 yr. 6 mo. at 5%.
7. \$825 for 3 yr. 3 mo. at 4%.
8. \$950 for 2 yr. 8 mo. at 6%.
9. \$305 for 1 yr. 9 mo. at 6%.
10. \$800 for 1 yr. 2 mo. at $4\frac{1}{2}$%.

Find the interest on \$875 for 2 yr. 3 mo. at

11. 5%. 12. 6%. 13. 3%. 14. 4%. 15. $5\frac{1}{2}$%.

FINDING INTEREST

1. Find the interest on \$1500 for 1 yr. 2 mo. at 5%; at 6%; at 4%; at $5\frac{1}{2}$%.

2. A dealer in automobiles buys 16 cars at \$1150 each and agrees to pay for them in 9 mo., with interest at 6%. How much does he pay, including principal and interest?

3. Mr. Russell hires Mr. Baker to make some repairs on his house, agreeing to pay him \$85 for the work. Mr. Baker agrees to wait 3 mo. for his money if he receives interest at 6%. How much is due Mr. Baker at the end of the 3 mo.?

4. Woodhull & Co. need \$3650 to buy holiday goods. They borrow it on Oct. 15 and agree to pay it back in 3 mo., with interest at 5%. How much do they owe on Jan. 15?

5. Pierson & Co. buy \$2250 worth of goods, agreeing to pay for the goods in 3 mo., with interest at 5%. How much do they owe at the end of the 3 mo.?

6. Mr. Holbrook buys from Mr. Jackson a second-hand automobile, agreeing to pay \$580 in 4 mo., with interest at 6%. How much interest is due in 4 mo.?

7. A man borrows \$2500, agreeing to pay interest at 6%. He pays the principal and interest at the end of 1 yr. $8\frac{1}{2}$ mo. How much does he pay, including principal and interest?

8. A man buys 36 head of cattle at \$90 a head and agrees to pay for them in 10 mo., with interest at 6%. How much does he pay, including principal and interest?

9. A man borrows \$950, agreeing to pay interest at 5%. He pays the principal and interest 2 yr. 10 mo. later. How much does he pay, including principal and interest?

10. The day Edward is 16 yr. old his father invests \$500 for him at 5%, the \$500 and interest being due the day he is 21 yr. old. How much will be due at that time?

Interest for Years, Months, and Days. 1. If you borrow \$5000 at 6%, how much interest is due 8 mo. 6 da. later?

Since 8 mo. 6 da. = 246 da., and there are 360 da. in 12 mo. of 30 da., we find the interest, as shown below, for $\frac{246}{360}$ yr.:

1 mo. = 30 da.
8
8 mo. = 240 da.
6 da.
8 mo. 6 da. = 246 da.

$$\frac{\overset{5}{\cancel{\$5000}} \times \overset{}{\cancel{6}} \times \overset{41}{\cancel{246}}}{\underset{}{\cancel{100}} \times \underset{60}{\cancel{360}}} = \$205$$

\$205. *Ans.*

2. Suppose that a man borrows \$400 from another man on Sept. 10, 1926, and agrees to pay the interest at 6% when he repays the principal. If he does not pay the loan until Aug. 7, 1927, how much interest is then due?

As shown below, we first find the time for which the \$400 draws interest. We then proceed in the usual way, as follows:

	yr.	mo.	da.
Aug. 7, 1927 =	1927	8	7
Sept. 10, 1926 =	1926	9	10
The time =		10	27

1 mo. = 30 da.
10
10 mo. = 300 da.
27 da.
10 mo. 27 da. = 327 da.

$$\frac{\overset{20}{\cancel{\$400}} \times \overset{}{\cancel{6}} \times \overset{109}{\cancel{327}}}{\underset{}{\cancel{100}} \times \underset{60}{\cancel{360}}} = \$21.80$$

\$21.80. *Ans.*

While we might have canceled further in this case, it is often better to leave the 100 in the denominator, because dividing by 100 is easily done by moving the decimal point.

Six-Per-Cent Method. Because the most common rate of interest is 6%, and money is often loaned by banks for periods of 30 da., 60 da., or 90 da., it is frequently convenient to find the interest by what is called the "six-per-cent," or "sixty-day," method.

For example, in finding the interest on \$420 for 60 da. at 6% by this method, since 60 da. = 2 mo. = $\frac{1}{6}$ yr., we have

$\frac{1}{6}$ of 6% of \$420 = 1% of \$420 = \$4.20.

Hence the interest at 6% for 60 da. is 0.01 of the principal, and, similarly, for 6 da. it is 0.001 of the principal. That is,

To find the interest at 6% for 60 da., move the decimal point two places to the left; for 6 da., move it three places to the left.

For other periods the interest can be found from that for 60 da. or 6 da.

For example, in finding the interest on \$420 for 5 mo. 10 da. at 6%, the work should be done mentally except that the amounts may, if necessary, be written down, as here shown, and then added to find the required interest.

2 mo.,	\$4.20
2 mo.,	4.20
1 mo.,	2.10
$\frac{1}{3}$ mo.,	.70
	\$11.20

SIX-PER-CENT METHOD

Numbers 1 to 5, oral

Find the interest at 6% for 60 da. and also for 30 da. on

1. \$250. **2.** \$370. **3.** \$1250. **4.** \$1720. **5.** \$2800.

Find the interest at 6% on

6. \$750 for 3 mo. **10.** \$920 for 4 mo. **14.** \$1500 for 165 da.
7. \$1250 for 5 mo. **11.** \$870 for 1 mo. **15.** \$2500 for 216 da.
8. \$1450 for 7 mo. **12.** \$940 for 18 da. **16.** \$2750 for 132 da.
9. \$1860 for 9 mo. **13.** \$875 for 24 da. **17.** \$3275 for 184 da.

Interest Table. Since interest tables, such as are used by banks, are generally made up in book form, it is impossible to give the complete tables in an arithmetic. Here is a small table which shows the interest on $100 at various rates:

Time	4%	5%	6%	7%	Time	4%	5%	6%	7%
1 da.	0.01	0.01	0.02	0.02	1 mo.	0.33	0.42	0.50	0.58
2 da.	.02	.03	.03	.04	2 mo.	.67	.83	1.00	1.17
3 da.	.03	.04	.05	.06	3 mo.	1.00	1.25	1.50	1.75
4 da.	.04	.06	.07	.08	4 mo.	1.33	1.67	2.00	2.33
5 da.	.06	.07	.08	.10	6 mo.	2.00	2.50	3.00	3.50
6 da.	.07	.08	.10	.12	12 mo.	4.00	5.00	6.00	7.00

Using the Table. To find the interest on $275 for 33 da. at 5%, we follow along the line in the table marked "1 mo." to the column marked "5%," and find 0.42. Then, on the line for 3 da. and under 5%, we find .04. We then proceed as follows:

Interest on $100 for 30 da. (1 mo.) at 5%,	$0.42
Interest on $100 for 3 da. at 5%,	.04
Interest on $100 for 33 da. at 5%,	$0.46
$275 = 2.75 × $100,	2.75
	230
	322
	92
Interest on $275 for 33 da. at 5%,	$1.2650

That is, the required interest is 1.26\frac{1}{2}$, or $1.27.

With such a small table, the result may not be exact. If we find the interest in the usual way, we shall find that it is nearer $1.26 than $1.27. This table serves simply to show how a larger table is used.

USING THE INTEREST TABLE ON PAGE 110

Using the table, find the interest when the principal, the time, and the rate are as follows:

1. \$100, 4 mo., 7%.
2. \$100, 6 mo., 4%.
3. \$100, 5 da., 6%.
4. \$200, 1 yr., 5%.
5. \$300, 3 mo., 6%.
6. \$400, 6 da., 7%.
7. \$700, 4 mo., 4%.
8. \$850, 3 mo., 6%.
9. \$925, 2 mo., 6%.
10. \$775, 6 mo., 5%.
11. \$1200, 33 da., 6%.
12. \$1250, 3 mo. 5 da., 6%.
13. \$1225, 2 mo. 6 da., 5%.
14. \$1500, 35 da., 6%.
15. \$1850, 64 da., 4%.
16. \$1625, 6 mo. 2 da., 5%.
17. \$2500, 1 yr., 7%.
18. \$2500, 3 mo. 4 da., 6%.
19. \$3000, 2 mo. 6 da., 6%.
20. \$3250, 2 mo. 4 da., 4%.

21. Using the six-per-cent method, find the interest on \$250 for 90 da. at 6%. Check the result by using the table.

22. Using the method shown on page 106, find the interest on \$450 for 93 da. at 7%; that is, for $\frac{93}{360}$ yr. Check the result by using the table.

23. Using the table, find the interest on \$700 for 95 da. at 6%. Check the result by using the six-per-cent method.

24. Using the table, find the interest on \$1250 for 64 da. at 5%. Check the result by using the method shown on page 106.

25. In Exs. 21–24 three methods have been employed in finding interest. Find the interest on \$450 for 90 da. at 6% by each of the three methods, and state which one took the least time.

26. As in Ex. 25, time each method in finding the interest at 6% on \$1200 for 36 da.; on \$1250 for 63 da.; on \$1600 for 84 da.; on \$1450 for 126 da.

REVIEW OF INTEREST

Copy and complete the following statements:

1. The interest at 6% on \$200 for 2 mo. is ——.
2. The interest at 5% on \$200 for 2 mo. is ——.
3. The interest at 5% on \$400 for 2 mo. is ——.
4. The interest at 5% on \$400 for 6 mo. is ——.
5. At 4% the interest on \$500 for 3 mo. is ——.
6. At 5% the interest on \$750 for 3 mo. is ——.
7. At 6% the interest on \$225 for 6 mo. is ——.
8. At 7% the interest on \$1250 for 8 mo. is ——.
9. For 3 mo. the interest on \$300 at 6% is ——.
10. For 2 mo. the interest on \$450 at 5% is ——.
11. The interest at 6% on \$1500 for 33 da. is ——.
12. The interest is —— on \$250 for 64 da. at 6%.
13. The interest is —— on \$425 for 92 da. at 5%.
14. On \$300 the interest for 30 da. at 6% is ——; and if the time is 60 da., the interest is ——.
15. At 5% the interest on \$500 for 60 da. is ——, and for half the time it is ——.
16. The interest on \$500 for 60 da. at 3% is ——, and for half that time and double that rate it is ——.
17. The interest on \$1000 for 6 mo. at 6% is ——; if the rate is half as much, if the time is twice as long, and if the principal is doubled, the interest is ——.
18. Doubling the principal —— the interest; doubling the time —— the interest; doubling the rate —— the interest; doubling any two of them multiplies the interest by ——; doubling all three multiplies the interest by ——.

PAYMENTS IN INSTALLMENTS

1. A man owes \$100 and is given the choice of paying it in 60 da. with interest at 6%, or of paying \$1.06 a day for 100 da. Which is the cheaper plan for him to take?

2. Mr. Roe buys a home costing \$5000, paying \$1000 down and the rest in eight installments of \$500 each. The installments are due at the end of each 6 mo., with interest at 6% on the unpaid balance; that is, at the end of the first 6 mo. he owes interest on \$4000, at the end of the second 6 mo. on \$3500, and so on. After the last payment, how much has Mr. Roe paid in principal and interest?

3. A man buys a store costing \$14,000 by paying \$4000 down and the rest in five payments of \$2000 each, due at the end of each year, with interest at 6% on the unpaid balance. How much does he pay in all for the store?

4. Suppose that you buy a radio set and have the choice of paying \$110 at once or \$10 a month for a year. If the use of the money is worth 6% to you, which is the cheaper plan for you? How much cheaper is it?

5. A dealer in radio supplies finds that if he sells for cash, he can put the money back into the business, where it will pay him 12% a year. Supposing that he is willing to wait 60 da. for payment, find the price that he should quote on a radio set which he ordinarily sells for \$50, cash; that is, find the price which is equal to \$50 plus the interest for 60 da. at 12% a year.

6. The dealer also sells the set mentioned in Ex. 5 on the installment plan, requiring \$25 "down" (in cash) and the balance in six payments of \$5 each, at the end of each month. If he puts each payment back into the business, how much better is this plan for him than the cash price of \$50?

Promissory Note. When a person borrows money, the details of the transaction should be put in writing for the protection of all concerned. This is generally done by means of a written promise on the part of the borrower to repay the money to the lender at a certain time, either with interest at a specified rate or without interest. Such a written promise is called a *promissory note*, or simply a *note*.

The sum borrowed is stated in the note, and is called the *face* of the note.

The face plus the interest on a given date is called the *amount* of the note on that date.

A note should state the date, face, rate, person to whom payable, and date of *maturity* (date when it is due to be paid). It is customary to have the note state that it has been given for value received, but this is not necessary to make the note legal in most states.

The following form shows a simple type of promissory note:

$75.50 TOPEKA, KANSAS, Aug. 23, 1928

Two months after date, for value received, I promise to pay to the order of John R. Thomas

Seventy-five and 50/100 Dollars,

with interest at 5%.

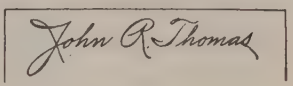
James P. Connolly

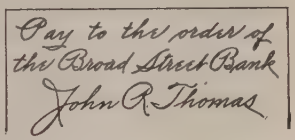
If, in the above note, the words "on demand" were substituted for "two months after date," the note would be called a *demand note*. Such notes are payable only after payment is demanded by the lender, and are generally used only in cases of borrowing for a few days or in cases where it is not feasible to set a definite date of maturity.

Parties to a Note. The person named in a note as the one to whom it is payable is called the *payee*. The person who signs a note is called the *maker*.

Indorsing a Note. If the payee wishes to sell a note which is payable to himself or order, so that he can get his money at once, he must *indorse* the note.

A note is indorsed by the payee's writing his name across the back. The indorser must pay the note if the maker does not. The two forms at the right show how Mr. Thomas might indorse the note on page 114.





The first indorsement is known as an *indorsement in blank*, and the second one as an *indorsement in full*.

Negotiable Note. A note that is payable to bearer, or that has been indorsed in blank, is a *negotiable note*; that is, it can be sold. If it is made or indorsed to the order of a particular person it is not negotiable without his indorsement.

Interest on Loans. The United States borrowed money before the World War at about 2%; during the war the rate began at $3\frac{1}{2}$% and increased to $4\frac{3}{4}$%. Savings banks pay depositors about 4% or 5%. In cities, borrowers usually pay from 4% to 6% for ordinary loans. Is this the rate where you live?

When a note bears interest, but the rate is not specified, the rate is the one fixed by the law of the state in which the transaction takes place. In many states this rate is 6%, and various penalties are prescribed by law for demanding a rate of interest higher than the legal rate. Since the law is liable to change, inquiry should be made as to whether or not a government stamp is still necessary on a note.

BORROWING MONEY ON NOTES

1. Suppose that Mr. R. J. Lee borrows $275 from you today, giving his note due in 3 mo. with interest at 6%. Write the note and find how much he will have to pay when the note becomes due.

2. A purchaser of an automobile is offered the choice of paying half of $2400 in cash and giving his note for the rest, due in 4 mo. at 5%, or of paying $2300 in cash. How much less is the cost of the car if he pays cash?

3. A salesman starting on a trip borrowed $750 on a 90-day note bearing interest at 5%. He returned on the day the note was due and immediately paid it. Write the note and find the amount of the note on the date of payment.

4. A merchant borrowed $900 on May 4 and $1150 on June 7, giving his note for each loan. He paid both notes the first of the following October, with interest at $4\frac{1}{2}$%. Counting 30 da. for each full month, from May 4 to Sept. 4 is 4 mo., or 120 da., and from Sept. 4 to Oct. 1 is 27 da. How many days did the first note run? How many days did the second one run? Find the total amount of the notes on Oct. 1.

5. A manufacturer held a note signed by one of his customers for $3500. Interest at 6% was due on the note from May 16, 1928, to Feb. 26, 1929. The manufacturer sent the note to a collector, who was to receive 5% of the amount collected, but the customer failed and paid only 40% on his debts. How much did the manufacturer receive from the collector?

6. Write a note like the one on page 114, dating it today and taking $150 as the principal and 6% as the rate. Use any names you choose for maker and payee and indorse the note to your order.

II. BANKING

A SILENT READING LESSON

If people had not earned and saved money, we should not have comfortable homes in which to live, mills and factories in which clothing and other things that we need are made, railroads to bring the city's food from the farms, or school buildings in which to obtain an education. All these things represent money which was saved by someone.

One of the best ways to save money is to make a *budget* in which we write down our allowance for a month, how much we shall spend, and how much we shall save. A family should do the same thing, making its budget for a year and doing its best to live up to the plan thus made.

We should all try to save at least a little of what we earn, and to see that what we have saved is kept safely. We may, for example, deposit our savings in a bank. There are several kinds of banks, and most people at some time have business dealings with one kind of bank or another. Only a part of the money deposited is kept by the bank; the rest is lent to business men or others who pay interest to the bank. On deposits of savings, banks generally pay a certain per cent of interest to the depositor.

One thing that is essential to everyone who hopes to succeed is a *bank account.* In most banks a person may open a savings account as soon as he begins to save even a small amount. When a person opens an account, he is given a *bank book* in which a record of his account is kept. Many schools organize school banks, electing officers and carrying on a banking business with imitation money.

We shall first learn more about a budget, which helps us to save money, and then shall learn about banks, which help to protect the money we have saved.

A SILENT READING LESSON (CONTINUED)

One advantage of a budget is that it places a check upon the tendency of a person, family, or business to spend money freely as it comes in, without thinking of future needs or of emergencies that are likely to arise.

In the case of families, experience has shown about what per cent of income may safely be assigned to various classes of expenditures. In a general way the expenses of the house may take about 30% of a small income, food 30%, clothing 10%, advancement 5%, health 5%, miscellaneous 10%, and investments 10%; but these figures will vary greatly.

We should prepare for our own use, but not for discussion in class, a budget for a month. At the end of the month we should see how closely the expenses come to our estimates and should then try to improve upon our budget.

There are many ways of making up a budget, but the following divisions of the expenses are typical:

1. *House*, including rent, taxes, insurance, heat, light, telephone. If a man owns his home, he does not pay rent but pays taxes and property insurance; if he rents a home, the owner pays for these items.

2. *Food*, including groceries, meat, milk, fruit, and so on.

3. *Clothing*, including suits, coats, underwear, shoes, hats, repairs, and similar items.

4. *Advancement*, including education, books, magazines, papers, travel, religion, charity.

5. *Health*, including recreation, bills of physicians and dentists, medicine, exercise, and so on.

6. *Miscellaneous*, including laundry, presents, small items.

7. *Savings*, including bank account, life insurance, and other ways of providing for future needs.

Form for a Budget. The model below is one of various convenient forms that may be used in preparing a budget.

I shall allow for my expenses during the next year the following amounts, and I will endeavor to keep within each allowance:

1. *House*

a. Rent (or taxes)			*d.* Light		
b. Insurance			*e.* Miscellaneous		
c. Heat			*Total*		

2. *Food*

a. Groceries			*d.* Fruit		
b. Meat			*e.* Other items		
c. Milk			*Total*		

3. *Clothing*

a. Suits and coats			*d.* Hats		
b. Underwear			*e.* Other items		
c. Shoes			*Total*		

4. *Advancement*

a. Education			*d.* Religion		
b. Reading matter			*e.* Charity		
c. Travel			*Total*		

5. *Health*

a. Recreation			*c.* Other expenses		
b. Doctor's bills			*Total*		

6. *Miscellaneous*

a. Laundry			*c.* Small items		
b. Presents			*Total*		

7. *Savings*

a. Bank account			*c.* Other kinds of saving		
b. Life insurance			*Total*		

BUDGETS

1. A family with an income of \$1500 a year allowed 20% of the income for house, 25% for food, 20% for clothing, 20% for advancement, 10% for other expenses, and the rest for savings. How much money was allowed for each purpose?

2. A family with an income of \$2500 a year allowed 20% for house, 25% for food, 15% for clothing, 25% for advancement, 10% for other expenses, and the rest for savings. How much money was allowed for each purpose?

3. The per cents stated in Exs. 1 and 2 are the results of careful studies of family expenses where the incomes range from \$1500 to \$4000. The changes in the cost of living do not materially affect the relative per cents of the income expended for various purposes. Solve Ex. 2, assuming that the income is \$2800; is \$3200; is \$4000.

4. A study of family expenditures in a manufacturing community showed that 19% of the income was paid for house, 43% for food, 11% for clothing, 22% for all other expenses, and the rest for savings. If a family has an income of \$1200 a year, how much money is spent for each purpose?

5. Using the form given on page 119, make up a budget for an income of \$3000 a year, allotting the specified per cent to each item. 1. House: rent, 18%; heat, 5%; light, 2%; miscellaneous, 2%. 2. Food: groceries, 12%; meat, $6\frac{1}{2}$%; milk, 2%; fruit, $1\frac{1}{2}$%; other items, 3%. 3. Clothing: suits and coats, 6%; underwear, 2%; shoes, 2%; hats, 1%; other items, 4%. 4. Advancement: education, 1%; reading matter, $2\frac{1}{2}$%; travel, $2\frac{1}{2}$%; religion, 2%; charity, 2%. 5. Health: recreation, 3%; doctor's bills, 2%; other expenses, 2%. 6. Miscellaneous: laundry, 2%; presents, 3%; small items, 2%. 7. Savings: bank account, 4%; life insurance, 3%; other kinds of saving, 2%.

A SILENT READING LESSON

A boy who puts 1¢ each day into a toy bank will save enough in 6 mo. to buy a fielder's glove or two baseball bats. A girl who saves 10¢ a day will save enough in 2 mo. to buy a dress or a pair of good shoes. A man who puts aside $1 a day will perhaps save enough in a year to make the first payment on a lot on which to build a home.

A boy who deposits $1 a week for 10 yr. in a bank, which pays interest at 4% a year and adds it to the amount on deposit every 6 mo., will have over $600 in 10 yr. A man who deposits $10 a week in the same bank will have over $6300 at the end of the 10 yr., — an amount well worth having.

People generally get the things that they need in order to live comfortably, or the things that help them to enjoy life, by saving a little at a time. There are many people who never learn how to save money wisely. Many spend their money as they earn it, because of the constant temptation to live extravagantly, which includes not only living beyond one's means but also spending money foolishly.

We should all begin early to save, no matter if it is but a few dollars or a few cents a year. Saving money is just as important as earning it.

After a depositor has opened a savings account at a bank, as described on page 117, he may make deposits whenever he wishes. Each time he takes his book and money to the bank. The bank clerk (or *teller*, as he is often called) takes the money and writes in the book the amount deposited. Similarly, when money is drawn out, the amount is entered in the book. Deposits may also be sent by mail.

We should inquire at home or of some banker about trust companies, which, in addition to doing a regular banking business, take charge of funds and manage and settle estates.

SAVING MONEY

1. Beginning on the day that he is 16 yr. old, a boy saves 30¢ a day for 300 da. a year and deposits it in a savings bank. Not counting interest, how much will he have in the bank when he becomes 21 yr. old?

2. A man gives his daughter $5 on the day that she is 1 yr. old, $10 on her next birthday, $15 on her next birthday, and so on until she is 21 yr. old, depositing the money for her in a savings bank. Not counting interest, how much will the girl have in the bank when she is 21 yr. old?

3. A man saved $5 a week for 50 wk. of the year in which he was 21 yr. old. In each following year he increased his savings 10% over the preceding year. How much did he save during the year in which he was 29 yr. old?

4. A merchant saves $4125 the first year he is in business. The second year he increases his savings one third. The third year they are only 85% as much as the second year. The fourth year they increase 30% over the third year. How much does he save in the 4 yr.?

5. A man is paid a salary of $45 a week for 50 wk. in a year. His monthly expenses are $35 for house rent, 60% as much for clothing, $60 for food, and 30% as much for other necessary expenses as for food. How much can the man deposit a year in the savings bank?

6. Two years ago a clerk in a store had a salary of $28 a week and was also paid a commission of 2% on the goods he sold. That year he worked 48 wk. and sold $6350 worth of goods. Last year his salary was increased 25%, but the rate of commission was not changed. If he worked 49 wk. and sold $7750 worth of goods, how much was his income last year? How much was his income increased?

Interest at Savings Banks. Savings banks usually pay interest every 6 mo. or every 3 mo. If not drawn out by the depositor, this interest is added to the principal, and the total amount then draws interest.

When interest due is added to the principal, and the total amount draws interest, the interest is called *compound interest.*

The method of finding compound interest is much like that of simple interest. For example, suppose that we wish to find the interest on \$2000 for 2 yr. at 4% a year, compounded semiannually.

Arranging the work as here shown, we find the amount to be \$2164.87 and the interest to be \$164.87.

The interest here is found exactly, but since savings banks pay interest only on the dollars and not on the cents, a bank would pay a depositor only \$164.84 in this case.

Disregard a fraction less than $\frac{1}{2}¢$ in any interest; call $\frac{1}{2}¢$ or more a full cent.

\$2000, first principal
0.02, rate for 6 mo.
\$40, int. first 6 mo.
2000
\$2040, amt. after 6 mo.
0.02
\$40.80, int. second 6 mo.
2040.00
\$2080.80, amt. after 1 yr.
0.02
\$41.62 (\$41.616)
2080.80
\$2122.42, amt. after $1\frac{1}{2}$ yr.
0.02
\$42.45 (\$42.4484)
2122.42
\$2164.87, amt. after 2 yr.
2000.00, original principal
\$164.87, int. for 2 yr.

Simple interest in the above example would be \$160, or \$4.87 less than the compound interest. If simple interest is collected when due and is at once reinvested at the same rate as the original principal, the investor receives the same as compound interest.

Savings-Bank Account. Some savings banks compute the interest due each Jan. 1 and July 1 on the smallest balance on deposit during the preceding 6 mo., and add it to the amount on deposit. The following is a specimen account at a bank where the interest rate is 4%, payable semiannually:

Date		Deposits		Interest		Payments		Balance	
1928									
July	1	600	50					600	50
July	20	75						675	50
Sept.	6					120		555	50
Dec.	7	60						615	50
Dec.	20					65		550	50
1929									
Jan.	1			11				561	50
May	9	200						761	50
July	1			11	22			772	72

The smallest balance during the first interest period (July 1, 1928, to Jan. 1, 1929) is \$550.50, and interest is computed on exact dollars only. At 2%, the interest on \$550 is \$11, and hence \$11 is credited in the interest column for Jan. 1 and added to the balance. In the second period the smallest balance is \$561.50, and the interest is 2% of \$561, or \$11.22.

Some banks allow deposits to begin drawing interest on the first of each month; others on the first of each quarter; others, as above, on the first of each half year. In a bank which allows interest from the first of each quarter and credits it every 6 mo., we should have, in the above case,

Oct. 1, 1928, 1% of \$555 = \$5.55
Jan. 1, 1929, 1% of \$550 = 5.50
Jan. 1, 1929, credit of interest = \$11.05

SIMPLE AND COMPOUND INTEREST

At simple interest, and also at interest compounded annually, find the amount (principal plus interest) of

1. \$2500 for 4 yr. at 5%.
2. \$4000 for 3 yr. at 6%.
3. \$3500 for 3 yr. at 4%.
4. \$2250 for 4 yr. at 3%.
5. \$1750 for 3 yr. at $3\frac{1}{2}$%.
6. \$625.50 for 4 yr. at $3\frac{1}{2}$%.
7. \$10,000 for 6 yr. at 4%.
8. \$425.50 for 4 yr. at $4\frac{1}{2}$%.

At interest compounded semiannually, find the amount of

9. \$200 for 3 yr. at 4%.
10. \$650 for 2 yr. at 6%.
11. \$700 for 3 yr. at 3%.
12. \$500 for 2 yr. at 4%.
13. \$1000 for 2 yr. at $4\frac{1}{2}$%.
14. \$2000 for 2 yr. at $3\frac{1}{2}$%.

15. John has \$500 on deposit in a savings bank on Jan. 1. Assuming that no money is withdrawn, how much interest is due on the following July 1 if the bank pays 4%? if it pays $3\frac{1}{2}$%? if it pays $3\frac{1}{4}$%? if it pays $3\frac{3}{4}$%?

16. A school has an account at a savings bank in which are deposited the savings of all the pupils, and a committee of the pupils keeps the account of each pupil. If \$475 is deposited on June 30, and the bank pays 4%, how much interest will be due on the following Jan. 1?

17. Edwin has \$50 in a savings bank on July 1, and deposits \$10 on the last day of each month for 6 mo., beginning July 31. The bank allows 4% on sums on deposit the first of each month, and credits interest on Jan. 1 and July 1; that is, on Jan. 1 the balance will be increased by the interest due Jan. 1, which is computed for 6 mo. on \$50, for 5 mo. on the first \$10, for 4 mo. on the second \$10, and so on to the December deposit, which draws no interest. Find Edwin's balance on Jan. 1 following the last deposit.

18. Which brings the more interest in 2 yr., \$1250 drawing simple interest at $4\frac{1}{2}\%$, or \$1250 deposited at the beginning of a year in a savings bank which pays 4%, compounded semiannually, and disregards the cents in the principal in figuring the interest? How much is the difference?

19. On Jan. 1 a man deposits \$2000 in a savings bank which pays interest at 4%, compounded semiannually. If he leaves the money in the bank for 2 yr., how much interest does he receive on his deposit?

20. Mrs. Hall deposits \$40 each month in a savings bank that pays 4%, compounded on Jan. 1 and July 1, on all sums on deposit on the first day of each month. If she deposits the money just before the first of the month, beginning the last of January, what will be her balance on the following Jan. 1?

21. In Ex. 20, if the bank credits interest on Jan. 1 and July 1 on all sums on deposit on Jan. 1, Apr. 1, July 1, and Oct. 1, what will be Mrs. Hall's balance on Jan. 1?

22. A man's account at a savings bank shows the following deposits: Jan. 2, 1928, \$950; Mar. 22, 1928, \$40; Apr. 12, 1928, \$65; Sept. 10, 1928, \$25; Mar. 8, 1929, \$125. The account shows also the following withdrawals: Feb. 1, 1928, \$35; Aug. 2, 1928, \$75; Sept. 7, 1928, \$125; Dec. 13, 1928, \$95; Jan. 2, 1929, \$385. The bank pays 4% on the smallest balance on deposit during an interest period, and compounds the interest on Jan. 1 and July 1. Find how much the man has to his credit on July 2, 1929.

23. A man deposits \$200 in a savings bank on Jan. 1, \$300 on Feb. 1, \$100 on May 1, \$400 on Aug. 1, and \$350 on Nov. 1. The bank allows interest at 4% from the first of each month, compounding it on July 1 and on Jan. 1. How much will the man have to his credit on Jan. 1 following these deposits? How much will he have on the following July 1?

Postal Savings System. The United States government conducts a savings bank, known as the *Postal Savings System*, in connection with the post office. Although every savings bank is supervised by the state in which it is located, many persons prefer a smaller rate of income from the Postal Savings System, because our national government guarantees the safety of their money.

Any person of the age of 10 yr. or over may open an account, and may make deposits (which must be in exact dollars), until the balance reaches $2500, exclusive of accumulated interest.

Interest is paid by the government at the rate of 2% for each full year that the money remains on deposit, beginning on the first day of the month following the one in which the deposit is made. Interest is not paid for any fraction of a year. The government does not compound the interest on these deposits, but if the interest is withdrawn when due and is at once deposited, the depositor will receive the equivalent of compound interest on his money.

A person may exchange his deposits for *postal savings bonds*, which are like promissory notes that bear interest. These bonds bear interest at $2\frac{1}{2}\%$, payable every 6 mo., and are issued in denominations of $20, $100, and $500. The exchange may be made on Jan. 1 or July 1 of any year.

The depositor receives a *postal savings certificate of deposit* for the amount of each deposit which he makes. These certificates are issued in the following denominations: $1, $2, $5, $10, $20, $50, $100, $200, and $500.

Since no deposit of less than $1 is allowed, the government sells *postal savings stamps* at 10¢ each. These stamps may be attached to a *postal savings card*, which has spaces for ten stamps. The card with one stamp already on it costs 10¢. When the card is filled up, it may be used as a deposit of $1, or the $1 may be collected in cash.

POSTAL SAVINGS SYSTEM

Numbers 1 to 16, oral

Find the interest for 1 yr. at 2% on a deposit of

1. \$20. **2.** \$50. **3.** \$100. **4.** \$400. **5.** \$800.

Find the interest for 2 yr. at 2% on a deposit of

6. \$10. **7.** \$40. **8.** \$200. **9.** \$700. **10.** \$1000.

Find the interest for 1 yr. at $2\frac{1}{2}\%$ received by a man who holds postal savings bonds for

11. \$20. **12.** \$100. **13.** \$300. **14.** \$500. **15.** \$1000.

16. Using as large denominations as possible, state the denominations of the certificates of deposit which you will receive for a deposit of \$87.

17. Saving 10¢ on each of the 312 week days of a certain year, a boy filled his postal savings card as fast as he could and then bought certificates of deposit. How much money did he have on deposit with the post office at the end of the year?

18. If you deposit \$4 on June 7 and \$16 on June 15, 1929, how much interest will be due on July 1, 1930?

19. If you deposit \$10 the last day of each month for 1 yr., beginning Jan. 31, 1928, how much do you have on deposit at the post office on Jan. 1, 1929? What is the first date on which any interest is due you, and how much is then due? How much interest is due on Feb. 1, 1930?

20. If in Ex. 19 you make no withdrawals of interest, how much interest is due you on Aug. 1, 1930?

21. On July 1 Mr. Watson exchanges \$500 in postal savings certificates for a postal savings bond paying $2\frac{1}{2}\%$. Find how much interest is due on this bond on the following Jan. 1.

A SILENT READING LESSON

Instead of requiring the bank book to be presented, some banks pay money from a depositor's account upon presentation of a written order called a *check*. Such a bank is sometimes called a *commercial bank*, to distinguish it from a savings bank. An account in such a bank is often called a *checking account*, and, if small, does not draw interest. The convenience of such an account is more important than the interest that the money might earn.

It is much safer to send a check from one place to another than to send cash.

When a man makes a deposit of money or of checks at his bank, he fills out a *deposit slip*, as here shown, and hands it in with his deposit. Most banks request that the name of the bank on which the check is drawn be given on the deposit slip when the two banks are located in the same city or town. In other cases the name of the place in which the other bank is located is generally sufficient.

Deposited to the Credit of
John P. Tait
in the
FIRST NATIONAL BANK
OF NEW ORLEANS
Date Feb. 7, 1929

Bills	31	—
Gold		
Silver	2	75
Checks:		
City National Bank	28	52
Lee Trust Co.	16	49
Washington, D.C.	7	75
	86	51

When the money to be deposited is taken to the bank, the teller who receives it checks the deposit with the items on the deposit slip to see that it is correct. When a deposit is made by mail, the bank fills out a printed receipt form and sends it to the depositor.

DEPOSIT SLIPS

Numbers 1 to 10, oral

State the amount of each of the following deposits:

1. $15.50	3. $12.75	5. $23.80	7. $12.00	9. $62.50
3.75	2.25	12.70	13.75	2.50
2. $10.00	4. $22.50	6. $45.00	8. $16.75	10. $75.50
5.75	2.50	5.75	10.00	4.50
1.25	5.00	1.25	3.25	20.00

Fill out a deposit slip for each of the following cases, using your name as depositor in some bank of which you know the name:

11. Bills, $254; silver, $40; check on First National Bank, $87.50; check on Traders Bank, $127.50.

12. Bills, $423; gold, $275; silver, $135.75; check on Cotton Exchange Bank, $342.60; check on City Trust Co., $67.50.

13. Bills, $135; check on Chemical National Bank, $325; check on Third National Bank, $63.73.

14. Bills, $1726; gold, $125; silver, $100; check on Traders Bank, $335.50; check on Second National Bank, $175.40.

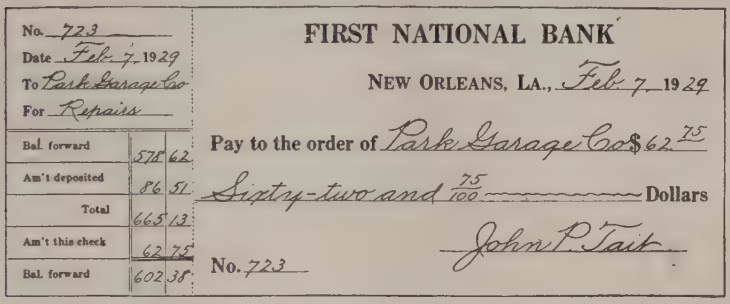
15. Bills, $145; gold, $480; silver, $350.75; check on Merchants National Bank, $255; check on Farmers Trust Co., $162.50; check on Second National Bank, $275.50.

16. Gold, $50; checks on First National Bank, $27.62, $41.75, $32.80; check on Lincoln Trust Co., $37.42; check on a Chicago bank, $67.85; check on a Detroit bank, $135.20.

17. Bills, $146; silver, $275.50; check on First National Bank, $43.50; check on State Trust Co., $250; check on a San Francisco bank, $125.60; check on a St. Louis bank, $37.65; check on a Rochester bank, $42.50.

Checks. As we have already learned, a check is an order written by the depositor directing his bank to pay out money from his account. A check is as good as money if the one who signs it has enough funds in the bank to pay the check.

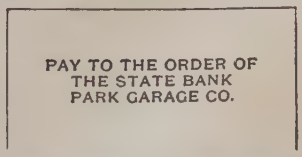
A *check book* containing blank checks and stubs is furnished to each depositor by the bank. When the check is filled out, it is detached, but the stub is left in the book. On the stubs the depositor puts down the amounts of the checks that he draws and of the deposits that he makes. He thus keeps a record of his bank balance, as shown in the following model:


No. 723
Date Feb. 7, 1929
To Park Garage Co
For Repairs
Bal. forward 578 62
Am't deposited 86 51
Total 665 13
Am't this check 62 75
Bal. forward 602 38
FIRST NATIONAL BANK
NEW ORLEANS, LA., Feb. 7, 1929
Pay to the order of Park Garage Co $62 75/100
Sixty-two and 75/100 Dollars
John P. Tait
No. 723

Stub **Check**

The person to whom a check is payable is called the *payee*. To collect the check, the payee must *indorse* it by writing his name across the back of the check.

If the payee wishes to make the check payable to some other person, for example, to the bank at which he wishes to deposit the check to his account, he may indorse it as here shown. Business houses often use a rubber stamp to indorse checks that are to be deposited. In this case only the State Bank can collect the check, and to do so must indorse it below the preceding indorsement.



DEPOSITS AND CHECKS

1. On Feb. 7 Mr. Tait, a page of whose check book is shown on page 131, makes out check No. 724 to the Corner Grocery Co. in payment of a bill of $14.63. Write the check and fill in the stub, showing his bank balance after paying this bill.

2. A man has $350 in the bank on the first of the month. During the month he pays by check the following bills: rent, $40; telephone, $3.50; milk, $4.27; groceries, $22.17; meat, $17.67; light, $3.86; coal, $72.20; insurance, $18.76. What is his balance on the last day of the month?

3. If your deposits in a bank have been $68.45, $92.30, $47.60, and $96.70, and you have drawn checks for $8.55, $23.65, $8.58, $48.75, and $34.60, what is your balance?

4. A merchant having $980.75 in the bank makes deposits during the next week of $185.50, $97.85, $135.50, $86.85, and $236.80. He draws checks for $89.65, $37.20, $93.60, $15.20, $248.70, and $39.80. What is his balance?

5. A man whose bank balance is $825.60 deposits checks for $75.80, $126.75, $234.80, and $42.80. He draws checks for $36.48, $7.85, $47.50, $128.75, $125.80, and $24.75. What is his balance?

6. A grocer having $828.50 in the bank deposits $567.80, $245.50, $89.65, $482.86, $429.50, and $376.50, and draws checks for $427.50, $38.95, $67.82, $568.70, and $122.58. He also pays by check one invoice for $173.96 and another for $86.75. What is his balance?

7. A restaurant keeper having $1026.92 in the bank makes deposits of $488.75, $928.75, $386.48, $442.80, $196.85, and $327.75, and draws checks for $96.75, $286.75, $342.80, and $438.50. He also pays back by check a loan of $230 on which interest is due for 4 mo. at 5%. What is his balance?

Pass Book. When an account is opened, the bank usually gives the depositor a *pass book*, in which is entered the amount deposited. Whenever an additional deposit is made at the bank, the book is handed to the receiving teller, who enters the amount and returns the book to the depositor.

Statement. While some banks balance the pass book, most large banks use it simply as the customer's receipt for money deposited and, at the end of each month, send the depositor a *statement* of his account in some such form as the following:

FIRST NATIONAL BANK
OF NEW ORLEANS, LA.

John P. Tait
624 Forsyth St., City

Statement for the month of FEBRUARY, 1929

CHECKS		DEPOSITS	DAY	BALANCE
Balance brought forward			1	686.72
108.10			2	578.62
		86.51	7	665.13
62.75	14.63		9	587.75
53.54		95.00	15	629.21
32.70	6.74		18	589.77
		167.87	25	757.64
12.00	4.66		26	740.98
Balance forward			28	740.98

The deposits and other credits are added to the balance from the previous month's statement, and the checks are deducted as they are paid by the bank. The column at the right shows the daily balance, and the last entry shows the balance to be carried to the next month's statement.

With the statement the bank returns the checks that it has paid during the month. The depositor checks this statement with his own record, as kept in his check book.

BANK STATEMENTS

1. The bank statement for October shows that a firm has a balance of $7632.96 on Oct. 31. The pass book shows the following deposits in November: *6*, $3758.60; *9*, $2843.85; *12*, $2976.42; *16*, $4826.75; *20*, $2928.85; *24*, $3174.30; *29*, $2672.29. The check book shows the following checks issued in November: *2*, $689.85; *3*, $392.90, $167.75; *5*, $3228.45, $7.90, $654.68; *9*, $850, $525.35; *14*, $796.80, $2285.40; *20*, $948.65; *22*, $3689.85; *29*, $34.80, $340, $10.26, $3.92, $52.30, $128.75, $68.38. What is the firm's balance on Nov. 30?

2. For Ex. 1 make out the statement which the bank sends the firm on Nov. 30, using a form like the one on page 133 and inserting the name of some bank and firm in your vicinity. Consider that the checks issued were paid by the bank on the day they were drawn, although they may actually have been paid some time thereafter.

3. A merchant's bank balance was $14,296.35 on Oct. 1. Deposits were made during the month as follows: *8*, $2978.30; *14*, $2285.85; *18*, $2862.75; *22*, $2438.70; *25*, $3162.62; *28*, $1657.37. Checks were drawn as follows: *6*, $697.28, $1128.67, $231.27; *13*, $268.92, $765.43, $1.47; *16*, $1428.83, $647.26, $2786; *19*, $11.28, $350, $591.23; *25*, $1100; *29*, $6.27, $285, $2.89, $15.75, $28.50, $528.92. Make out the bank statement for October as in Ex. 2.

4. Make out a bank statement for December, as in Ex. 2, using the following items: Balance, *Dec. 1*, $2385.60. Deposits: *2*, $495.65; *3*, $229.86; *6*, $308.75; *8*, $445.65; *10*, $638.80; *28*, $1275.30. Checks: *3*, $50, $74.28, $61.97; *7*, $221.62, $45; *8*, $4.27, $940, $78.26; *14*, $16.30, $28.16, $97.80; *18*, $1225, $87.30; *25*, $1.47, $6.89, $3.26; *29*, $41.60, $12.30, $2.48, $75.

A SILENT READING LESSON

Business houses usually have need of large sums of money at certain times of the year, and it is better for them to borrow it for two or three months than to keep such an amount on hand for a whole year. Borrowing money, therefore, is often good business policy for the merchant, contractor, or manufacturer, although it is poor policy for any person to borrow money simply to gratify some extravagant desire.

It is now the custom for business houses and for many private individuals to borrow money at banks. The banks obtain the money for these loans from their depositors, who find it safer and more convenient to keep their money in a bank than to lock it up in their homes or offices. The banks are paid for their work by charging borrowers a higher rate of interest than they pay to depositors.

If a business man wishes to borrow money from a bank, the bank will sometimes lend it to him on the strength of his financial standing alone.

A reputation for honesty and promptness in business dealings is a very valuable asset for an individual or for a company. Start life with a good reputation and never lose it.

If a man's financial standing is not such as to permit an unsecured loan, he may deposit with the bank sufficient *collateral,* which means property of value such as stocks and bonds about which we shall learn more later. The bank has the right to sell the collateral in case the note is not paid.

Instead of depositing collateral, the borrower may get some financially sound person to indorse his note, the indorser thereby agreeing to pay it if the maker does not. Some large city banks have about a third of their loans secured by collateral, and the rest of their loans are made on notes either with or without indorsers.

Borrowing on a Note. Suppose that Mr. Taylor wishes to borrow $1000 for about 2 mo. If the prospects for his business are satisfactory, and if he has a good reputation in financial matters, the bank with which he does business may lend him the money without requiring a deposit of collateral. Mr. Taylor will then fill out a note like the following model:

$1000 00 LOUISVILLE, KY., May 15, 1929

Sixty days after date I promise to pay

to the order of The Exchange National Bank

One thousand no/100 Dollars,

value received.

No. 547 Due July 14, 1929 William F. Taylor

The bank keeps this note until the date of maturity, which is 60 da. after May 15, 1929, or July 14, 1929. The bank then charges the note to Mr. Taylor's account, unless he has made some other arrangement for paying it, notifies him that it has done so, and returns the note to him.

If Mr. Taylor wishes to keep the money for longer than 60 da., the bank will probably have him make out a new note for 30 da., 60 da., or 90 da. more, and will cancel the old one.

If Mr. Taylor's financial standing is not such as to warrant lending the money to him without some security, he may be asked to get an indorser. In such a case he would make the note payable to some person known to the bank, and this person would indorse it to the bank, thus promising to pay the money if Mr. Taylor failed to do so.

Sometimes, instead of indorsing a note in this way, the indorser signs his name below that of the maker; that is, there are then two makers. Such a note is called a *joint note.*

Note Secured by Collateral. The following model shows a typical form of a note secured by the deposit of collateral:

$5600 NEW YORK, Oct 1, 1929

Ninety days after date, FOR VALUE RECEIVED, the undersigned promises to pay to the order of

WELLS NATIONAL BANK OF THE CITY OF NEW YORK

(hereinafter called the Bank), at its Banking Office in New York City,

Fifty-six hundred no/100 Dollars,

in United States gold coin or its equivalent, having deposited with the Bank, as collateral security for the payment of this note, or any note given in extension or renewal thereof, as well as for the payment of any other obligation or liability, direct or contingent, of the undersigned to the Bank, due or to become due, whether now existing or hereafter arising, the following property, viz.:

Nos 4769-77 Southern Railway, 5% bonds, Series A, due 1956

of a market value estimated by the undersigned at $7240.—; and the undersigned agrees to deliver to the Bank additional securities, or to make payments on account to its satisfaction, should the market value of the said securities, as a whole, suffer any decline. The undersigned hereby gives to the Bank a lien for the amount of all such obligations and liabilities upon all the property or securities now or at any time hereafter given unto or left in the possession of the Bank by the undersigned, whether for the express purpose of being used by the Bank as collateral security, or for any other or different purpose, and also upon any balance of the deposit account of the undersigned, with the Bank.

Philip R. Shaw

In the above case the collateral deposited with the bank consists of nine Southern Railway bonds for $1000 each. When we study bonds, we shall understand why they are property of value that can be pledged in this way. Banks usually require that the face of a note shall be not greater than about 80% of the market value of the collateral.

It is not necessary to try to remember much about such a note. We simply need to know what it means when we hear some one speak of "putting up collateral" to secure a loan.

Interest by the Six-Per-Cent Method. The six-per-cent method, mentioned on page 109, is so commonly used in banking, where the notes often bear 6% and run for 30 da., 60 da., or 90 da., that it will help us to review it briefly and to learn a little more about it.

Briefly stated, the principle of the method is as follows:

To find the interest at 6% for 60 da., move the decimal point two places to the left; for 6 da., move it three places to the left.

In applying such a rule, we, of course, remember that a sum like \$250 is the same as \$250. or \$250.00; that is, any whole number may be thought of as having a decimal point.

For example, the interest at 6% for 60 da. on \$250 is \$2.50, and on \$765.50 it is \$7.655, or \$7.66. Similarly, at 6% the interest for 6 da. on \$1250 is \$1.25, and on \$1875 it is \$1.875, or \$1.88.

For 30 da. the interest is half that for 60 da.; for 90 da. it is one and a half times that for 60 da. Thus, the interest on \$1200 at 6% for 30 da. is $\frac{1}{2}$ of \$12, or \$6, and for 90 da. it is \$12 + \$6, or \$18.

NOTES BEARING INTEREST AT 6%

Find the interest at 6% on a note for

1. \$120 for 60 da.
2. \$270 for 90 da.
3. \$525 for 30 da.
4. \$2575 for 90 da.
5. \$3625 for 90 da.
6. \$4250 for 30 da.
7. \$3850 for 60 da.
8. \$4000 for 60 da.
9. \$7500 for 90 da.
10. \$160 for 2 mo. 6 da.
11. \$450 for 2 mo. 12 da.
12. \$725 for 2 mo. 15 da.
13. \$850 for 2 mo. 20 da.
14. \$550 for 1 mo. 15 da.
15. \$1275 for 1 mo. 20 da.
16. \$6250 for 1 mo. 10 da.
17. \$7500 for 3 mo. 15 da.
18. \$8000 for 3 mo. 20 da.

Six-Per-Cent Method for Other Rates. We may use the six-per-cent method to find interest at other rates in common use.

For example, since 5 is $\frac{1}{6}$ less than 6, the interest at 5% on a given principal is $\frac{1}{6}$ less than the interest at 6%. The work at the right shows how we apply this principle in computing the interest due on \$1260 for 90 da. at 5%.

Interest for 60 da. at 6%,	\$12.60
Interest for 30 da.,	6.30
Interest for 90 da. at 6%,	\$18.90
Deduct $\frac{1}{6}$,	3.15
Interest for 90 da. at 5%,	\$15.75

Similarly, the interest at $5\frac{1}{2}$% is $\frac{1}{12}$ less than the interest at 6%, the interest at $4\frac{1}{2}$% is $\frac{1}{4}$ less than the interest at 6%, the interest at 4% is $\frac{1}{3}$ less than the interest at 6%, and so on for other rates of interest.

SIX-PER-CENT METHOD FOR OTHER RATES

Find the interest at 5% for these principals and periods:

1. \$780, 30 da.
2. \$840, 60 da.
3. \$86.40, 60 da.
4. \$79.20, 30 da.
5. \$1200, 90 da.
6. \$1560, 90 da.

Find the interest at 4% for these principals and periods:

7. \$240, 30 da.
8. \$630, 60 da.
9. \$68.80, 60 da.
10. \$72.24, 30 da.
11. \$1800, 90 da.
12. \$2436, 90 da.

Find the interest at $5\frac{1}{2}$% for these principals and periods:

13. \$720, 30 da.
14. \$960, 60 da.
15. \$60.72, 60 da.
16. \$97.68, 30 da.
17. \$5600, 90 da.
18. \$4048, 90 da.

Find the interest at $4\frac{1}{2}$% for these principals and periods:

19. \$160, 30 da.
20. \$920, 60 da.
21. \$54.80, 60 da.
22. \$62.48, 30 da.
23. \$6000, 90 da.
24. \$8648, 90 da.

Bank Discount. When a man borrows money from a bank the bank may require him to pay the interest in advance, although not all banks do so. Interest paid in advance is called *discount*, and the period for which discount is charged is called the *term of discount*.

The word "discount" is thus used both in banking and in buying goods. The arithmetic involved is the same in both cases; that is, it is finding a per cent of a number.

Proceeds of a Note. The face of a note less the discount is called the *proceeds* of the note.

For example, suppose that a 90-day note for $500 is discounted by a bank at 6%.

By the six-per-cent method, as shown in the first step at the right, the discount (interest) is found to be $7.50.

As shown in the second step, the proceeds are then found to be $492.50.

$5.00	$500.00
2.50	7.50
$7.50	$492.50

BANK DISCOUNT

When discounted at 6%, find the proceeds of a note for

1. $75 for 60 da.	4. $250 for 90 da.	7. $1500 for 60 da.
2. $90 for 30 da.	5. $275 for 90 da.	8. $1800 for 63 da.
3. $50 for 90 da.	6. $425 for 60 da.	9. $4500 for 93 da.

Find the discount at 5% when the face of the note and the term of discount are as follows:

10. $40, 30 da.	15. $900, 36 da.	20. $3560, 30 da.
11. $70, 60 da.	16. $1250, 30 da.	21. $4550, 60 da.
12. $80, 90 da.	17. $2275, 60 da.	22. $7725, 90 da.
13. $175, 66 da.	18. $4750, 90 da.	23. $8250, 60 da.
14. $250, 78 da.	19. $6200, 30 da.	24. $6575, 90 da.

BANK DISCOUNT

1. Make out a note for \$1200, dated today and payable to your order in 60 da. with interest at 6%, and sign it with the initials X.Y.Z.

2. Make out a 30-day note without interest for \$675, dated today and payable to the order of the County National Bank, and sign it A.B.C. If the bank discounts the note today at 6%, what are the proceeds?

3. Make out a 90-day note without interest for \$1250, dated today and payable to the order of the Blank Trust Co. If the note is discounted today at 6%, what are the proceeds?

4. A man's bank account shows deposits of \$37.50, \$82.50, \$75, \$100, \$50, \$300, \$40, \$125, \$500, and checks drawn for \$13.75, \$62, \$5, \$125.50, \$82.75. He needs \$2000 to start him in business and wishes to keep about \$500 in the bank. How much, to the nearest \$100, should he borrow?

5. If, in Ex. 4, the man's bank discounts his note for this sum for 90 da., what are the proceeds?

6. A merchant has to pay a debt of \$1275 less 10%. His balance in the bank is \$672.80, and he has \$107.50 in cash in his safe. He wishes to leave about \$300 in the bank and about \$65 in his safe. How much, to the nearest \$50 must he borrow? If he gives a 90-day note for this sum, and the note is discounted the same day at 6%, the proceeds are how much more or how much less than the exact sum which he needs?

Find the proceeds when the face of a note, the period of discount, and the rate of discount are as follows:

7. \$875, 30 da., 6%.

8. \$975, 90 da., 5%.

9. \$425, 30 da., 6%.

10. \$2565, 90 da., 5%.

11. \$3250, 90 da., $5\frac{1}{2}$%.

12. \$4250, 30 da., $4\frac{1}{2}$%.

Discounting an Interest-bearing Note. A note is like any other commodity, something that may be sold. Banks, being dealers in money, are the institutions to which to sell notes. Thus, if you hold a note given you by someone else and need the money before the note is due, you may sell it to a bank. The bank will, of course, charge you discount for advancing the money on the note.

For example, suppose that you hold T. F. Hoyt's note for $500, dated Sept. 1 and due Dec. 1, with interest at 5%. If you discount the note at your bank on Sept. 26 at 6%, what are the proceeds?

Since the interest on the note will be paid to the bank, the first step is to find the amount of the note on Dec. 1. From a table the bank finds that there are 66 da. from Sept. 26 to Dec. 1 (Sept., 4 da.; Oct., 31 da.; Nov., 30 da.; Dec., 1 da.), and computes the discount for this term on the amount of the note. Deducting this discount, as shown in the last step of the following solution, the proceeds paid you are $500.68:

Face of note,	$500	60 da.,	$5.0625
Rate of interest,	0.05	6 da.,	.50625
	4)$25.00	Discount,	$5.56875
Interest due,	$6.25	Amount,	$506.25
Face,	$500.00	Discount,	5.57
Amount, Dec. 1,	$506.25	Proceeds,	$500.68

Federal Reserve Banks. If banks wish to borrow money, they may do so, subject to certain restrictions, from the *Federal Reserve Banks* which have been established by the government in different parts of the country. That is, briefly stated, these government banks may be said to act as banks for other banks.

DISCOUNTING NOTES

Find the discount and the proceeds for each of the following notes, which draw no interest, when the face, the term of discount, and the discount rate are as follows:

1. \$750, 60 da., 5%.
2. \$225, 60 da., 5%.
3. \$500, 60 da., 6%.
4. \$450, 60 da., 5%.
5. \$225, 90 da., 5%.
6. \$5575, 90 da., 6%.
7. \$7500, 30 da., 5%.
8. \$4750, 30 da., 6%.
9. \$42.50, 30 da., 6%.
10. \$125.50, 60 da., 6%.
11. \$275.25, 60 da., 6%.
12. \$375.50, 90 da., 6%.
13. \$457.50, 30 da., 5%.
14. \$296.50, 30 da., 5%.
15. \$287.60, 90 da., 5%.
16. \$375.40, 90 da., 5%.
17. \$2750, 90 da., $4\frac{1}{2}$%.
18. \$3275, 90 da., $4\frac{1}{2}$%.

19. The discount at 6% on a note for 90 da. is \$18.75, and on another note for 30 da. the discount is \$32.50. If neither note draws interest, what is the face of each note?

20. On Oct. 4 a dealer bought \$1750 worth of goods, giving a 90-day note bearing interest at 5%. On Oct. 10 the holder of the note discounted it at 6%. Find the proceeds.

Find the proceeds of each of the following notes:

	Face of Note	Date of Note	Date Due	Rate of Interest	Date of Discount	Rate of Discount
21.	\$300	July 1	July 31	4%	July 16	6%
22.	\$800	Sept. 1	Oct. 31	5%	Sept. 11	6%
23.	\$550	Oct. 1	Dec. 30	4%	Dec. 2	6%
24.	\$325	Mar. 1	Apr. 30	5%	Mar. 17	6%
25.	\$1000	Nov. 1	Dec. 1	6%	Nov. 1	6%
26.	\$1250	June 1	Aug. 30	5%	July 23	5%

REVIEW OF BANKING

1. Find the interest on \$1750 for 60 da. at 6%, first by the six-per-cent method, then by cancellation, and finally by the ordinary method of finding the interest for 1 yr. and then for the fractional part of a year. Write a statement telling the advantage of the six-per-cent method over the other methods.

2. Using the three methods mentioned in Ex. 1, find the interest on \$675 for 90 da. at 6%; on \$1425 for 30 da. at 6%.

3. A firm wishes to borrow about \$5000, and a bank offers to lend the money at 6%. If the firm discounts a 60-day note for \$5100, how much more than \$5000 will the firm actually receive after the discount is paid?

4. A real-estate dealer buys some property for \$15,000. He pays \$4800 down and borrows the rest from his bank on a 90-day note at 6%. How much discount does he pay?

5. How much greater, if any, is the discount on a note for \$1250 discounted for 60 da. at 6% than the discount on a note for \$2500 discounted for 30 da. at 6%?

6. A man who wishes to borrow \$4875 gives his bank a 30-day note for \$4900. If the bank discounts the note at 6%, will the proceeds be more or will they be less than the amount he needs? How much more or how much less will they be?

7. On Apr. 1 a dealer buys 16 automobiles at \$1285, paying \$8060 cash and giving a 90-day note, bearing interest at 6%, for the balance. If the manufacturer who holds the note discounts it at 6% on Apr. 16, how much discount does he pay? What are the proceeds of the note?

8. If a discount of 2% is allowed for cash on a bill of \$1850, and a bank will discount a 30-day note for \$1850 at 6%, how much does a merchant gain by borrowing the money at the bank and taking advantage of the cash discount?

A SILENT READING LESSON

If Mr. Brasch, who has an account at the Central National Bank, sends a check to Mr. Locke, who has an account at the West End Trust Co. in the same city, Mr. Locke would probably deposit the check at the West End Trust Co.

In a business day the West End Trust Co. may thus take in a good many checks which are drawn on the Central National Bank. Each day representatives from the different city banks meet at a place known as the *clearing house*, where they exchange checks and settle the balances due one another. In this way Mr. Brasch's check is returned to the Central National Bank, where it is charged to his account.

A similar procedure takes place in case the two persons live in different places. For example, a man in San Francisco might deposit at his bank a check drawn on a New York bank. If the San Francisco bank wished to increase its deposit with a Chicago bank with which it does business, the check might be sent to the Chicago bank and there credited to the account of the San Francisco bank. The Chicago bank, in turn, would send it to a New York bank with which it does business and receive credit for the amount. The New York bank on which the check is drawn would receive the check through the clearing house, as described above.

Some idea of the importance to modern business of checks and similar forms of transmitting funds may be gained from the fact that the New York City "clearings" alone often total more than a billion dollars a day.

Sometimes banks make a small collection charge to pay for the expense and work of collecting checks drawn on banks at distant points. Checks drawn on banks in large commercial centers, such as New York, Chicago, Philadelphia, and so on, are often collected for a depositor without any charge.

Paying Bills by Bank Draft. Suppose that Joseph T. Martin of Austin, Texas, wishes to order farm tools from the Farmers Specialty Co. of Chicago. He might send his personal check, but if the company does not know him it may not be willing to accept it. Accordingly he asks his bank in Austin, the Traders National Bank, for a "draft on Chicago" for the desired amount. The Traders Bank has money on deposit in the West National Bank of Chicago, and so issues the following order, called a *bank draft*, against this account:

TRADERS NATIONAL BANK

No. 1824 AUSTIN, TEXAS, July 10, 1929

Pay to the order of Joseph T. Martin $76.35

Seventy-six and 35/100 Dollars

To the West National Bank
Chicago, Ill.

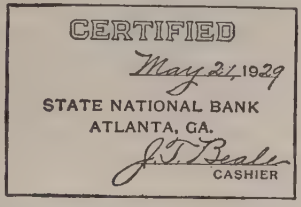
Arnold L. Russell
Cashier

Mr. Martin then indorses the draft as here shown and sends it to the Farmers Specialty Co. with his order. Since the draft is really a check of the Traders National Bank instead of a personal check, there will be no question of its payment. Drafts on the large financial centers are accepted freely in all parts of the country and are used extensively in paying bills.

Pay to the order of the
Farmers Specialty Co.
Joseph T. Martin

A draft made by a cashier upon his own bank is usually called a *cashier's check*, and may be used for transmitting funds in the same way as a bank draft. While a personal check is a kind of draft, because it directs a bank to pay money, the word "draft" is not used in this connection. In fact, the word "check" is now often used in place of "draft."

Certified Check. To avoid any doubt as to a man's having sufficient money in the bank with which to pay a personal check when it is presented, he may have the check *certified*, which means that the bank guarantees the payment of the check. To do this the man takes the check to his bank and has the cashier, or some other officer, stamp it across the face in some such way as here shown. The bank at once takes the money from the man's account and holds it in a separate account until the check is returned for payment.



Letter of Credit. If a person is traveling, either in this country or abroad, it is necessary for him to obtain money in various places. It is difficult for him to cash personal checks in places where he is not known, and if he has to carry with him sufficient money to cover all his expenses, he runs the risk of theft or loss. One of the methods of avoiding such difficulties is that of carrying a *letter of credit*.

For example, a person may deposit $5000 with a bank that sells letters of credit, and this bank will, for a reasonable compensation (usually 1% or less), give him a letter, signed by the bank officials and addressed to a large number of other banks, telling them that it guarantees the payment of any drafts drawn by the holder of the letter up to a total of $5000.

When a draft is drawn on a letter of credit, the bank that pays the money writes on the letter a statement of the amount drawn. The holder of the letter is identified by his signature, which is either on the letter or, more commonly, in a book of instructions accompanying the letter.

Express money orders and *travelers' checks* are often used in place of the letter of credit, but the principle of identification by means of a comparison of signatures is the same.

Paying Bills by Money Order. In case you do not have a checking account at some bank and wish to send money to a distant place, you can easily do so by means of a *money order*. Such orders may be purchased at a post office, or from an express company, by payment of a small fee in addition to the face of the order. The order is mailed to the person to whom you are sending the money, and he can obtain the money at his post office or express office.

The fees for issuing postal money orders for use in this country are as follows:

For sums not exceeding \$2.50, 5¢; for amounts from \$2.51 to \$5, 7¢; \$5.01 to \$10, 10¢; \$10.01 to \$20, 12¢; \$20.01 to \$40, 15¢; \$40.01 to \$60, 18¢; \$60.01 to \$80, 20¢; \$80.01 to \$100, 22¢.

MONEY ORDERS

Numbers 1 to 16, oral

State the cost (face plus fee) of a postal money order for

1. \$25.	**5.** \$31.50.	**9.** \$92.30.	**13.** \$92.75.
2. \$60.	**6.** \$19.90.	**10.** \$69.95.	**14.** \$60.32.
3. \$80.	**7.** \$52.75.	**11.** \$16.30.	**15.** \$80.41.
4. \$100.	**8.** \$86.50.	**12.** \$90.50.	**16.** \$33.70.

17. When the United States government charges 22¢ for issuing a postal money order for \$100, what per cent of the face does it charge?

18. If you wish to send \$345 by means of postal money orders, you will have to buy three orders for \$100 each and one for \$45. How much, including both the face and the fee in each case, do you have to pay for the four orders?

19. When an express company charges 18¢ for issuing an express money order for \$75, what per cent of the face does the company charge?

III. MEASUREMENTS

A SILENT READING LESSON

Last year we learned how to measure the lengths of lines and how to find the area of each one of the following figures:

Square Rectangle Parallelogram Triangle

Before we study more about measurements, we should review this work. In the cases of the square, the rectangle, and the parallelogram, as we already know,

The area is the product of the base and height,

or briefly, $Area = base \times height.$

For example, for a square that is 6 in. on a side, we have

$$6 \times 6 \text{ sq. in.} = 36 \text{ sq. in., the area.}$$

Similarly, if we wish to find the area of a rectangle that is 8 in. long and 6 in. wide, we have

$$8 \times 6 \text{ sq. in.} = 48 \text{ sq. in., the area.}$$

In the same way, if we wish to find the area of a parallelogram with a base of 9 in. and a height of $2\frac{1}{2}$ in., we have

$$9 \times 2\tfrac{1}{2} \text{ sq. in.} = 22\tfrac{1}{2} \text{ sq. in., the area.}$$

In finding the area of a triangle, we have this relation:

$$Area = \tfrac{1}{2} \times base \times height.$$

Thus, in finding the area of a triangle with a base of 7 in. and a height (or altitude) of 3 in., we have

$$\tfrac{1}{2} \times 7 \times 3 \text{ sq. in.} = 10\tfrac{1}{2} \text{ sq. in., the area.}$$

In the following review of the common measures we should refer to the tables on pages 483–484, if necessary.

REVIEW DRILL CHART ON THE TABLES OF MEASURES

State rapidly the number of inches in

1. 1 ft. **2.** 6 ft. **3.** 1 yd. **4.** $\frac{1}{2}$ yd. **5.** $\frac{3}{4}$ yd.

State rapidly the number of feet in

6. 24 in. **7.** 1 yd. **8.** 1 rd. **9.** 1 mi. **10.** 7 yd.

State rapidly the number of yards in

11. 3 ft. **12.** 9 ft. **13.** 1 rd. **14.** 36 in. **15.** 18 in.

State rapidly the number of rods in

16. $16\frac{1}{2}$ ft. **17.** $5\frac{1}{2}$ yd. **18.** 11 yd. **19.** 33 ft. **20.** 1 mi.

State rapidly the number of ounces in

21. 1 lb. **22.** 2 lb. **23.** $\frac{1}{2}$ lb. **24.** $\frac{1}{4}$ lb. **25.** $\frac{3}{4}$ lb.

State rapidly the number of pounds in

26. 16 oz. **27.** 32 oz. **28.** 24 oz. **29.** 8 oz. **30.** 1 T.

State rapidly the number of pints in

31. 1 qt. **32.** $\frac{1}{2}$ qt. **33.** $2\frac{1}{2}$ qt. **34.** 1 gal. **35.** $\frac{1}{2}$ gal.

State rapidly the number of quarts in

36. 1 gal. **37.** 6 pt. **38.** 1 pk. **39.** 3 gal. **40.** 1 bu.

State rapidly the number of pecks in

41. 1 bu. **42.** $\frac{1}{2}$ bu. **43.** $1\frac{1}{2}$ bu. **44.** 8 qt. **45.** 32 qt.

State rapidly the number of square rods in

46. 1 A. **47.** $\frac{1}{2}$ A. **48.** $\frac{1}{4}$ A. **49.** $1\frac{1}{2}$ A. **50.** 2 A.

State rapidly the number of square feet in

51. 144 sq. in. **52.** 1 sq. yd. **53.** 5 sq. yd. **54.** $1\frac{1}{2}$ sq. yd.

REVIEW DRILL CHART IN ADDING MEASURES

Perform the following addition rapidly:

1. 2 ft. 7 in.
   3 ft. 9 in.

2. 5 ft. 4 in.
   1 ft. 8 in.
   3 ft. 7 in.

3. 2 lb. 7 oz.
   3 lb. 9 oz.

4. 5 lb. 4 oz.
   1 lb. 8 oz.
   3 lb. 7 oz.

5. 2 yd. 7 in.
   3 yd. 9 in.

6. 5 yd. 4 in.
   2 yd. 3 in.
   1 yd. 8 in.
   3 yd. 7 in.

7. 5 ft. 2 in.
   3 ft. 8 in.
   7 ft. 6 in.
   7 in.
   3 ft. 5 in.
   2 ft. 8 in.

8. 2 qt. 1 pt.
   5 qt. 1 pt.

9. 2 qt. 1 pt.
   3 qt. $1\frac{1}{2}$ pt.
   4 qt. $\frac{3}{4}$ pt.

10. 2 bu. 1 pk.
    5 bu. 1 pk.

11. 2 bu. 1 pk.
    3 bu. $1\frac{1}{2}$ pk.
    4 bu. $\frac{3}{4}$ pk.

12. 2 pk. 1 qt.
    5 pk. 1 qt.

13. 2 pk. 1 qt.
    3 pk. $2\frac{1}{2}$ qt.
    3 pk. $1\frac{1}{2}$ qt.
    4 pk. $\frac{3}{4}$ qt.

14. 2 qt. 1 pt.
    2 qt. $\frac{1}{2}$ pt.
    6 qt. $1\frac{1}{2}$ pt.
    5 qt. 1 pt.
    $\frac{3}{4}$ pt.
    2 qt. $1\frac{1}{4}$ pt.

15. 4 T. 1700 lb.
    6 T. 1500 lb.

16. 5 T. 1400 lb.
    6 T. 1200 lb.
    9 T. 1400 lb.

17. 4 mi. 1700 yd.
    6 mi. 1500 yd.

18. 5 mi. 1400 yd.
    6 mi. 1200 yd.
    9 mi. 1400 yd.

19. 4 mi. 1700 ft.
    6 mi. 1500 ft.

20. 5 mi. 1400 ft.
    2 mi. 1800 ft.
    6 mi. 1200 ft.
    9 mi. 1400 ft.

21. 5 T. 800 lb.
    1 T. 1200 lb.
    1800 lb.
    3 T. 1600 lb.
    4 T. 1700 lb.
    1800 lb.

REVIEW DRILL CHART IN SUBTRACTING MEASURES

Perform the following subtractions rapidly:

1. 7 ft. 2 in. 3 ft. 7 in.	11. 9 bu. 20 qt. 6 bu. 25 qt.	21. 4 yr. 170 da. 1 yr. 200 da.
2. 7 yd. 2 in. 3 yd. 7 in.	12. 9 yd. 20 in. 6 yd. 25 in.	22. 4 mi. 170 yd. 1 mi. 200 yd.
3. 7 rd. 2 in. 3 rd. 7 in.	13. 9 rd. 20 in. 6 rd. 25 in.	23. 4 mi. 170 rd. 1 mi. 200 rd.
4. 7 mi. 2 ft. 3 mi. 7 ft.	14. 9 mi. 20 rd. 6 mi. 25 rd.	24. 4 T. 170 lb. 1 T. 200 lb.
5. 7 lb. 2 oz. 3 lb. 7 oz.	15. 9 mi. 20 ft. 6 mi. 25 ft.	25. 4 T. 200 lb. 1 T. 170 lb.
6. 7 bu. 2 qt. 3 bu. 7 qt.	16. 9 T. 20 lb. 6 T. 25 lb.	26. 4 mi. 170 ft. 1 mi. 200 ft.
7. 7 gal. 2 pt. 3 gal. 7 pt.	17. 9 hr. 20 min. 6 hr. 25 min.	27. 4 mi. 200 ft. 1 mi. 170 ft.
8. 8 rd. 2 yd. 5 rd. 3 yd.	18. 9 mo. 20 da. 6 mo. 25 da.	28. 4 sq. yd. 170 sq. in. 1 sq. yd. 200 sq. in.
9. 6 ft. 1 in. 3 ft. 9 in.	19. 8 yd. 19 in. 28 in.	29. 3 sq. ft. 1 sq. ft. 100 sq. in.
10. 8 ft. 2 in. 7 ft. 8 in.	20. 9 mi. 30 ft. 6 mi. 75 ft.	30. 3 sq. yd. 1 sq. yd. 100 sq. in.

REVIEW DRILL CHART IN MULTIPLYING MEASURES

Perform the following multiplications rapidly:

1. 2 ft. 9 in. × 2	11. 3 ft. 2 in. × 24	21. 2 mi. 1800 ft. × 3
2. 2 yd. 9 in. × 2	12. 3 lb. 2 oz. × 24	22. 2 T. 1800 lb. × 3
3. 2 rd. 9 in. × 2	13. 3 yd. 2 in. × 24	23. 5 hr. 48 min. × 6
4. 2 lb. 9 oz. × 2	14. 3 bu. 2 pk. × 24	24. 5 mi. 48 rd. × 6
5. 4 ft. 1 in. × 20	15. 7 ft. 3 in. × 48	25. 5 T. 48 lb. × 6
6. 4 lb. 1 oz. × 20	16. 7 gal. 3 qt. × 48	26. 5 min. 48 sec. × 6
7. 4 gal. 1 qt. × 20	17. 7 yd. 3 in. × 48	27. 5 yr. 48 wk. × 6
8. 4 bu. 1 pk. × 20	18. 7 gal. 3 pt. × 48	28. 5 wk. 48 hr. × 6
9. 7 ft. 6 in. × 15	19. 9 ft. 3 in. × 50	29. 5 hr. 48 sec. × 6
10. 8 lb. 2 oz. × 24	20. 7 ft. 9 in. × 12	30. 5 da. 48 min. × 6

REVIEW DRILL CHART IN DIVIDING MEASURES

Perform the following divisions rapidly:

1. 18 ft. 6 in. by 2.
2. 18 lb. 6 oz. by 2.
3. 19 ft. 8 in. by 4.
4. 19 lb. 8 oz. by 4.
5. 23 yd. 9 in. by 5.
6. 23 ft. 9 in. by 5.
7. 14 bu. 3 pk. by 6.
8. 14 ft. 3 in. by 6.
9. 14 lb. 3 oz. by 6.
10. 14 yd. 3 in. by 6.
11. 14 gal. 3 qt. by 6.
12. 14 gal. 3 pt. by 6.
13. 23 lb. 2 oz. by 8.
14. 23 yd. 2 ft. by 8.
15. 23 yd. 2 in. by 8.
16. 23 bu. 2 pk. by 8.
17. 23 ft. 2 in. by 8.
18. 23 rd. 2 ft. by 8.
19. 23 rd. 2 in. by 8.
20. 23 gal. 2 qt. by 8.
21. 23 gal. 2 pt. by 8.
22. 23 mi. 2 yd. by 8.
23. 23 mi. 2 rd. by 8.
24. 23 mi. 2 ft. by 8.
25. 8 ft. by 2.
26. 8 ft. by 2 ft.
27. 8 ft. 2 in. by 2.
28. 8 ft. 2 in. by 2 ft.
29. 8 ft. 2 in. by 2 in.
30. 8 lb. 2 oz. by 2 lb.
31. 8 lb. 2 oz. by 2 oz.
32. 8 yd. 2 in. by 2 yd.
33. 8 yd. 2 in. by 2 in.
34. 8 gal. 2 pt. by 2 gal.
35. 8 gal. 2 pt. by 2 pt.
36. 27 lb. 2 oz. by 12 lb.
37. 27 lb. 2 oz. by 12 oz.
38. 27 ft. 2 in. by 12 ft.
39. 27 ft. 2 in. by 12 in.
40. 27 yd. 2 ft. by 12 yd.
41. 27 yd. 2 ft. by 12 ft.
42. 47 yd. 2 ft. by $\frac{1}{2}$ yd.
43. 47 yd. 2 ft. by $\frac{1}{4}$ ft.
44. 47 yd. 2 ft. by $\frac{1}{8}$ in.
45. 50 lb. 8 oz. by $\frac{1}{2}$ lb.
46. 50 lb. 8 oz. by $\frac{1}{2}$ oz.
47. 50 ft. 8 in. by $2\frac{1}{2}$ ft.
48. 50 ft. 8 in. by $2\frac{1}{2}$ in.

Ruler and Compasses. We can draw a straight line by means of a ruler, and a circle by means of compasses. If we do not have a ruler, we can fold a piece of paper and use the folded edge. If we do not have compasses, we can draw the circle around the rim of a coin.

The following figures illustrate *perpendicular lines*, which are straight lines that form *right angles* with each other like any two adjoining sides of a rectangle or of a square:

The easiest way of drawing two perpendicular lines is to draw a right angle, and there are several ways of doing this. A carpenter's square may be used, but those who have much drawing to do use a triangle of celluloid or other stiff substance, one angle of which is a right angle. If you fold a piece of paper, and then fold it again so that one part of the crease lies along the other part, the two creases will form a right angle, and you can use this for drawing perpendicular lines. Try it and see how it works.

DRAWING FIGURES

1. Draw a straight line 1 in. long and at either end draw a line perpendicular to the 1-inch line.

2. From any point draw two straight lines, each 1 in. long, and each perpendicular to the other.

3. Draw a triangle with its base 1 in. long.

4. Draw a square with each side 1 in. long.

5. Draw a rectangle with a base of 2 in. and a height of 1 in.

6. Draw a four-sided figure with its opposite sides as nearly parallel as you can make them.

RECTANGLES, TRIANGLES, AND PARALLELOGRAMS

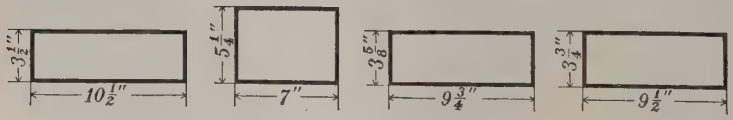
1. Draw a rectangle 3″ long and 2″ high; that is, one which is 3 in. by 2 in. Find the area. Divide the rectangle into square inches and show that the answer you give is correct.

2. A rectangle is $2\frac{1}{2}$″ long and $1\frac{1}{4}$″ wide. Find the perimeter (the distance around the rectangle) and the area.

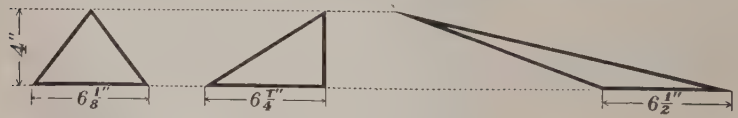
3. A parallelogram has a base of 2″ and a height of $1\frac{1}{2}$″. If possible, find both the perimeter and the area.

4. What do you think we mean when we say that we "draw a figure to scale"? If we represent 10′ (10 ft.) by 1″, by what should we represent 5′? 1′? 20′?

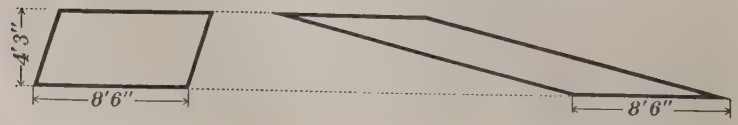
5. Here are four rectangles, drawn to the same scale. Which one has the largest area? Which one has the smallest area?



6. Here are three triangles, drawn to the same scale. Which one has the largest area? Find the area of each triangle.

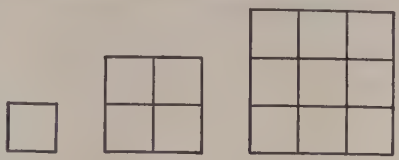


7. Here are two parallelograms, drawn to scale. Which, if either, has the larger area? Find the area of each one.



8. If the side of the square A here shown is 1″, what is the area of A? Find the side and the area of B; of C.

9. If we double the side of a square, what does this do to the perimeter? What does it do to the area?



10. If we multiply the side of a square by 3, what does this do to the perimeter? to the area?

11. Here are two rectangles, in which the base of X is half that of Y. Does this tell us anything about the perimeter of X as compared with that of Y? the area of X as compared with that of Y? What more must we know in order to compare the perimeters? the areas? How does this case differ from the one discussed in Exs. 8–10?

Copy the following statements, and complete each one by numbers or by words, drawing a figure to illustrate each case:

12. If we multiply the side of a square by 4, we multiply the perimeter by —— and the area by ——.

13. If we multiply the base of a rectangle by 2, but leave the height the same, we multiply the area by ——, but of the —— we can only say that it is greater.

14. If we multiply the height of a rectangle by 3, but leave the base the same, we multiply the area by ——, but we can only say that the perimeter is ——.

15. If we multiply both the base and height of a rectangle by 2, we multiply the perimeter by —— and the area by ——.

16. If we multiply both the base and height of a rectangle by 3, we multiply the perimeter by —— and the area by ——.

Copy the following statements and complete each one in the same manner as Exs. 12–16:

17. If we double the perimeter of a square, we multiply each side by —— and the area by ——.

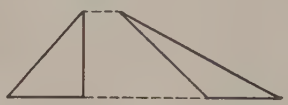
18. If we double the side of an equilateral (equal-sided) triangle, we multiply the perimeter by ——.

19. If we double the base of any triangle, leaving the height unchanged, we multiply the area by ——.

20. If we double the height of any triangle, leaving the base unchanged, we multiply the —— by ——.

21. If we double both the base and the height of any triangle, we multiply the —— of the triangle by ——.

22. If we increase the perimeter of a triangle, without changing either the base or the height, the area of the triangle ——.



23. If we multiply the perimeter of the first triangle in Ex. 22 by 10, and in so doing increase both the base and the height, the area of the triangle ——.

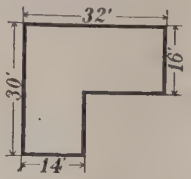
24. If the area of a square is 4 sq. in., the side is —— in. and the perimeter is —— in.

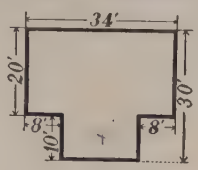
25. If the area of a triangle is 4 sq. in., the base may be 4 in. and the height —— in., or any other two numbers representing inches such that half their —— is ——; for example, the numbers may be —— and ——.

26. If we know only that the area of a rectangle is 4 sq. in., we —— tell what the perimeter is.

27. A rectangle 4 in. long and 1 in. high has an area equal to that of a square whose side is —— in., but the perimeter of the rectangle is —— than that of the square.

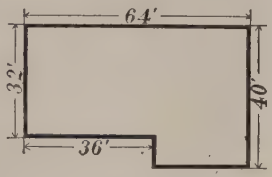
28. This picture shows the outlines of two cellars. Which cellar has the larger floor space, and how much more floor space has it?





29. In Ex. 28, what would be the answer if each of the sides were doubled?

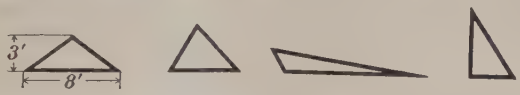
30. Here is the outline of the floor plan of a garage. How many feet is it around the inside of the walls of the garage?



31. In Ex. 30, how many square feet of concrete will be needed to floor the garage? How many square yards of concrete will be needed?

32. If a width of only 32′ can be allowed in building a garage, how long must it be built in order to have the same floor space as the one shown in Ex. 30?

33. These triangles are drawn to the scale of $\frac{1}{16}'' = 1'$.



Measure the figures carefully, find the lengths of the sides of the triangles which they represent, and then find the area of each triangle in square feet. Compare the results.

34. Draw figures to show that increasing the perimeter of a triangle may leave the area unchanged.

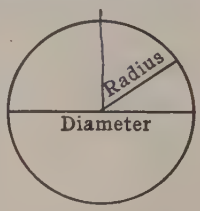
35. Draw figures to show that increasing the perimeter of a square must increase the area.

36. Draw figures to show that increasing the perimeter of a rectangle may decrease the area.

A SILENT READING LESSON

There is one kind of figure that we see very often but have not yet studied; namely, the *circle*. Familiar illustrations of circles are coins, wheels, the cross section of an orange, and similar objects. If we try to imagine what the world today would be without wheels, gears, wires, and all our machines that depend upon the circle, we can see how important the circle is to man.

A circle is a curve every point of which is at the same distance from a point within called the *center*. The distance around a circle is called the *circumference*. A straight line drawn through the center and terminated at each end by the circle is called a *diameter*, and a straight line drawn from the center to any point on the circle is called a *radius*.



A circle may be compared to the tire of a wheel; a circumference is like the length of such a tire; a radius is like a spoke of the wheel; a diameter is like two opposite spokes, and is, of course, twice the radius. The word "circumference" is often used for the line which forms the circle as well as for the length of the line, and the word "circle" is often used for the space inclosed by the curve line.

When we study geometry, we shall learn that a circumference of a circle is about $3\frac{1}{7}$ times the length of the diameter. It is more nearly 3.1416 times this length, but in our work we shall generally use $3\frac{1}{7}$, or $\frac{22}{7}$; that is, briefly stated,

$$\textit{Circumference} = \tfrac{22}{7} \times \textit{diameter}.$$

Thus, if the diameter of a circle is 10 in., the circumference is $3\frac{1}{7} \times 10$ in., or $31\frac{3}{7}$ in. Since in measuring we seldom use fractions like $\frac{3}{7}$ in., such a result means more if given decimally to the nearest 0.1 in.; that is, as 31.4 in.

Finding the Circumference. **1.** If a boy's bicycle wheel has a diameter of 21 in., what is the circumference?

Since the circumference is $3\frac{1}{7}$ times the diameter, we have

$$\tfrac{22}{7} \times 21 \text{ in.} = 66 \text{ in., the circumference.}$$

2. Find the circumference of a circle whose radius is 7 in.

The diameter is 2×7 in., and so we have

$$\tfrac{22}{7} \times 2 \times 7 \text{ in.} = 44 \text{ in., the circumference.}$$

FINDING THE CIRCUMFERENCE

1. If your bicycle wheel has a diameter of 28 in., what is the circumference?

Find the circumference when the diameter is

2. 7 in.	**6.** 105 ft.	**10.** 12 in.	**14.** $3\frac{1}{2}$ in.	**18.** 3 ft. 6 in.
3. 42 in.	**7.** 119 yd.	**11.** 15 ft.	**15.** 4.9 in.	**19.** 4 ft. 1 in.
4. 35 in.	**8.** 112 ft.	**12.** 62 in.	**16.** 6.3 in.	**20.** 7 ft. 7 in.
5. 63 in.	**9.** 133 yd.	**13.** 51 ft.	**17.** 2.7 in.	**21.** 8 ft. 2 in.

22. If the radius of a circle is $3\frac{1}{2}$ in., what is the diameter? What is the circumference? If you are given the radius, how do you find the circumference?

Find the circumference when the radius is

23. 7 in.	**25.** 28 in.	**27.** 6 in.	**29.** $10\frac{1}{2}$ in.	**31.** 2 ft. 1 in.
24. 14 in.	**26.** 42 in.	**28.** 8 in.	**30.** 4.9 in.	**32.** 10 ft. 6 in.

33. How far will a bicycle wheel which is 2 ft. 3 in. in diameter go in 1 revolution? in 5000 revolutions?

34. If convenient, measure the diameter of a wheel on an automobile, find the circumference, compute how far the car will go while the wheel is revolving once, and then find how many revolutions the wheel makes in going 1 mi.

Finding the Diameter and the Radius. Sometimes it is easier to measure the circumference than the diameter of a circle. For example, a farmer who has a silo with a circular base can easily measure the circumference. From this, as shown in the following examples, he can then find the diameter, which he will need to know if he is computing the capacity:

1. A farmer measures the circumference of a silo and finds that it is 132 ft. What is the length of the diameter?

Since the circumference is $\frac{22}{7}$ times the diameter, we divide the circumference by $\frac{22}{7}$ to find the diameter, and have

$$132 \text{ ft.} \div \frac{22}{7} = \overset{6}{\cancel{132}} \text{ ft.} \times \frac{7}{\cancel{22}} = 42 \text{ ft., the diameter.}$$

2. The distance around a circular flower bed in a park is 286 ft. Find the radius.

We first find the diameter; that is, we first find that

$$286 \text{ ft.} \div \tfrac{22}{7} = 286 \text{ ft.} \times \tfrac{7}{22} = 91 \text{ ft., the diameter.}$$

The radius of the bed is then $\frac{1}{2}$ of 91 ft., or $45\frac{1}{2}$ ft.

FINDING THE DIAMETER AND THE RADIUS

1. The circumference of a round water tank is 75 ft. What is the diameter? What is the radius?

Find the diameter of a circle with a circumference of

2. 22 in.	**5.** 110 ft.	**8.** 30 in.	**11.** 2.2 in.	**14.** 1 ft. 2 in.
3. 44 in.	**6.** 154 ft.	**9.** 80 in.	**12.** 8.8 in.	**15.** 2 ft. 1 in.
4. 88 in.	**7.** 198 ft.	**10.** 150 ft.	**13.** 13.2 in.	**16.** 2 ft. 8 in.

Find the radius of a circle with a circumference of

17. 66 in.	**19.** 4.4 in.	**21.** 270 ft.	**23.** 10 in.	**25.** 7 ft. 4 in.
18. 132 ft.	**20.** 165 ft.	**22.** 176 ft.	**24.** 11.55 in.	**26.** 5 ft. 6 in.

MEASURING CIRCLES

1. Frank measures the diameter of the drive wheel of a locomotive and finds it to be 5 ft. 3 in. What is the circumference of the wheel?

2. What radius should be used in laying out a pattern for a pulley which is to have a circumference of 154 in.?

3. If an iron water pipe has a circumference of 4.2 in., what is the diameter?

Find the circumference of a circle with a diameter of

4. 427 ft. **6.** 5.11 ft. **8.** 4.69 ft. **10.** 64.4 in.

5. 68.2 in. **7.** 6.37 in. **9.** 6.02 in. **11.** 10 ft. 6 in.

Find the diameter of a circle with a circumference of

12. 132 ft. **13.** 176 in. **14.** 68.2 in. **15.** 198 in.

Find the circumference of a circle with a radius of

16. 49 in. **17.** 91 in. **18.** 9.8 in. **19.** 154 ft.

Using 3.1416 instead of $3\frac{1}{7}$, and giving the result to the nearest 0.1 in., find the circumference of a circle with a diameter of

20. 17 in. **21.** 6.28 in. **22.** 3.42 in. **23.** 6 ft. 2 in.

24. The outside diameter of an iron water pipe is 4 in., and the iron is $\frac{1}{4}$ in. thick. Using $3\frac{1}{7}$, or $\frac{22}{7}$, as usual unless otherwise directed, find the inside circumference.

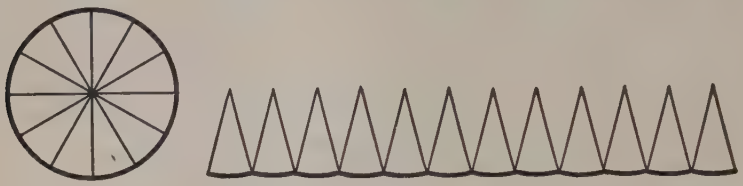
25. The globe in a schoolroom has a diameter of 30 in. How long is the equator on the globe?

26. If there is a globe in your schoolroom, measure the length of the equator and then find the radius of the circle of the equator.

27. If convenient, measure the circumference of the school flagpole or of some tree near the school and find the diameter.

A SILENT READING LESSON

As shown below, a circle can be separated into figures which are nearly triangles. The height of each triangle is the radius of the circle, and the sum of the bases is the circumference.



If they were exact triangles, the *area* of the circle, by which we mean the area inclosed by the circle, would be

$$\tfrac{1}{2} \times \text{radius} \times \text{circumference}.$$

If, later on, we study geometry, we shall prove that this is the true area of a circle. Since the circumference is $\tfrac{22}{7}$ times the diameter, we may now write "$\tfrac{22}{7} \times$ diameter" for "circumference" in the above expression, and have

$$\tfrac{1}{2} \times \text{radius} \times \tfrac{22}{7} \times \text{diameter}.$$

Then, writing "$2 \times$ radius" for "diameter," we have

$$\tfrac{1}{2} \times \text{radius} \times \tfrac{22}{7} \times 2 \times \text{radius},$$

or

$$\frac{\text{radius} \times 22 \times 2 \times \text{radius}}{2 \times 7},$$

from which, canceling the common factor 2, we have

$$\tfrac{22}{7} \times \text{radius} \times \text{radius}.$$

If we multiply a number by itself, the result is called the *square* of the number. We may therefore write "square of the radius" for "radius $\times$ radius"; that is, in finding the area of a circle, we have the following relation:

$$\textit{Area} = \tfrac{22}{7} \times \textit{square of the radius}.$$

Finding the Areas of Circles. 1. Find the area of a circle which has a radius of 5 in.

Using the relation given on page 164, we have

$$\frac{22}{7} \times 5 \times 5 \text{ sq. in.} = \frac{550}{7} \text{ sq. in.} = 78\frac{4}{7} \text{ sq. in.}$$

That is, to the nearest 0.1 sq. in., which is a more sensible form for the answer, the area is 78.6 sq. in.

2. Find the area of a circle with a diameter of 8 in.

Since the diameter is 8 in., the radius is 4 in., and we have

$$\frac{22}{7} \times 4 \times 4 \text{ sq. in.} = \frac{352}{7} \text{ sq. in.} = 50.28 \cdots \text{ sq. in.},$$

the dots $(\cdots)$ indicating that the division by 7 can be carried further. That is, to the nearest 0.1 sq. in. the area is 50.3 sq. in.

FINDING THE AREAS OF CIRCLES

1. In laying out a circular flower bed a man fastens two stakes to a string, 7 ft. apart. He drives one stake into the ground and marks the circle with the other as the string revolves about the first stake. What is the area of the bed?

Find the area of a circle with a radius of

2. 7 in.	5. $3\frac{1}{2}$ in.	8. 2 in.	11. 7 ft. 7 in.
3. 14 in.	6. 6.3 in.	9. 5 in.	12. 5 ft. 6 in.
4. 35 in.	7. $10\frac{1}{2}$ in.	10. 9 ft.	13. 2 yd. 6 in.

14. What is the radius of a circular table-top of which the diameter is 4 ft. 6 in.? What is the area?

Find the area of a circle with a diameter of

15. 42 in.	18. 14 ft.	21. 1 yd.	24. 4 ft. 8 in.
16. 28 in.	19. 5.6 in.	22. 2 yd.	25. 3 ft. 6 in.
17. 56 in.	20. 9.8 in.	23. 3 ft.	26. 10 ft. 6 in.

MEASURING CIRCLES

1. If we draw a circle with a radius half as long as that of a given circle, the area of the circle which we draw is what part of the area of the given circle?

2. If we draw a circle with double the radius of a given circle, the area of the circle which we draw is how many times as large as that of the given circle? Make and solve a problem to illustrate your answer.

3. Find the area of the base of a round water tank which is 42 ft. in diameter.

4. A horse is tethered by a rope fastened to a post in an open field. If the rope is 17 ft. 6 in. long, over approximately how many square feet of ground can the horse graze?

5. A flagpole has a circumference of 24.2 in. at the base. Find the diameter, the radius, and the area of the cross section of the flagpole at the base.

6. Using 3.1416 instead of $3\frac{1}{7}$, what is the area of the cross section of a circular iron shaft the circumference of which is 31.416 in.?

7. The outside diameter of an iron water pipe is 3 ft. 6 in. If the iron is 1 in. thick, how many square inches of iron are there in the cross section of the pipe?

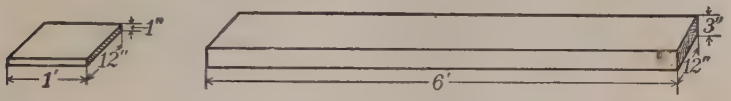
8. If Texas has 4,660,000 inhabitants, and 25% of them live within 100 mi. of Dallas, what, to the nearest unit, is the average population per square mile in this circle?

9. If the water pressure is 125 lb. to the square inch, what is the total pressure exerted on the end of a closed water pipe which has an inside diameter of 3.15 in.?

10. Measure the circumference of a baseball. From this find the diameter, the radius, and the cross-section area.

Measuring Lumber. In measuring lumber the unit used is the *board foot* (bd. ft.), which is a piece that is 1 sq. ft. on one surface and 1″ or less in thickness.

Thus, a board 2′ long, 1′ wide, and 1″ or less thick contains $2 \times 1 \times 1$ bd. ft., or 2 bd. ft., and one that is 12′ long, 8″ wide, and 1″ or less thick contains $12 \times \frac{8}{12} \times 1$ bd. ft., or 8 bd. ft.



The figure at the left shows a piece of board that contains exactly 1 bd. ft., while the plank shown at the right contains $6 \times \frac{12}{12} \times 3$ bd. ft., or 18 bd. ft.

In finding the number of feet, board measure (B.M.), in a plank that is $12' \times 6'' \times 2\frac{1}{2}''$, we may arrange the work as shown at the right, cancel, and find that the plank contains 15 bd. ft.

$$12 \times \tfrac{1}{2} \times 2\tfrac{1}{2} = \frac{\overset{3}{\cancel{12}} \times 5}{\underset{}{2} \times \underset{}{\cancel{2}}} = 15$$

In billing lumber, a fraction of a board foot is always counted as equal to 1 bd. ft.

In finding the number of feet B.M. in several pieces we have the following convenient relation:

Number of feet B.M. = number of pieces × length in feet × width in feet × thickness in inches.

This relation is particularly convenient in finding the cost of an order for several pieces of lumber. For example, in finding the cost of 18 pieces, 16′ long, 4″ wide, and 3″ thick, at \$50 per M (that is, per 1000 bd. ft.), we have

$$18 \times 16 \times \frac{\overset{1}{\cancel{4}}}{\underset{4}{\cancel{12}}} \times \cancel{3} \times \frac{\$5\cancel{0}}{100\cancel{0}} = \$14.40, \text{ the cost.}$$

Building a House. In building the house shown in the picture above, a builder estimates the cost of materials and labor as stated in the following problems. The results of some of the computations that have to be made are given; the rest is left for the pupils to complete.

The right-hand figure in the picture shows the meaning of the terms used in the problems.

BUILDING A HOUSE

1. For sills and plates the builder needs the pieces indicated below. Fill in the parentheses and find the total number of feet B.M. required for the sills and plates:

3 pieces, $6'' \times 8''$, 16′ long . . . . . 192 bd. ft.
4 pieces, $6'' \times 8''$, 14′ long . . . . . (?) bd. ft.
6 pieces, $6'' \times 8''$, 12′ long . . . . . (?) bd. ft.
4 pieces, $4'' \times 6''$, 14′ long . . . . . (?) bd. ft.
6 pieces, $4'' \times 6''$, 18′ long . . . . . (?) bd. ft.

2. Find the number of feet B.M. required for the floor timbers and rafters, for which the builder needs

30 pieces, $2'' \times 10''$, 16′ long
18 pieces, $2'' \times 8''$, 12′ long
13 pieces, $2'' \times 8''$, 6′ long
27 pieces, $2'' \times 8''$, 16′ long
27 pieces, $2'' \times 8''$, 12′ long
27 pieces, $2'' \times 6''$, 16′ long

3. Find the number of feet B.M. required for the studding, for which the builder needs

5 pieces, $4'' \times 6''$, 14′ long	35 pieces, $2'' \times 4''$, 10′ long
45 pieces, $3'' \times 4''$, 12′ long	25 pieces, $2'' \times 4''$, 18′ long
40 pieces, $3'' \times 4''$, 18′ long	50 pieces, $2'' \times 4''$, 8′ long
100 pieces, $2'' \times 4''$, 14′ long	50 pieces, $2'' \times 4''$, 14′ long

4. For the roof the builder needs 1050 bd. ft. of roofing boards. Find the total amount of lumber needed in Exs. 1–4. How much will this lumber cost the builder at an average price of \$68 per M?

5. The builder needs also 2000 bd. ft. of sheathing at \$60 per M; 1950 bd. ft. of flooring at \$85 per M; 325 bd. ft. of porch flooring at \$75 per M; 2500 clapboards at \$14.25 per M; 7000 shingles at \$12 per M. What will this cost?

6. He needs also gutters, cornice, corner boards, etc., \$75; porch columns, \$14; 16 doors at an average cost of \$9.75 each; 16 windows at an average cost of \$8.50 each; hardware, \$115; chimney and plastering, \$475; painting, \$320; tin work, \$64.50; labor, 6 workmen for 32 da. at \$6 a day each; excavating and foundation, \$300; grading, \$65. What is the total cost of these items?

7. He must pay \$485 for the plumbing and \$645 for the heating plant. Taking all the material and labor as found in Exs. 4–7, what is the total cost of the house?

Find the number of feet B.M. in each of the following cases:

8. 32 planks, 3 in. thick, 8 in. wide, and 16 ft. long.

9. 36 joists, 3 in. by 4 in., and 16 ft. long.

10. 12 beams, 10 in. by 12 in., and 14 ft. long.

11. 120 boards, 6 in. wide, $\frac{3}{4}$ in. thick, and 16 ft. long.

12. 48 timbers, 6 in. by 8 in., and 18 ft. long.

PLASTERING AND PAPERING

1. Allowing 14 sq. yd. for openings, how many square yards of plastering are needed for the walls and ceiling of a room 21 ft. long, 15 ft. wide, and 9 ft. high?

2. How much will it cost, at 60¢ a square yard, to plaster a schoolroom $34\frac{1}{2}$ ft. long, 18 ft. wide, and 12 ft. high, making an allowance of 30% of the total surface to be plastered for blackboards, baseboard, ventilators, doors, and windows?

3. The walls of a room are 8 ft. 6 in. high above the baseboard, and the total length of the four walls, allowing for openings, is 72 ft. Allowing 6 in. on each strip for matching, how many rolls of paper, 18 in. wide and containing 16 yd. of paper, will be needed? Find the cost of the paper at 65¢ a roll.

4. Deducting 2 rolls for openings, how many rolls of 24-inch plain paper, 16 yd. long, will it take to cover the four walls of a room 25 ft. long, 20 ft. wide, and 12 ft. high? If, in buying the paper, parts of rolls cannot be bought, what will be the cost of the paper at 85¢ a roll?

5. The walls of a room are to be papered with plain paper costing 60¢ a roll. The walls are 9 ft. high above the baseboard, and their total length after deducting for windows and doors is 60 ft. Allowing $1\frac{1}{2}$ rolls to 100 sq. ft., what will be the cost of the paper?

6. Measure the length, width, and height of your schoolroom. Find the area of the ceiling in square yards, as you would have to do in estimating the cost of plastering.

7. Either by measuring or estimating the sizes of the openings or other unplastered parts of the walls of your schoolroom, find the approximate number of square yards of plastering in the walls, using the measurements which you made in Ex. 6.

WALLS AND FLOORS OF A HOUSE

1. A bathroom 9 ft. long and 6 ft. wide has a tile floor. The lower part of the walls also is tiled to a height of 3 ft. Deducting 12 sq. ft. for door and fixture openings, find the number of square feet of tiling used.

2. The rest of the walls and the ceiling of the room in Ex. 1, which is 9 ft. high, are plastered and painted. Find the number of square yards of painted surface, deducting 3 sq. yd. as the allowance for openings.

3. The walls of a cellar $24\frac{1}{2}$ ft. by $37\frac{1}{4}$ ft. by 8 ft. are to be whitewashed. Allowing 3 sq. yd. for door and window openings, find the number of square yards to be covered.

4. At \$2.75 per square yard, including the cost of laying, find the cost of covering a kitchen floor, 15 ft. long and 12 ft. wide, with linoleum.

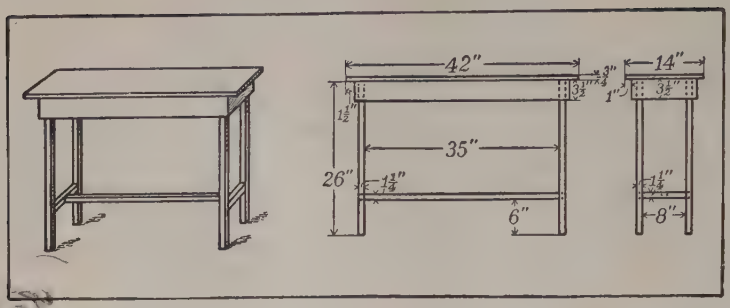
5. A rug 12 ft. square, placed in a dining room that is 16 ft. long and 14 ft. wide, will cover what per cent of the floor space?

6. How many strips of plain carpeting $\frac{3}{4}$ yd. wide are needed for a room 18 ft. wide?

7. A carpet has an 18-inch pattern. Is there any loss in matching the strips if they are 15 ft. long? if they are 14 ft. long? Illustrate each answer by a plan drawn to scale.

8. In allowing for waste in matching carpet patterns no allowance need be made on the first strip, since the other strips are cut to match that one. If a 27-inch carpet with a $\frac{1}{2}$-yard pattern is to be laid lengthwise of a room 17 ft. 6 in. long and 14 ft. wide, how much is wasted in matching on each strip after the first? Find the amount of carpet needed.

9. Would it be more economical if the strips in Ex. 8 ran across the room? If so, how many yards might be saved?



WORKING DRAWINGS

1. The above plan shows that a $\frac{3}{4}$-inch board, 14″ wide and 42″ long, is needed for the top of the stand. Not allowing for waste in cutting, how many board feet are needed?

2. Pieces $1\frac{1}{4}$″ square, all sawed out, can be bought for the legs. How long a piece is needed for each leg? Allowing 6″ for waste in cutting, how long a piece of this square stock is needed for the four legs?

3. The $1\frac{1}{4}$-inch square stock is also used for the braces at the bottom. Allowing another 6″ for waste in cutting, how many running feet of this stock are needed for the stand?

4. The strip around the top of the legs is $\frac{3}{4}$″ thick and $3\frac{1}{2}$″ wide. The side pieces are 3″ shorter than the top of the stand, and the end pieces are 2″ shorter than the width of the top. How many board feet are needed for this strip?

5. In making a tool cabinet, a boy uses a $\frac{3}{4}$-inch board 10″ wide. He cuts off 3 pieces 14″ long, 2 pieces 18″ long, and 5 pieces 20″ long. Allowing 6″ for waste in cutting, how many board feet did he use?

6. At $95 per M, what is the cost of the lumber that the boy uses, as described in Ex. 5?

Longitude and Time. In geography we learn about the longitude of places, and find that longitude is reckoned in degrees (°), minutes (′), and seconds (″) from the prime meridian, which passes through Greenwich, England.

Correspondence of Longitude to Time. Since the earth makes a complete revolution about its axis once in every 24 hr., the place in which we live will pass through 360° between now and this time tomorrow. That is, 360° of longitude corresponds to 24 hr., and so

1° of longitude corresponds to $\frac{1}{360}$ of 24 hr., or $\frac{1}{15}$ hr., or 4 min.;

1′ of longitude corresponds to $\frac{1}{60}$ of 4 min., or $\frac{1}{15}$ min., or 4 sec.;

1″ of longitude corresponds to $\frac{1}{60}$ of 4 sec., or $\frac{1}{15}$ sec.

In studying these relations, which belong partly to geography, it will help us to use the globe.

Earlier West, Later East. Because the earth revolves 360° in 24 hr., the sun appears to move 15° an hour from east to west. Therefore, when it is noon here it is an hour later, or 1 P.M., 15° east of here; an hour earlier, or 11 A.M., 15° west of here; and 6 hr. earlier, or 6 A.M., 90° west of here.

LONGITUDE AND TIME

All work oral

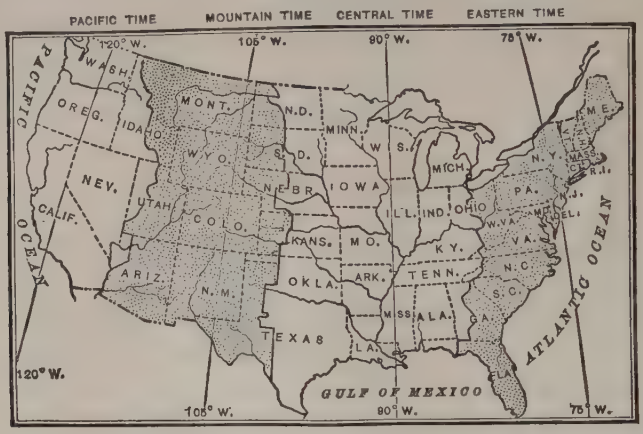
1. When it is 9 A.M. here, what time is it 15° east of here?

2. When it is noon here, what time is it directly north of here? 30° east of here? 90° west of here? 150° east of here?

3. What difference in time corresponds to a difference in longitude of $7\frac{1}{2}$°? 45°? 60°? 105°? 120°? 165°? 180°?

4. Since 1′ of longitude corresponds to 4 sec. of time, to what does 7′ correspond? 11′? 15′? 30′? 45′? 60′? 120′?

Standard Time. The map given below shows how the United States has been divided into four sections in order to regulate the time. Every place in each section uses *standard time*, which is the time of a certain meridian in that section.



In going from one section to the next, the time changes 1 hr. Thus, when it is noon by Eastern time (75° W.), it is 11 A.M. by Central time (90° W.), 10 A.M. by Mountain time (105° W.), 9 A.M. by Pacific time (120° W.), and 5 P.M. by Greenwich time (0°).

STANDARD TIME

1. What time is it now in your school? in California?
2. When it is 10 A.M. in Illinois, what time is it in Maine?
3. When it is 10 A.M. in New York, what time is it in Dallas? in Memphis? in Denver? in New Orleans?
4. When it is 9 A.M. in Seattle, what time is it in Savannah?
5. When it is noon in Maryland, what time is it in Wyoming?
6. When it is 9 A.M. in Albany, what time is it in Nevada?

Difference in Time. The relation of time to longitude is more apparent when we consider two places that are far apart. For example, if we wish to know the difference in local (not standard) time on two ships which are 65° 7′ 30″ of longitude apart, we may proceed as follows:

LONGITUDE	TIME	
65°	$65 \times \frac{1}{15}$ hr. $= 4\frac{1}{3}$ hr.	$= 4$ hr. 20 min.
7′	$7 \times \frac{1}{15}$ min. $= \frac{7}{15}$ min.	$=$ 28 sec.
30″	$30 \times \frac{1}{15}$ sec.	$=$ 2 sec.
	Difference in local time	$= 4$ hr. 20 min. 30 sec.

DIFFERENCE IN TIME

1. The figures of the answer in the case above can be obtained by this division. Do you see why we divide by 15? Write the reason.

$$15\overline{)65\ \ 7\ 30}\ \ 4\ 20\ 30$$

2. A difference of 27° 4′ 15″ in longitude corresponds to what difference in local time?

3. If two ships are 75° 30′ 30″ of longitude apart, what is the difference in local time?

4. The local time of Rome is that of 15° E. longitude, and that of Chicago is the time of 90° W. longitude. When it is 2 P.M. in Chicago, what time is it in Rome?

5. The local time of San Francisco is that of 120° W. longitude. When it is noon in London (on the prime meridian), what time is it in San Francisco?

6. At 8 30 A.M., local time, a ship at 68° 10′ 30″ W. longitude sends a wireless message that is picked up by a lightship at 70° W. longitude. At what hour, local time, is it received at the lightship? At what hour (local time, 75° W. longitude) is the message received in New York?

Difference in Longitude. Since 24 hr. of time corresponds to 360° of longitude, we have the following relations:

1 hr. corresponds to $\frac{1}{24}$ of 360°, or 15°, of longitude;
1 min. corresponds to $\frac{1}{60}$ of 15°, or $\frac{1}{4}$°, or 15′, of longitude;
1 sec. corresponds to $\frac{1}{60}$ of 15′, or $\frac{1}{4}$′, or 15″, of longitude.

Thus, in finding the difference in longitude between two ships on which the local time differs by 3 hr. 7 min. 3 sec., we may proceed as follows:

TIME	LONGITUDE
3 hr.	$3 \times 15° = 45°$
7 min.	$7 \times 15' = 105' = 1° 45'$
3 sec.	$3 \times 15'' = 45''$
	Difference in longitude $= 46° 45' 45''$

DIFFERENCE IN LONGITUDE

1. The figures of the answer in the case above can be obtained by this multiplication. Do you see why in this solution we may multiply by 15? Write a statement of the reason involved.

3	7	3
		15
46	45	45

2. If the difference in local time between two steamships is 2 hr. 3 min. 10 sec., what is the difference in longitude?

3. When it is noon in Boston, it is 5 P.M. at Greenwich (0°). From what longitude does Boston take its time?

4. When it is noon at Greenwich, it is 6 A.M. in Iowa. From what longitude does Iowa take its time?

5. At 10 10 A.M., local time, a steamer at 26° 30′ W. longitude sends a wireless message to another steamer. If the message is received at 10 25 A.M., local time, what is the longitude of the second steamer?

IV. REVIEW AND DRILL

MINIMUM ESSENTIALS

1. Write the number of next year in Roman numerals.

2. Write the largest whole number that you can write in the usual way with six figures.

Copy the following examples and supply the missing numbers indicated by the stars:

3.	4.	5.	6.	7.
7.96	9.96	\$5.53	\$4.78	4 ft. 8 in.
3.75	7.28	***	3.96	3 ft. 9 in.
***	.63	2.46	.28	2 ft. 10 in.
8.82	4.07	5.90	5.00	10 in.
.77	2.70	2.75	2.75	7 ft. 3 in.
25.00	***	\$25.00	***	***

8. From $3\frac{5}{8} + 6\frac{3}{4}$ subtract $2\frac{5}{32} + 1\frac{3}{16}$.

9. Multiply 9.46 by 0.38 and 17.245 by 0.07, giving each result to the nearest 0.01.

10. Divide 42.634 by 0.93 and 67.855 by 0.017, giving each result to the nearest 0.01.

11. Find the area of a circle having a diameter of 1.4 in.

12. The number 8.25 is 3% of what number? 30% of what number? 0.3% of what number?

13. At a discount of $12\frac{1}{2}\%$, what is the amount of the discount on an order amounting to \$49.60 at list prices? What is the net amount of the order?

14. What is the interest on \$275 for 9 mo. 15 da. at 6%?

15. On what sum is \$14.40 the interest for 1 yr. at 5%?

16. Which is larger, the interest on \$1500 for 6 mo. at $6\frac{1}{2}\%$ or that on \$1498 for 6 mo. at 6%? How much larger is it?

PROBLEMS WITHOUT NUMBERS

All work oral

1. How do you find a per cent of a number?

2. If you know what a certain per cent of a number is, how do you find the number?

3. If you have two numbers, how do you find what per cent the first is of the second? what per cent the second is of the first? what per cent the first is of the sum of the numbers?

4. If you wish to borrow a certain sum of money, and can do so for a certain length of time at a certain rate of interest, how do you find the interest?

5. If you can buy a house for a certain cash payment and a certain number of equal payments at definite intervals, with interest on the unpaid balance at a certain rate, how do you find how much the house has cost you in all after you make the last payment?

6. If you know the result of adding two numbers, and one of the numbers, how do you find the other number?

7. Consider Ex. 6, supposing that you know the result of subtracting one number from another instead of the result of adding them. Does it make any difference whether the given number is the larger one or the smaller one?

8. Consider Ex. 6, supposing that you are given the result of multiplying one number by another.

9. Consider Ex. 6 for the result of dividing one number by another. Are there two cases, as in Ex. 7?

10. If you borrow money for a certain time at a certain rate, how will the interest compare with the interest on the same sum for half the time at double the rate? with the interest on double the sum for half the time at half the rate?

PROBLEMS FOR COMPLETION

1. Mr. Russell has borrowed \$250 at 6% for 60 da. Complete the problem in any reasonable way and solve it.

Complete and solve the following problems:

2. Mr. Lodge has borrowed \$500 for 8 mo. at 6%. After 2 mo. he borrows \$250 more for 6 mo. at 5%.

3. At a sale Edward's father bought a post-card projector for showing colored post cards on a screen. The regular price was \$8.50, but it was reduced 20% for the sale.

4. William has a small electric motor. It cost him \$4.76 at a sale at which a discount of 15% was given.

5. The Stars played 16 games and lost a quarter of them.

6. Last season the Hillside team lost 7 games, which were $33\frac{1}{3}\%$ of the games on its schedule.

7. The Green Sox lost $33\frac{1}{3}\%$ of their games last season, but they won or tied 22 games.

8. Mr. Redfield wishes to borrow \$7500 for 90 da. His bank will discount his note at $5\frac{1}{2}\%$.

9. The diameter of the wheels on our automobile is 32 in. It is 1 mi. from our house to the post office.

10. Tom earns 75¢ every Saturday that he works. He has worked long enough to earn \$8.25.

11. At 4 P.M. I sent a telegram from New York to Chicago. It was delivered 45 min. later. There is a difference of 1 hr. in time between New York and Chicago.

12. A cowboy outfit is marked \$3.20; a small moving-picture machine, \$12.30; and a small phonograph, \$11.75. At a sale each of these articles is marked down 20%. A boy has \$27 in the bank.

FOOD VALUES

1. In planning properly balanced meals we often estimate food values in *calories*; that is, in terms of their heat value to the body. A calorie is the amount of heat needed to raise the temperature of 1 pt. of water about 4°. If children of your age need 30 calories a day for each pound of weight, how many calories do you need?

2. If 1 lb. of dried beef yields 790 calories, and you have bought 25¢ worth of dried beef costing 50¢ a pound, how many calories have you bought?

3. If 1 lb. of granulated sugar yields 1750 calories, what part of an ounce do you need to eat so as to get 50 calories?

4. If there are 1620 calories to 1 lb. of rice, how many calories do you get when you eat 5¢ worth of rice at 15¢ a pound?

5. If there are 1250 calories in a loaf of bread costing 12¢, how much do 250 calories cost at this rate?

6. If there are 3410 calories to 1 lb. of butter, how many calories do you get from $\frac{1}{2}$ oz. of butter?

THE COST OF WASTEFULNESS

1. If a farm wagon that cost \$90 is left out in the yard instead of being kept in the shed, it will last about 6 yr. If kept under cover, it will last about twice as long. How much does a farmer pay for his carelessness per year if he neglects to put his wagon under cover?

2. If farm machinery depreciates (loses value) about $8\frac{1}{2}\%$ a year, what is the annual loss on a machine costing \$145? If due to carelessness the rate is 15%, what is the annual loss?

3. Each of two farmers bought a two-horse cultivator, and each paid \$45. One of them took good care of his cultivator and made it last 15 yr. The other left his cultivator out during all kinds of weather and had to buy a new one at the end of every 5 yr. Not considering interest, how much did the careless farmer lose in the 15 yr.?

4. After threshing his wheat, a farmer had 16 T. of straw left. A ton of this straw contains 12 lb. of nitrogen, worth 30¢ a pound, and 10 lb. of potash and 2 lb. of phosphoric acid, each worth 5¢ a pound. If the farmer wastes the straw instead of using it as fertilizer, how much money does he waste?

HAULING BY MOTOR TRUCK

1. In a recent year the cost of a one-ton motor truck was \$1050 for the chassis and \$125 for the body. If the freight amounted to \$25, what was the total cost?

2. For the first year, the different items of upkeep of the motor truck in Ex. 1 were as follows: repairs, \$75; tires, \$480; insurance, \$61.05; depreciation due to wear, 10% of the original cost of the truck. Find the total upkeep.

3. The truck ran 14,400 mi. the first year, averaging 9 mi. to a gallon of gasoline. Find the total cost of the gasoline for the year at an average price of 22¢ a gallon.

4. The motor truck averaged 1 qt. of lubricating oil for each 45 mi. that it ran. Find the total cost of the oil for the first year at 75¢ a gallon.

5. The total cost of operating the truck for the first year was \$2957, including wages. Find to the nearest cent the average cost per mile.

6. In a recent year the cost of a two-ton motor truck was \$1875 for the chassis and \$150 for the body. If the freight amounted to \$40, what was the total cost?

7. In Ex. 6 the yearly depreciation due to wear was estimated at \$225. Find to the nearest 0.1 the per cent of the original cost charged as depreciation.

8. During a recent year 158,400 hogs were delivered to the Omaha market by motor truck. Allowing 80 hogs to a freight car, how many freight cars would it take to transport all these hogs to market?

9. In a recent year 138,000 cattle were delivered to the Cincinnati stockyards by motor truck. Allowing 30 cattle to a car, how many freight cars would it take to transport these cattle?

BUILDING A BARN

1. A farmer employed a carpenter to build a barn. As part of the materials needed, the carpenter gave the following order for lumber: 12 pieces $8'' \times 8'' \times 18'$; 8 pieces $4'' \times 6'' \times 10'$; 8 pieces $2'' \times 8'' \times 24'$; 36 pieces $2'' \times 10'' \times 14'$; 60 pieces $2'' \times 8'' \times 22'$; 12 pieces $2'' \times 8'' \times 20'$; 10 pieces $2'' \times 8'' \times 18'$; 30 pieces $2'' \times 8'' \times 16'$; 220 pieces $2'' \times 8'' \times 12'$; 20 pieces $2'' \times 4'' \times 10'$; 40 pieces $2'' \times 4'' \times 8'$; 22 pieces $4'' \times 4'' \times 24'$; 8 pieces $4'' \times 4'' \times 12'$; 5000 bd. ft. of matched siding; 22,000 shingles. Find the number of feet of lumber B.M. ordered, its cost at an average price of \$68 per M, and the cost of the shingles at \$9.10 per M.

2. The carpenter figured on needing also 100 lb. of small nails, 140 lb. of board nails, and 650 lb. of spikes of various sizes. At an average price of 7¢ per pound, what was the cost of the nails and spikes that he needed?

3. The carpenter bought ten ready-made windows at \$5.60 each, and allowed \$50 for building the foundation of the barn. He bought also the necessary hardware, such as door hinges, latches, hangers, tracks for the large doors, and window fasteners, the bill amounting to \$32.75. What was the cost of all these items?

4. The carpenter figured his work at \$7.50 a day and the work of another man at \$5.25 a day, both for a period of 37 da. How much was allowed for labor?

5. The farmer decided to put a concrete floor in the barn. He found that he would have to cover a space 58 ft. long and 36 ft. wide. At \$3.60 per square yard, what was the cost of laying the floor?

6. The farmer also paid \$185 for painting the barn. Using the results of Exs. 1–5, find the total cost of the barn.

V. LITTLE EXAMINATIONS

I. 1. $33\frac{1}{3}\% = \frac{\quad}{100} = \frac{\quad}{3}$.
2. $1\frac{2}{3} = 1 + \frac{\quad}{100} = (?)\%$.
3. $0.62\frac{1}{2} = \frac{\quad}{100} = (?)\%$.
4. $330\% = 3 + \frac{\quad}{10} = \frac{\quad}{10}$.
5. $125\% = 1 + \frac{\quad}{100} = 1\frac{\quad}{\quad}$.
6. $40\% \div 20\%$.
7. $33\frac{1}{3}\%$ of \$75.
8. $87\frac{1}{2}\%$ of \$240.
9. 125% of \$400.
10. 3.6% of \$1260.

II. 1. Find $4\frac{1}{2}\%$ of \$50, 9% of \$25, and $2\frac{1}{4}\%$ of \$100.
2. At 15%, what is the discount on a bill for \$275?
3. If the discount on \$85 is \$4.25, what is the rate?
4. Find the circumference when the radius is 8 in.
5. In Ex. 4 find the area of the circle.

III. 1. Doubling the diameter does what to the area?
2. Find the diameter when the circumference is 12 in.
3. In Ex. 2, find the radius and the area.
4. Find the area of a rectangle 18 in. by 2 ft. 6 in.
5. An acre is what per cent of a square mile?

IV. 1. Find the interest on \$1250 for 90 da. at 6%.
2. On what sum is \$17.25 the interest for 30 da. at 6%?
3. Find the interest on \$625 for 2 mo. 10 da. at 5%.
4. At $5\frac{1}{2}\%$, find the discount on a 90-day note for \$450.
5. Find the interest on \$1850 for 2 yr. 3 mo. at $4\frac{1}{2}\%$.

V. 1. Write a check for \$5; for \$10.37.
2. Write a promissory note for \$150 due in 60 da.
3. Write a deposit slip of not more than five items.
4. Make up and solve a problem in square measure.
5. Make up and solve a problem in board measure.

CHAPTER III

I. EFFICIENCY TESTS

A SILENT READING LESSON

This is our last year in the elementary school. The most important part of arithmetic is the work in addition, subtraction, multiplication, and division of whole numbers, common fractions, and decimals, including per cents. If we can do this work, we can usually solve any ordinary problem.

Why do people so often fail to solve correctly the ordinary problems that arise in daily life? *Because they cannot add, subtract, multiply, and divide correctly.*

Why do business men complain that the boys and girls they hire do not know arithmetic? *Because these boys and girls cannot add, subtract, multiply, and divide correctly.*

Why do so many people have trouble in doing simple examples like those which require finding the value of $3\frac{1}{2} \times 48¢$ or of $2\frac{7}{8} \times \$1.60$? *Because they did not learn these things thoroughly when they were in school.*

Do we wish to be counted among the failures when we begin our life's work? If not, it will be a good thing for us to begin this year with a rapid review of that part of arithmetic which we are most likely to need in everyday life. We can thus find out how efficient we are in computation. Another advantage of such a review is that it will help us to increase not only the speed at which we work but also our accuracy, because work that is done with a reasonable degree of rapidity is usually the most accurate.

ORAL SPEED TEST CHART IN ADDITION

Allowing 30 sec. to a row, add the following rows from left to right, and check by adding from right to left:

1.	3	8	4	9	5	6	3	6	8	7	3	7	5	8	4
2.	7	5	3	4	7	8	6	5	3	9	2	9	7	7	4
3.	5	6	4	5	7	8	2	6	9	7	6	5	6	4	5
4.	6	5	6	7	4	5	8	7	5	8	4	2	6	6	5
5.	4	6	5	6	5	6	9	6	6	7	5	1	7	5	6
6.	2	7	5	8	6	5	4	5	9	6	4	6	6	7	5
7.	1	4	4	3	8	7	7	4	4	8	3	8	8	6	5
8.	0	3	5	2	9	6	8	3	5	7	4	7	9	5	6
9.	8	2	6	1	9	6	9	2	6	6	5	8	8	4	7
10.	9	5	6	5	6	5	9	6	7	6	6	0	8	4	7

Allowing 15 sec. to a column, add these columns upward, and check by adding downward:

11.	12.	13.	14.	15.	16.	17.	18.
2	7	5	3	9	8	4	6
6	2	6	8	7	8	7	6
8	8	4	5	8	9	3	2
7	3	7	6	6	5	8	7
3	7	3	4	3	4	9	8
4	2	8	9	4	3	6	7
9	6	4	3	2	3	8	7
5	9	6	8	6	5	2	1
8	8	9	7	4	5	8	9
2	2	1	3	4	5	9	6
3	6	5	9	2	7	0	3
6	7	4	6	3	2	1	0

SELF-CHECKING ADDITION

1. Copy the following table; add each horizontal line and write the result in the right-hand column; add each column and write the result below it; then add the sums of the lines and also of the columns. The last two results should be the same, and the number should be written in the space marked *:

4	3	4	2	8	6	9	4	8	3	8	5	
3	9	7	3	2	1	7	2	6	5	2	4	
7	2	2	9	7	4	5	9	4	7	9	3	
6	8	8	6	5	2	6	8	2	9	7	2	
4	6	3	4	3	5	3	1	7	6	4	6	
3	7	2	1	8	6	2	9	5	4	3	8	
												*

2. A traveling man's expense account for a week is given in the table below. Perform the additions as directed in Ex. 1:

Date	Fares		Board		Telephone		Postage		Miscellaneous		Total	
1	2	20	3	90		20				55		
2	1	40	3	85		30		12		60		
3		60	4	20		75		08	1	10		
4	1	95	4	10	1	20		16		75		
5	2	60	3	95		45		24		45		
6	1	46	4	25		25		12	1	00		
7	2	32	4	35		20		18		75		
Total												

ORAL DRILL CHART IN RAPID ADDITION

1. Add 39 and 26 as a business man would; that is, think of $39 + 26$ as $39 + 20$ and $59 + 6$.

Do these additions by the method suggested in Ex. 1:

2.	22	22	22	22	22	42	65	47
	10	16	18	28	29	39	38	18
3.	27	34	45	55	78	58	47	68
	36	27	35	26	12	14	27	53
4.	38	68	27	47	57	68	78	85
	18	18	19	19	29	12	13	68
5.	90	95	95	96	98	57	67	72
	10	10	15	15	64	13	13	37
6.	85	85	86	88	64	89	99	33
	20	25	25	37	36	42	56	81
7.	58	58	58	58	65	65	67	83
	50	52	56	57	40	45	48	68
8.	77	77	78	78	78	78	78	92
	20	23	30	32	35	56	68	61
9.	64	64	64	74	89	89	89	75
	30	49	39	59	40	44	74	39
10.	37	37	37	47	47	67	78	97
	66	50	56	66	69	79	56	99
11.	85	92	32	86	27	38	94	68
	27	38	59	44	59	46	87	39

REVIEW DRILL CHART IN COLUMN ADDITION

Copy the following, add, and check each result:

1.	3.	5.	7.	9.
$2.75	$4.80	$5.00	$2.88	$1.47
1.46	2.69	2.70	1.49	2.93
3.90	1.47	.69	3.76	4.12
.75	3.29	3.82	4.05	.76
1.00	4.16	4.01	3.71	1.47

2.	4.	6.	8.	10.
$3.27	$5.75	$5.80	$7.28	$5.96
.42	2.86	2.60	2.96	6.82
1.27	.92	3.07	.39	3.48
2.68	3.00	4.26	8.72	7.56
.39	4.05	.39	.48	8.29
4.00	.75	.57	6.75	3.07
7.68	2.10	2.61	3.29	6.40
1.29	3.76	1.75	4.36	3.36
3.00	4.24	6.00	9.64	3.00

Copy the following, add, check each result, and write the number of minutes taken for all the columns:

11.	12.	13.	14.
$2864.75	$5873.96	5738.2	473.82
3904.08	2981.44	698.3	698.87
628.37	3837.62	4000.0	438.37
5072.99	897.68	783.6	507.96
832.87	3278.48	3408.8	296.81
6271.03	687.49	629.3	300.09
876.53	853.61	4426.3	41.76
273.06	3400.07	3900.0	391.63
81.27	3.25	15.6	.72
4269.34	289.63	862.3	512.19

ORAL DRILL CHART IN SUBTRACTION

1. How do you subtract 28¢ from 54¢? Do you have to take pencil and paper to do what every business man does in his head? If it were 54¢ − 30¢, you would give the result at once as 24¢; but 28¢ lacks only 2¢ of being 30¢, and so the result is 24¢ + 2¢. So think simply "28¢ + 2¢ = 30¢, 54¢ + 2¢ = 56¢, and 56¢ − 30¢ = 26¢." Try this in subtracting 38¢ from 54¢.

2. In 61 − 34, subtract by thinking first "34 + 6 = 40" and then "61 + 6 = 67." What is the result?

Do these subtractions orally:

3.	47	47	47	57	56	5.4	0.92
	12	27	28	38	38	3.7	0.49
4.	60	61	61	71	82	8.3	0.85
	32	32	35	37	36	4.5	0.58
5.	72	71	70	73	73	8.4	2.92
	38	42	42	47	48	5.8	1.96
6.	98	92	90	90	92	9.2	4.93
	52	52	52	61	63	6.8	2.95
7.	87	87	86	84	84	9.2	5.93
	28	37	37	37	39	3.9	1.75
8.	96	96	95	93	92	9.1	6.90
	36	48	48	58	67	7.6	2.89
9.	58	58	98	84	73	8.8	8.75
	27	29	69	46	27	6.9	6.85

RAPID SUBTRACTION

Numbers 1 to 8, oral

1. Suppose that you make a purchase amounting to \$2.46 and give the dealer a 5-dollar bill. Think as the dealer does: "\$2.46 + 4¢ = \$2.50, \$2.50 + 50¢ = \$3, \$3 + \$2 = \$5." Then add mentally 4¢, 50¢, and \$2, and state the amount of change that is due you.

Make change from \$5 when you buy something that costs

2. \$2.57. **3.** \$1.72. **4.** \$2.61. **5.** \$0.36. **6.** \$3.52.

7. In subtracting 4726 from 9000, some business men begin at the left, thinking of 9000 as 8990 + 10, written as here shown. Using this method of subtraction, how much is 9000 − 3268?

8	9	9	10
4	7	2	6
4	2	7	4

8. How do you check your work in subtraction? Illustrate by the case of 42 − 17 = 25; by the case of 100 − 72.

See if you can do all these subtractions in 5 min.:

9. 3000	**13.** 3000	**17.** 5726	**21.** 52,123	**25.** 12.8300
1111	1786	3987	18,068	6.9426
10. 4000	**14.** 4000	**18.** 8209	**22.** 61,023	**26.** 301.220
2222	2938	6138	52,836	49.726
11. 6000	**15.** 5000	**19.** 7012	**23.** 72,133	**27.** 8023.08
3999	2673	3648	29,239	4768.29
12. 8000	**16.** 6000	**20.** 7000	**24.** 70,000	**28.** 8000.00
4852	2673	4728	34,820	5290.76

29. From \$1,000,000 subtract \$750,920 + \$124,480.

30. From \$2,500,000 subtract \$425,000 + \$75,000 + \$16,250.

REVIEW DRILL CHART IN SUBTRACTION

1. Beginning with 50, subtract orally the following numbers, each from the preceding result: 5, 2, 3, 4, 6, 4, 2, 4, 5, 7, 6, 2. That is, say "45, 43, 40," and so on.

In each of the following cases begin at the left and subtract orally from 50, as in Ex. 1, until you reach zero:

2.	5	7	3	5	4	6	5	2	3	4	2	4
3.	3	7	2	8	5	6	5	4	3	2	2	3
4.	7	5	2	3	8	1	8	6	5	4	0	1
5.	5	8	7	6	5	1	2	3	4	2	4	3

Copy and subtract, checking each result:

6. \$1768 − 492	**11.** \$2089.70 − 379.69	**16.** \$8273.86 − 4969.98	**21.** \$12,782.68 − 698.98
7. \$4107 − 298	**12.** \$8231.21 − 4681.96	**17.** \$7234.32 − 2989.75	**22.** \$23,072.08 − 16,789.79
8. \$5123 − 1768	**13.** \$9210.82 − 7876.98	**18.** \$6000.00 − 4783.81	**23.** \$30,292.72 − 17,693.49
9. \$8234 − 3968	**14.** \$8023.81 − 6789.96	**19.** \$7000.00 − 4383.69	**24.** \$52,293.37 − 16,820.98
10. \$9230 − 4768	**15.** \$7333.03 − 5768.79	**20.** \$8000.00 − 3270.08	**25.** \$75,000.00 − 968.39

26. From \$432.86 + \$981.08 subtract \$42.38 + \$681.96.

27. Subtract \$292.07 + \$57.36 from \$75.82 + \$378.42.

28. From \$1,500,000 subtract \$32,750.25 + \$125,875.50.

REVIEW DRILL CHART IN MULTIPLICATION

After copying the examples, see if you can do these ten multiplications in 6 min.:

1. 32,236 × 24	3. 45,008 × 37	5. 41,433 × 53	7. 77,088 × 87	9. 40,756 × 86
2. 21,408 × 46	4. 48,808 × 49	6. 30,098 × 72	8. 59,960 × 36	10. 27,824 × 58

Multiply as follows, and check each result:

11. $10 \times \$0.35$.	26. $25 \times \$160$.	41. $33\frac{1}{3} \times 66$.
12. $10 \times \$7251$.	27. $25 \times \$640$.	42. $33\frac{1}{3} \times 48$.
13. $100 \times \$2.87\frac{1}{2}$.	28. $25 \times \$75.25$.	43. $16\frac{2}{3} \times \$636$.
14. $1000 \times \$0.25$.	29. $12\frac{1}{2} \times \$6400$.	44. $5 \times 47{,}664$.
15. $50 \times \$64{,}224$.	30. $125 \times \$640$.	45. $125 \times \$480$.
16. 401×3664.	31. $283 \times \$48.25$.	46. 704×6.25.
17. 503×2763.	32. $342 \times \$29.75$.	47. 172×31.7.
18. 630×4876.	33. $276 \times \$49.42$.	48. 776×0.125.
19. 580×6924.	34. $428 \times \$75.19$.	49. 0.175×424.
20. 847×6476.	35. 3574×0.05.	50. 1.75×424.
21. 972×3502.	36. 0.25×536.	51. 175×424.
22. 707×8763.	37. 0.25×5.36.	52. 175×4.24.
23. 5002×7983.	38. 0.25×0.536.	53. 0.425×584.
24. 6107×6107.	39. 0.75×576.	54. 375×504.
25. 4004×4004.	40. 7.5×57.6.	55. 3.75×5.04.

56. Find the product of $24.7 + 16.8$ times $12.07 - 10.69$.

REVIEW DRILL CHART IN DIVISION

After copying the examples, see if you can do all these divisions in 7 min.:

1. $4\overline{)3368}$	$5\overline{)1725}$	$6\overline{)3942}$	$7\overline{)5264}$	$8\overline{)4456}$
2. $3\overline{)7779}$	$2\overline{)5550}$	$9\overline{)4446}$	$8\overline{)7464}$	$7\overline{)6566}$
3. $4\overline{)9232}$	$8\overline{)9048}$	$7\overline{)4844}$	$9\overline{)6669}$	$5\overline{)4405}$
4. $2\overline{)3425}$	$3\overline{)4177}$	$4\overline{)6283}$	$5\overline{)7291}$	$6\overline{)3337}$
5. $7\overline{)6441}$	$8\overline{)9649}$	$9\overline{)3470}$	$7\overline{)8507}$	$5\overline{)9643}$
6. $6\overline{)4445}$	$8\overline{)7327}$	$9\overline{)5555}$	$7\overline{)1004}$	$7\overline{)3005}$

Perform the following divisions, finding the results in Exs. 27–36 to the nearest cent:

7. $482 \div 5$.	**17.** $50{,}568 \div 602$.	**27.** $\$48.72 \div 16$.
8. $507 \div 4$.	**18.** $13{,}020 \div 620$.	**28.** $\$39.54 \div 12$.
9. $732 \div 6$.	**19.** $19{,}251 \div 629$.	**29.** $\$57.81 \div 15$.
10. $810 \div 3$.	**20.** $76{,}109 \div 629$.	**30.** $\$34.51 \div 17$.
11. $477 \div 9$.	**21.** $33{,}750 \div 125$.	**31.** $\$208.65 \div 65$.
12. $743 \div 2$.	**22.** $26{,}288 \div 106$.	**32.** $\$282{,}127 \div 75$.
13. $347 \div 3$.	**23.** $153{,}186 \div 422$.	**33.** $\$359{,}333 \div 78$.
14. $607 \div 7$.	**24.** $237{,}975 \div 501$.	**34.** $\$463{,}481 \div 55$.
15. $555 \div 8$.	**25.** $972{,}360 \div 222$.	**35.** $\$582{,}603 \div 82$.
16. $702 \div 9$.	**26.** $520{,}251 \div 303$.	**36.** $\$4283.25 \div 25$.

37. Divide $98 \times 2871 \times 0.628$ by $49 \times 33 \times 3.14$.

REVIEW DRILL CHART IN DIVISION

See if you can do the following six divisions in 12 min., finding each result to the nearest 0.01:

1. $177\overline{)3427.81}$
2. $42.3\overline{)286.94}$
3. $239\overline{)7341.68}$
4. $63.8\overline{)427.52}$
5. $125\overline{)3492.50}$
6. $39.7\overline{)797.97}$

Divide as indicated:

7. $62.9 \div 37.$
8. $6.29 \div 37.$
9. $62.9 \div 3.7.$
10. $6.29 \div 3.7.$
11. $80.5 \div 35.$
12. $80.5 \div 3.5.$
13. $8.05 \div 3.5.$
14. $46.2 \div 22.$
15. $46.2 \div 2.2.$
16. $4.62 \div 2.2.$
17. $600 \div 0.6.$
18. $600 \div 0.06.$
19. $6 \div 0.0006.$
20. $627 \div 0.3.$
21. $6.27 \div 0.3.$
22. $7.14 \div 0.42.$
23. $62.32 \div 7.6.$
24. $6.232 \div 0.76.$
25. $8190 \div 0.9.$
26. $61.20 \div 1.2.$
27. $7.200 \div 1.2.$
28. $78.4 \div 0.08.$
29. $3.12 \div 0.04.$
30. $7.81 \div 1.1.$
31. $8.91 \div 0.11.$
32. $4.55 \div 0.07.$
33. $2.73 \div 0.03.$
34. $8.96 \div 12.8.$
35. $4.41 \div 0.21.$
36. $17.28 \div 14.4.$
37. $1231.2 \div 2.4.$
38. $1.2312 \div 2.4.$
39. $62.75 \div 123.$
40. $6.275 \div 1.23.$
41. $36.08 \div 32.8.$
42. $3.608 \div 0.328.$
43. $1.036 \div 0.560.$
44. $882.32 \div 0.82.$
45. $4.4116 \div 0.41.$
46. $11{,}300 \div 5.65.$
47. $6.51021 \div 3.207.$
48. $651.021 \div 320.7.$
49. $176.4 \div 0.0072.$
50. $5.44 \div 0.0068.$
51. $717.75 \div 4.125.$
52. $88.232 \div 8.2.$
53. $633.775 \div 12.3.$
54. $24.5970 \div 0.027.$
55. $928.444 \div 0.44.$
56. $1.24792 \div 1.54.$
57. $0.56943 \div 33.3.$
58. $0.78210 \div 68.7.$
59. $62.351 \div 31.678.$
60. $489.246 \div 24.335.$

REVIEW DRILL CHART IN THE OPERATIONS

Add rapidly, recording the time taken:

1.	2.	3.	4.	5.
1276	9089	$12.75	$23.07	$371.42
2309	1627	26.42	41.68	126.09
5462	2314	3.97	33.99	83.54
1526	5403	40.21	8.27	102.07
7384	8675	16.95	65.45	29.68
4892	2756	7.34	57.29	32.67

Subtract rapidly, recording the time taken:

6.	8.	10.	12.	14.
7023	6021	$29.43	$80.12	$928.30
4681	5236	18.75	9.47	429.72

7.	9.	11.	13.	15.
8274	9231	$36.71	$92.20	$875.00
5328	4763	19.82	46.60	39.57

Multiply the first of these numbers by the second, and then divide the first by the second, giving the results in inexact divisions only to the nearest 0.01:

16. 75 by 840.	25. 144 by 12.	34. 3714 by 39.
17. 960 by 25.	26. 240 by 144.	35. 2806 by 73.
18. 125 by 960.	27. 288 by 24.	36. 4026 by 123.
19. 880 by 125.	28. 288 by 48.	37. 2708 by 236.
20. 625 by 440.	29. 1728 by 12.	38. 7432 by 247.
21. 625 by 640.	30. 1728 by 48.	39. 6296 by 793.
22. 640 by 375.	31. 1500 by 25.	40. 1225 by 326.
23. 480 by 375.	32. 25 by 1280.	41. 3265 by 472.
24. 800 by 625.	33. 1240 by 75.	42. 2766 by 395.

REVIEW DRILL CHART IN FRACTIONAL PARTS

Read, supplying the missing numbers:

1. $12\frac{1}{2}¢ = \frac{1}{8}$ of \$1.
2. $25¢ =$ —— of \$1.
3. $66\frac{2}{3}¢ =$ —— of \$1.
4. $6\frac{1}{4}¢ =$ —— of \$1.
5. $87\frac{1}{2}¢ =$ —— of \$1.
6. $40¢ =$ —— of \$1.
7. $37\frac{1}{2}¢ =$ —— of \$1.
8. $16\frac{2}{3}¢ =$ —— of \$1.
9. $62\frac{1}{2}¢ =$ —— of \$1.
10. $60¢ =$ —— of \$1.
11. $75¢ =$ —— of \$1.
12. $33\frac{1}{3}¢ =$ —— of \$1.
13. $33\frac{1}{3}¢ =$ —— of \$2.
14. $75¢ =$ —— of \$2.
15. $60¢ =$ —— of \$2.
16. $62\frac{1}{2}¢ =$ —— of \$2.
17. $6\frac{1}{4}¢ =$ —— of \$2.
18. $66\frac{2}{3}¢ =$ —— of \$2.
19. $25¢ =$ —— of \$2.
20. $12\frac{1}{2}¢ =$ —— of \$2.
21. $37\frac{1}{2}¢ =$ —— of \$3.
22. $62\frac{1}{2}¢ =$ —— of \$3.
23. $87\frac{1}{2}¢ =$ —— of \$3.
24. $\frac{1}{6}$ of \$1 $= 16\frac{2}{3}¢$.
25. $\frac{7}{8}$ of \$1 = ——.
26. $\frac{2}{3}$ of \$1 = ——.
27. $\frac{5}{8}$ of \$1 = ——.
28. $\frac{3}{8}$ of \$1 = ——.
29. $\frac{1}{3}$ of \$1 = ——.
30. $\frac{1}{20}$ of \$1 = ——.
31. $\frac{1}{25}$ of \$1 = ——.
32. $\frac{7}{8}$ of \$1 = ——.
33. $\frac{5}{6}$ of \$1 = ——.
34. $\frac{1}{2}$ of \$2 = ——.
35. $\frac{1}{3}$ of \$2 = ——.
36. $\frac{3}{4}$ of \$2 = ——.
37. $\frac{2}{3}$ of \$3 = ——.
38. $\frac{1}{2}$ of \$3 = ——.
39. $\frac{3}{4}$ of \$3 = ——.
40. $\frac{1}{8}$ of \$3 = ——.
41. $\frac{3}{8}$ of \$3 = ——.
42. $\frac{1}{8}$ of \$4 = ——.
43. $\frac{7}{8}$ of \$4 = ——.
44. $\frac{1}{6}$ of \$4 = ——.
45. $\frac{1}{6}$ of \$15 = ——.
46. $\frac{1}{6}$ of \$25 = ——.

REVIEW DRILL CHART IN FRACTIONS

Add the following:

1. $\frac{1}{4}, \frac{3}{8}$.
2. $\frac{1}{4}, \frac{3}{16}$.
3. $\frac{1}{4}, \frac{5}{16}$.
4. $\frac{1}{4}, \frac{9}{16}$.
5. $\frac{3}{4}, \frac{13}{16}$.
6. $\frac{1}{8}, \frac{3}{16}$.
7. $\frac{3}{8}, \frac{7}{16}$.
8. $\frac{3}{8}, \frac{15}{16}$.
9. $\frac{5}{8}, \frac{15}{16}$.
10. $\frac{7}{8}, \frac{13}{16}$.
11. $\frac{1}{2}, \frac{7}{32}$.
12. $\frac{1}{2}, \frac{15}{32}$.
13. $\frac{1}{4}, \frac{25}{32}$.
14. $\frac{3}{4}, \frac{31}{32}$.
15. $\frac{3}{8}, \frac{5}{32}$.
16. $\frac{3}{4}, \frac{7}{24}$.
17. $\frac{3}{4}, \frac{17}{24}$.
18. $\frac{5}{8}, \frac{5}{24}$.
19. $\frac{7}{8}, \frac{23}{24}$.
20. $\frac{5}{16}, \frac{1}{64}$.

21. Add the fractions $\frac{1}{2}, \frac{1}{4}, \frac{3}{8}, \frac{3}{4}, \frac{7}{16}, \frac{1}{8}, \frac{5}{8}$, and $\frac{1}{2}$.
22. Add the fractions $\frac{3}{4}, \frac{5}{8}, \frac{3}{16}, \frac{1}{2}, \frac{7}{8}, \frac{1}{4}, \frac{1}{16}, \frac{3}{16}$, and $\frac{9}{16}$.

Add the following:

23. $8\frac{3}{4}, 5\frac{1}{3}$.
24. $6\frac{3}{4}, 8\frac{2}{3}$.
25. $2\frac{1}{3}, 7\frac{1}{8}$.
26. $6\frac{1}{3}, 5\frac{3}{8}$.
27. $8\frac{2}{3}, 9\frac{5}{8}$.
28. $7\frac{2}{3}, 8\frac{7}{8}$.
29. $62\frac{5}{6}, 29\frac{3}{5}$.
30. $75\frac{7}{8}, 62\frac{15}{16}$.
31. $92\frac{1}{3}, 75\frac{4}{15}$.
32. $73\frac{2}{3}, 68\frac{7}{15}$.
33. $52\frac{4}{5}, 75\frac{13}{15}$.
34. $48\frac{9}{10}, 73\frac{11}{12}$.
35. $5\frac{4}{5}, 6\frac{2}{3}, 4\frac{2}{15}$.
36. $6\frac{7}{8}, 3\frac{1}{2}, 5\frac{15}{16}$.
37. $4\frac{1}{3}, 3\frac{3}{5}, 5\frac{1}{30}$.
38. $4\frac{7}{8}, 3\frac{2}{3}, 5\frac{11}{24}$.
39. $2\frac{7}{8}, 5\frac{13}{16}, 6\frac{5}{32}$.
40. $3\frac{5}{6}, 2\frac{5}{12}, 6\frac{7}{24}$.

Subtract as follows:

41. $\frac{3}{4} - \frac{1}{8}$.
42. $\frac{5}{8} - \frac{1}{4}$.
43. $\frac{5}{8} - \frac{3}{16}$.
44. $\frac{7}{8} - \frac{5}{16}$.
45. $\frac{1}{2} - \frac{5}{32}$.
46. $\frac{7}{8} - \frac{5}{32}$.
47. $\frac{7}{8} - \frac{11}{32}$.
48. $\frac{1}{2} - \frac{5}{16}$.
49. $\frac{5}{32} - \frac{1}{8}$.
50. $\frac{5}{8} - \frac{7}{32}$.
51. $\frac{1}{2} - \frac{11}{32}$.
52. $\frac{15}{32} - \frac{3}{64}$.
53. $\frac{13}{16} - \frac{5}{8}$.
54. $\frac{29}{64} - \frac{1}{4}$.
55. $32 - 4\frac{7}{8}$.
56. $63 - 5\frac{3}{4}$.
57. $72 - 8\frac{9}{10}$.
58. $48 - 3\frac{7}{32}$.
59. $67 - 5\frac{3}{16}$.
60. $71 - 6\frac{5}{24}$.
61. $80 - 9\frac{5}{48}$.
62. $9\frac{7}{16} - 7\frac{3}{8}$.
63. $8\frac{9}{16} - 3\frac{3}{8}$.
64. $6\frac{11}{16} - 4\frac{5}{8}$.
65. $7\frac{15}{16} - 3\frac{7}{8}$.
66. $9\frac{31}{32} - 6\frac{3}{4}$.
67. $8\frac{29}{32} - 5\frac{3}{8}$.
68. $23\frac{1}{8} - 8\frac{3}{16}$.

REVIEW DRILL CHART IN FRACTIONS

See if you can do these additions in $1\frac{1}{4}$ min.:

1. $\frac{1}{2}+\frac{1}{4}$. 3. $\frac{1}{2}+\frac{1}{8}$. 5. $\frac{1}{2}+\frac{5}{8}$. 7. $\frac{1}{4}+\frac{1}{8}$. 9. $\frac{3}{4}+\frac{3}{8}$.
2. $\frac{1}{2}+\frac{3}{4}$. 4. $\frac{1}{2}+\frac{3}{8}$. 6. $\frac{1}{2}+\frac{7}{8}$. 8. $\frac{1}{4}+\frac{7}{8}$. 10. $\frac{3}{4}+\frac{5}{8}$.

See if you can do these subtractions in $1\frac{1}{4}$ min.:

11. $\frac{1}{2}-\frac{1}{4}$. 13. $\frac{1}{2}-\frac{1}{8}$. 15. $\frac{7}{8}-\frac{1}{2}$. 17. $\frac{1}{4}-\frac{1}{8}$. 19. $\frac{3}{4}-\frac{3}{8}$.
12. $\frac{3}{4}-\frac{1}{2}$. 14. $\frac{1}{2}-\frac{3}{8}$. 16. $\frac{5}{8}-\frac{1}{2}$. 18. $\frac{7}{8}-\frac{1}{4}$. 20. $\frac{3}{4}-\frac{5}{8}$.

See if you can do these multiplications in $1\frac{1}{4}$ min.:

21. $\frac{1}{2}\times\frac{1}{4}$. 23. $\frac{1}{2}\times\frac{1}{8}$. 25. $\frac{5}{8}\times\frac{1}{2}$. 27. $\frac{1}{4}\times\frac{1}{8}$. 29. $\frac{3}{4}\times\frac{3}{8}$.
22. $\frac{1}{2}\times\frac{3}{4}$. 24. $\frac{1}{2}\times\frac{3}{8}$. 26. $\frac{7}{8}\times\frac{1}{2}$. 28. $\frac{7}{8}\times\frac{3}{4}$. 30. $\frac{3}{4}\times\frac{5}{8}$.

See if you can do these divisions in $1\frac{1}{4}$ min.:

31. $\frac{3}{4}\div\frac{3}{8}$. 33. $\frac{3}{4}\div\frac{5}{8}$. 35. $\frac{3}{4}\div\frac{7}{8}$. 37. $\frac{2}{3}\div\frac{3}{4}$. 39. $\frac{2}{3}\div\frac{7}{8}$.
32. $\frac{3}{8}\div\frac{3}{4}$. 34. $\frac{5}{8}\div\frac{3}{4}$. 36. $\frac{7}{8}\div\frac{3}{4}$. 38. $\frac{3}{4}\div\frac{2}{3}$. 40. $\frac{3}{5}\div\frac{2}{3}$.

Multiply as follows:

41. $34\times 3\frac{1}{2}$	44. $96\times 15\frac{7}{8}$	47. $29\frac{1}{2}\times 36$	50. $5\frac{1}{2}\times 2\frac{1}{2}$	53. $8\frac{1}{4}\times 2\frac{3}{8}$
42. $48\times 23\frac{1}{4}$	45. $64\times 35\frac{5}{8}$	48. $38\frac{2}{3}\times 45$	51. $7\frac{3}{4}\times 3\frac{1}{2}$	54. $6\frac{3}{4}\times 5\frac{5}{8}$
43. $56\times 38\frac{3}{4}$	46. $84\times 22\frac{2}{3}$	49. $52\frac{7}{8}\times 25$	52. $9\frac{7}{8}\times 6\frac{1}{2}$	55. $7\frac{2}{3}\times 3\frac{3}{8}$

Divide as follows:

56. $7\frac{3}{4}\div 2\frac{1}{2}$. 58. $8\frac{2}{3}\div 3\frac{1}{4}$. 60. $2\frac{1}{2}\div 1\frac{1}{4}$. 62. $8\frac{5}{8}\div 2\frac{7}{8}$.
57. $9\frac{7}{8}\div 3\frac{3}{4}$. 59. $3\frac{1}{4}\div 8\frac{2}{3}$. 61. $1\frac{1}{4}\div 2\frac{1}{2}$. 63. $2\frac{7}{8}\div 8\frac{5}{8}$.

REVIEW DRILL CHART IN FRACTIONS

Add rapidly, recording the time taken:

1.	2.	3.	4.	5.	6.
$\frac{1}{2}$	$1\frac{1}{2}$	$3\frac{1}{2}$	$8\frac{7}{8}$	$9\frac{3}{8}$	$7\frac{3}{16}$
$\frac{3}{4}$	$3\frac{3}{4}$	$5\frac{1}{4}$	$2\frac{3}{4}$	$7\frac{5}{16}$	$2\frac{5}{8}$
$\frac{5}{8}$	$2\frac{5}{8}$	$6\frac{7}{8}$	$5\frac{1}{2}$	$9\frac{3}{4}$	$9\frac{1}{2}$

Subtract rapidly, recording the time taken:

7.	8.	9.	10.	11.	12.
$\frac{7}{8}$	$\frac{7}{8}$	$9\frac{1}{2}$	$8\frac{1}{16}$	9	$8\frac{1}{3}$
$\frac{3}{4}$	$\frac{1}{2}$	$3\frac{3}{4}$	$3\frac{7}{8}$	$3\frac{2}{3}$	$3\frac{3}{5}$

Multiply rapidly, recording the time taken:

13.	14.	15.	16.	17.	18.
$1\frac{1}{2}$	$3\frac{3}{4}$	$5\frac{7}{8}$	$6\frac{3}{4}$	$7\frac{7}{8}$	$4\frac{2}{3}$
$\frac{3}{4}$	$\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{3}$	$4\frac{2}{3}$

Divide rapidly, recording the time taken:

19. $2 \div \frac{3}{4}$.
20. $\frac{3}{4} \div 2$.
21. $\frac{1}{2} \div \frac{7}{8}$.
22. $\frac{7}{8} \div \frac{1}{2}$.
23. $\frac{2}{3} \div \frac{3}{5}$.
24. $\frac{3}{5} \div \frac{2}{3}$.
25. $2\frac{7}{8} \div 3\frac{3}{4}$.
26. $3\frac{3}{16} \div 3\frac{3}{4}$.

27. Add $\frac{2}{3}$ and $\frac{3}{4}$ and divide the result by $2\frac{1}{2}$.
28. Multiply $3\frac{1}{2} - 1\frac{3}{4}$ by $5\frac{2}{3}$.
29. From $5\frac{3}{4} + 2\frac{7}{8}$ subtract $2\frac{3}{8} - \frac{5}{16}$.
30. Find the sum of $1\frac{1}{2}$, $2\frac{7}{8}$, $1\frac{3}{4}$, $5\frac{1}{2}$, and $6\frac{5}{8}$.
31. Find the difference between $3\frac{1}{2} + 2\frac{7}{8}$ and $1\frac{1}{4} + 2\frac{5}{8}$.
32. Find the product of $1\frac{1}{2} + 9\frac{7}{8}$ and $3\frac{1}{4} + 2\frac{7}{8}$.
33. Divide $8\frac{1}{2} + 3\frac{5}{8} + 2\frac{3}{4}$ by $4\frac{1}{4} + 1\frac{13}{16} + 1\frac{3}{8}$.
34. Add $2\frac{3}{4}$ and $5\frac{7}{8}$ and subtract $1\frac{3}{8}$ from the sum.
35. What number multiplied by $2\frac{5}{8}$ becomes $5\frac{1}{4}$? $7\frac{7}{8}$?
36. What number subtracted from $2\frac{1}{2}$ becomes $\frac{7}{8}$? $\frac{21}{32}$?

REVIEW DRILL CHART IN DECIMALS

Add rapidly, recording the time taken:

1. 12.07	3. 16.08	5. 231.4	7. 4.008
3.00	12.72	126.9	2.079
16.21	24.00	39.8	.630
4.09	76.75	107.0	8.000
2. 4.036	4. 173.2	6. 42.39	8. 3.675
2.750	52.9	17.62	2.473
6.382	610.3	51.87	5.214
1.571	725.1	2.46	3.279

Subtract rapidly, recording the time taken:

9. 32.01	11. 87.00	13. 501.21	15. 9.000
4.96	9.63	96.42	.078
10. 48.72	12. 75.05	14. 612.30	16. 8.123
19.39	62.78	149.79	4.690

Multiply rapidly, recording the time taken:

17. 24.09	19. 87.62	21. 722.31	23. 4.126
2.72	14.68	48.79	3.528
18. 362.1	20. 15.03	22. 52.613	24. 5.392
30.2	6.97	30.105	3.274

Divide, giving each result to the nearest 0.01:

25. $9273 \div 8.7$.	29. $40.89 \div 28$.	33. $8.289 \div 6.7$.
26. $896.8 \div 9.2$.	30. $49.72 \div 0.39$.	34. $9.763 \div 0.52$.
27. $42.76 \div 47$.	31. $7.362 \div 0.66$.	35. $5738 \div 8.74$.
28. $684.9 \div 7.3$.	32. $87.34 \div 5.7$.	36. $788.5 \div 0.35$.

LADDER TEST DRILL CHART

Copy each of these three ladders, and, beginning at the foot, see if you can climb rapidly to the top without making a single mistake:

I	II	III
20. $\frac{7}{8}$ mi. = (?) ft.	20. $8\frac{1}{2}$ gal. = (?) qt.	20. 27 in. ÷ 3 yd.
19. $\frac{3}{4}$ mi. = (?) ft.	19. $8\frac{1}{2}$ qt. = (?) pt.	19. $3\frac{5}{8}$ yd. + 7 in.
18. $\frac{1}{2}$ mi. = (?) ft.	18. $2\frac{1}{2}$ bu. = (?) pk.	18. $3\frac{1}{2}$ yd. ÷ 8 in.
17. 68 ft. = (?) in.	17. $3\frac{3}{4}$ ft. = (?) in.	17. $2\frac{1}{2}$ ft. ÷ 6 in.
16. 68 in. = (?) ft.	16. $3\frac{3}{4}$ rd. = (?) ft.	16. 6 yd. − 6 in.
15. $\frac{3}{4}$ yd. = (?) ft.	15. $3\frac{3}{4}$ yd. = (?) in.	15. 6 yd. ÷ 6 ft.
14. $\frac{3}{4}$ ft. = (?) in.	14. $3\frac{7}{8} \div 4\frac{1}{2}$.	14. 0.03×0.25.
13. $2\frac{3}{4} \div 3\frac{7}{8}$.	13. $4\frac{1}{2} \div 3\frac{7}{8}$.	13. 0.03×2.5.
12. $3\frac{7}{8} \div 2\frac{3}{4}$.	12. $4\frac{1}{2} \times 3\frac{7}{8}$.	12. 0.3×16.8.
11. $3\frac{7}{8} \times 2\frac{3}{4}$.	11. $4\frac{1}{2} - 3\frac{7}{8}$.	11. $3\frac{3}{4} \div 6\frac{7}{8}$.
10. $3\frac{7}{8} - 2\frac{3}{4}$.	10. $4\frac{1}{2} + 3\frac{7}{8}$.	10. $6\frac{7}{8} \div 3\frac{3}{4}$.
9. $3\frac{7}{8} + 2\frac{3}{4}$.	9. 5.94 ÷ 198.	9. $6\frac{7}{8} - 3\frac{3}{4}$.
8. 38.4 ÷ 19.2.	8. 594 ÷ 1.98.	8. $3\frac{3}{4} \times 6\frac{7}{8}$.
7. 38.4 × 19.2.	7. 59.4 ÷ 19.8.	7. $\frac{3}{4} \times \frac{1}{2} \times \frac{7}{8}$.
6. 38.4 − 19.2.	6. 594 ÷ 198.	6. $\frac{3}{4} - \frac{1}{2} + \frac{7}{8}$.
5. 38.4 + 19.2.	5. 5.94 × 19.8.	5. $\frac{3}{4} + \frac{1}{2} - \frac{7}{8}$.
4. 484 ÷ 363.	4. 5.94 × 1.98.	4. $\frac{3}{4} + \frac{1}{2} + \frac{7}{8}$.
3. 484 × 363.	3. 59.4 × 19.8.	3. $\frac{3}{4} + \frac{1}{2} + \frac{1}{8}$.
2. 484 − 363.	2. 59.4 − 19.8.	2. $\frac{1}{4} + \frac{1}{2} + \frac{1}{8}$.
1. 484 + 363.	1. 59.4 + 19.8.	1. $\frac{1}{2} + \frac{5}{8} + \frac{1}{4}$.

II. APPLICATIONS OF PERCENTAGE

A SILENT READING LESSON

Last year we learned a great deal about how per cents were used. We know that the merchant or business man finds many uses of per cents in figuring discounts, commissions, profit or loss, and interest. We shall find also in everyday life that per cents, because of their particular value in helping us to see the relations between numbers, are used in all kinds of situations in which we wish to make comparisons.

For example, we may say that Kate's standing in arithmetic is higher this month than it was last month; but if we wish to state precisely how much higher it is, we may say that it is 15% higher, 20% higher, or whatever per cent it may be. In the same way, we may say that the population of the place in which we live has increased $12\frac{1}{2}\%$ in 10 yr.; that the number of pupils in our school has increased 2% over last year; that our class averages 5% more in weight than it did last year, and so on.

Before we take up these uses of per cents, however, it will be a good plan for us to review rapidly the operation of finding a per cent of a number, of finding what per cent one number is of another, and of finding the number of which a given number is a certain per cent. At the same time we can review the short cuts in multiplication and division that we found were helpful in saving time and labor in making computations.

For example, we know that 25% means 0.25, $\frac{25}{100}$, or $\frac{1}{4}$. Hence, instead of finding 25% of a number by multiplying by 0.25, it is much easier to find it by dividing the number by 4. It often happens that discounts are stated as per cents which are equivalent to certain easy fractions. For example, $12\frac{1}{2}\%$ (or $\frac{1}{8}$), $16\frac{2}{3}\%$ (or $\frac{1}{6}$), $33\frac{1}{3}\%$ (or $\frac{1}{3}$), and $37\frac{1}{2}\%$ (or $\frac{3}{8}$) are often used.

REVIEW DRILL CHART IN PERCENTAGE

State the results as rapidly as you can:

1. 1% of \$10; of \$15.
2. 4% of \$10; of \$15.
3. 40% of \$10; of \$15.
4. 25% of \$4; of \$8.
5. 25% of \$5; of \$10.
6. 20% of \$50; of \$80.
7. 20% of \$60; of \$70.
8. 20% of \$8; of \$20.
9. 5% of \$10; of \$60.
10. 5% of \$3; of \$30.
11. 6% of \$40; of \$48.
12. 6% of \$4; of \$12.
13. 10% of \$10; of \$18.
14. 10% of \$25; of \$50.
15. 30% of \$10; of \$30.
16. 30% of \$40; of \$45.
17. $33\frac{1}{3}$% of \$60; of \$90.
18. $33\frac{1}{3}$% of \$9; of \$12.
19. $66\frac{2}{3}$% of \$9; of \$27.
20. $66\frac{2}{3}$% of \$60; of \$90.
21. $37\frac{1}{2}$% of \$80; of \$120.
22. 60% of \$2500.
23. 60% of \$2000.
24. 2% of \$1200.
25. 2% of \$2700.
26. 1% of \$35.40.
27. 2% of \$33.50.
28. $\frac{1}{2}$ of \$4000.
29. $\frac{1}{2}$% of \$4000.
30. $\frac{1}{4}$% of \$8000.
31. $\frac{3}{4}$% of \$8000.
32. $3 \times$ \$6000.
33. 3% of \$6000.
34. 0.3% of \$6000.
35. 100% of 1000.
36. 100% of 0.01.
37. 200% of 1.
38. 4% of \$2000.
39. 0.4% of \$2000.
40. $\frac{4}{5}$% of \$2000.
41. 0.8% of \$20.
42. 400% of \$2000.

REVIEW DRILL CHART IN PERCENTAGE

Read, supplying the missing numbers:

1. 32 is ——% of 64.
2. 64 is ——% of 32.
3. 64 is ——% of 64.
4. 8 is ——% of 100.
5. 8 is ——% of 200.
6. 80 is ——% of 100.
7. 80 is ——% of 200.
8. 72 is ——% of 144.
9. 72 is ——% of 720.
10. 15 is ——% of 90.
11. 1 is ——% of 2.
12. 2 is ——% of 2.
13. 100 is ——% of 200.
14. 200 is ——% of 100.
15. 125 is ——% of 100.
16. 125 is ——% of 200.
17. 3 is ——% of 8.
18. 33 is ——% of 88.
19. 330 is ——% of 990.
20. 990 is ——% of 330.

Copy, supplying the missing numbers:

21. 25 is 20% of ——.
22. 60 is 10% of ——.
23. 48 is $37\frac{1}{2}$% of ——.
24. $12\frac{1}{2}$ is 25% of ——.
25. 27 is $37\frac{1}{2}$% of ——.
26. 70 is $87\frac{1}{2}$% of ——.
27. 80 is $16\frac{2}{3}$% of ——.
28. 20 is $8\frac{1}{3}$% of ——.
29. 25 is $12\frac{1}{2}$% of ——.
30. 80 is 40% of ——.
31. 25 is $\frac{1}{5}$ of ——.
32. 25 is $\frac{1}{5}$% of ——.
33. 60 is $\frac{1}{10}$% of ——.
34. 48 is $\frac{3}{8}$% of ——.
35. $12\frac{1}{2}$ is $\frac{1}{4}$% of ——.
36. 27 is $\frac{3}{8}$% of ——.
37. 70 is $\frac{7}{8}$% of ——.
38. 80 is $\frac{1}{6}$% of ——.
39. 20 is $\frac{1}{12}$% of ——.
40. 150 is $\frac{1}{3}$% of ——.

SHORT CUTS IN MULTIPLICATION AND DIVISION

All work oral

1. How do you multiply by $\frac{1}{2}$? How do you divide by $\frac{1}{2}$?

2. How do you multiply by $\frac{3}{4}$? How do you divide by $\frac{3}{4}$?

3. Where do you place the decimal point in multiplying \$15.20 by 0.05? by 5%? by 5? by 50?

4. How do you multiply by 10? by 100? by 1000?

5. How do you divide by 100? by 1000? by 2000?

6. How do you multiply by 25? Have you given the shortest method you know?

7. What is the shortest method you know of finding 25% of a number?

8. How do you find 75% of 88? Is this the shortest way?

9. How do you multiply by 50? Illustrate by multiplying 22 by 50; 122 by 50; 4800 by 50.

10. What is the short method of multiplying by $33\frac{1}{3}\%$? Illustrate by finding $33\frac{1}{3}\%$ of \$36.

11. State the shortest method you know of finding $16\frac{2}{3}\%$ of a number. Illustrate by finding $16\frac{2}{3}\%$ of \$480.

12. Which is the shorter method of multiplying by 125, to multiply by $\frac{1000}{8}$ or by $\frac{500}{4}$? Explain your answer.

13. What short cut do you use in dividing by 125? Illustrate by dividing 2000 by 125.

14. Find $12\frac{1}{2}\%$ of \$160 by the shortest method you know.

15. If you buy $3\frac{1}{2}$ yd. of silk at \$4 a yard, how much does it cost? How much change do you receive from \$20?

16. If you buy $5\frac{3}{8}$ yd. of velvet at \$3.20 a yard, must you take a pencil and paper to find the cost? If you find it mentally, what steps do you take? What is the cost?

REVIEW DRILL CHART IN SHORT CUTS

By first multiplying by 10 and then dividing by 2, multiply each of the following numbers by 5:

1. 36.	3. 326.	5. 316.	7. 1122.	9. 1896.
2. 72.	4. 724.	6. 254.	8. 1096.	10. 1752.

By first multiplying by 100 and then dividing by 2, multiply each of the following numbers by 50:

11. 34.	13. 842.	15. 1122.	17. 1762.	19. 1824.
12. 28.	14. 788.	16. 1238.	18. 1848.	20. 1426.

Using short cuts, multiply the following numbers by 25:

21. 160.	25. 720.	29. 540.	33. 170.	37. 70.
22. 180.	26. 900.	30. 680.	34. 110.	38. 700.
23. 280.	27. 940.	31. 360.	35. 330.	39. 530.
24. 310.	28. 860.	32. 220.	36. 320.	40. 640.

Using short cuts, multiply each of the following numbers in turn by 125 and 500:

41. 48.	44. 720.	47. 130.	50. 642.	53. 3972.
42. 64.	45. 736.	48. 133.	51. 961.	54. 7234.
43. 32.	46. 232.	49. 637.	52. 165.	55. 4610.

Using short cuts, divide each of the following numbers in turn by 5, 25, and 125:

56. 38.	59. 160.	62. 888.	65. 362.	68. 2750.
57. 47.	60. 220.	63. 270.	66. 588.	69. 5635.
58. 98.	61. 940.	64. 385.	67. 934.	70. 9999.

REVIEW DRILL CHART IN SHORT CUTS

State rapidly the value of 1% of

1. 35.	4. 325.	7. 1575.	10. 3000.	13. 15,000.
2. 75.	5. 320.	8. 1570.	11. 3050.	14. 15,100.
3. 90.	6. 305.	9. 1500.	12. 3005.	15. 15,125.

By first taking 1% of each of the following numbers, find the value of

16. 5% of 800.	21. 5% of 300.	26. 5% of 850.
17. 8% of 500.	22. 6% of 500.	27. 6% of 975.
18. 7% of 700.	23. 5% of 600.	28. 6% of 2000.
19. 8% of 800.	24. 3% of 750.	29. 6% of 2225.
20. 3% of 500.	25. 4% of 725.	30. 7% of 4280.

Find the value in each of the following cases:

31. $33\frac{1}{3}$% of 900.	37. $2\frac{1}{2}$% of 336.	43. 10% of 2750.
32. $12\frac{1}{2}$% of 800.	38. $1\frac{1}{4}$% of 672.	44. 100% of 275.
33. $12\frac{1}{2}$% of 960.	39. $37\frac{1}{2}$% of 96.	45. $62\frac{1}{2}$% of 800.
34. $8\frac{1}{3}$% of 600.	40. $87\frac{1}{2}$% of 88.	46. $66\frac{2}{3}$% of 720.
35. $16\frac{2}{3}$% of 960.	41. $62\frac{1}{2}$% of 72.	47. 125% of 96.
36. $9\frac{1}{4}$% of 176.	42. 250% of 40.	48. $87\frac{1}{2}$% of 968.

Using a short method, find the interest when the principal, time, and rate are as follows:

49. $500, 60 da., 6%.	52. $1200, 24 da., 6%.
50. $320, 30 da., 4%.	53. $2500, 66 da., 6%.
51. $650, 90 da., 6%.	54. $4750, 83 da., 5%.

MISCELLANEOUS PROBLEMS

Copy and complete the following problems:

1. From a piece of silk containing $42\frac{1}{8}$ yd. a clerk sold me $8\frac{3}{4}$ yd. The clerk had —— yd. left.

2. At \$4.20 a yard, I paid —— for the $8\frac{3}{4}$ yd. of silk which I bought.

3. After the holidays the silk in Ex. 2 was marked down 15%. The price of the silk was then —— a yard.

4. If my scores on four arithmetic tests were 60, 72, 80, and 84, my average score was ——.

5. In an arithmetic test Fred tried 48 problems and solved 36 correctly, while Frank tried 52 problems and solved 39 correctly. Of the problems tried, Fred solved ——% correctly, and Frank solved ——% correctly.

6. On another arithmetic test Fred solved 48 problems correctly, which was 80% of the total number given. If Frank solved 70% correctly, he solved —— problems.

7. The players on the Stars' basketball team weighed 124 lb., 132 lb., 118 lb., 120 lb., and 131 lb., and those on the Crescents' team weighed 115 lb., 125 lb., 128 lb., 132 lb., and 130 lb. The average weight of the —— team was —— lb. greater than that of the —— team.

8. A man uses 8 gal. of gasoline in making a trip of 100 mi. in his car. He estimates that on a trip of 375 mi. over much the same kind of road he will use —— gal.

9. A salesman who is paid a commission of $12\frac{1}{2}$% on goods that he sells, earns \$—— on a sale amounting to \$140.

10. A commission merchant sells a consignment of farm produce for \$480. If he charges a commission of 8%, the farmer receives \$——.

USING PER CENTS IN THE STORE

A department store advertises a January mark-down sale. Find the sale prices of overcoats which are advertised at

	Regular Price	Discount
1.	$67.50	20%
2.	$65.00	20%
3.	$55.00	25%
4.	$75.00	30%
5.	$50.00	20%
6.	$62.50	25%

Find the sale prices of suits which are advertised at

	Regular Price	Discount
7.	$47.00	25%
8.	$45.00	25%
9.	$42.00	25%
10.	$40.00	25%
11.	$37.50	15%
12.	$35.00	15%
13.	$32.50	15%
14.	$30.00	15%

Find the per cent of discount offered on the regular price of fur-trimmed coats for women which are advertised at

	Regular Price	Sale Price
15.	$125.00	$75.00
16.	$115.00	$70.00
17.	$110.00	$68.00
18.	$100.00	$60.00
19.	$90.00	$59.50
20.	$85.00	$49.90

Find the per cent of discount on the regular price of

21. Men's suits, regular price $45.00, reduced to $39.50.

22. Boys' suits, regular price $15.00, reduced to $12.75.

USING PER CENTS IN COMMISSION

In each of the following cases, which show the amounts of sales and the rates of commission paid, find the amount of the commission and the proceeds:

1. \$650, 2%.
2. \$1300, 3%.
3. \$1650, 4%.
4. \$1895, 5%.
5. \$3200, $3\frac{1}{2}$%.
6. \$3520, $4\frac{1}{2}$%.
7. \$1675, 8%.
8. \$2250, $8\frac{1}{2}$%.
9. \$8500, $3\frac{1}{2}$%.
10. \$12,000, $4\frac{1}{4}$%.
11. \$12,500, $3\frac{1}{4}$%.
12. \$16,550, $3\frac{3}{4}$%.
13. \$18,500, $5\frac{1}{2}$%.
14. \$22,500, $6\frac{7}{8}$%.
15. \$18,625, $5\frac{1}{4}$%.
16. \$20,750, $4\frac{3}{4}$%.
17. \$80.50, 8%.
18. \$69.35, 7%.
19. \$51.27, 10%.
20. \$98.25, $4\frac{1}{2}$%.
21. \$67.20, $12\frac{1}{2}$%.
22. \$86.40, 15%.
23. \$112.25, 12%.
24. \$150.60, 17%.

In each of the following cases, which show the amounts of sales and the net amounts received from the sales, find the amount and rate of commission paid:

25. \$500, \$450.
26. \$600, \$576.
27. \$700, \$665.
28. \$650, \$624.
29. \$320, \$302.40.
30. \$480, \$441.60.
31. \$640, \$598.40.
32. \$730, \$697.15.
33. \$10,000, \$9650.
34. \$12,500, \$11,875.
35. \$14,225, \$13,656.
36. \$16,250, \$14,950.

37. If possible, find out the customary rates of commission charged by local real-estate agents for selling various classes of property. Make up and bring to class two or three problems about selling pieces of property near the school.

38. Find out, if possible, the rates of commission charged by commission merchants in your city or in a neighboring city for handling farm produce. Make up and bring to class several problems about selling farm produce on commission.

39. Find out about architects' fees and the customary rates, and make up two or three problems about them.

FINDING PROFIT OR LOSS

Disregarding the expenses of doing business, which must, of course, be subtracted from the gain or added to the loss, state rapidly, in each of the following cases, the per cent of gain or loss on the cost, stating also whether it is a gain or a loss:

	Cost	Selling Price
1.	1¢	2¢
2.	5¢	10¢
3.	4¢	3¢
4.	$1\frac{1}{2}$¢	3¢
5.	$10	$15

	Cost	Selling Price
6.	$10	$12
7.	$10	$25
8.	$25	$20
9.	$50	$30
10.	$75	$100

11. Cost $1000, selling price $500; selling price $1500.

12. Cost $800, selling price $600; selling price $1000.

As above, state the per cent of gain or loss, but base it on the selling price in each of the following cases:

13. Cost 1¢, selling price $1\frac{1}{2}$¢; selling price 2¢.

14. Cost 5¢, selling price 6¢; selling price 8¢.

15. Cost 90¢, selling price 80¢; selling price $1.

16. Cost $800, selling price $750; selling price $1000.

Find the amount of gain or loss and also the selling price when the cost and the per cent of gain or loss based on the cost are

17. Cost $500, gain 20%.

18. Cost $250, loss 20%.

19. Cost $600, loss 25%.

20. Cost $800, gain 10%.

21. Cost $75, loss $33\frac{1}{3}$%.

22. Cost $1260, loss 10%.

23. Cost $100, gain 30%.

24. Cost $1.48, loss 10%.

Copy the following table, which shows amounts and per cents of gain figured on the selling price, and fill the blank columns, checking the selling price in each case by taking the given per cent of it and comparing the result with the given amount of gain:

	Gain	Per Cent	Selling-Price	Cost
25.	\$25	50%		
26.	\$50	$33\frac{1}{3}$%		
27.	\$25	25%		
28.	\$20	$16\frac{2}{3}$%		
29.	\$20	20%		
30.	\$20	25%		
31.	\$5	$6\frac{1}{4}$%		
32.	\$75	$33\frac{1}{3}$%		
33.	\$125	$62\frac{1}{2}$%		
34.	\$13.94	17%		
35.	\$213.44	28%		
36.	\$668.25	45%		

37. If you buy articles at 5 for 3¢ and sell them at 5 for 10¢, what is your per cent of profit figured on the selling price? figured on the cost?

38. If $\frac{1}{2}$¢ is a gain of $33\frac{1}{3}$%, figured on the selling price, what is the selling price? What is the cost?

39. If $3\frac{1}{3}$¢ is a gain of $33\frac{1}{3}$%, figured on the selling price, what is the selling price? What is the cost? What is the per cent of gain on the cost?

40. Solve Ex. 39 for $3\frac{3}{4}$¢ and $37\frac{1}{2}$%; for $7\frac{1}{2}$¢ and 75%; for $16\frac{2}{3}$¢ and $33\frac{1}{3}$%.

USING PER CENTS IN COMPARISONS

1. In Boston, consumers pay higher prices for brown eggs, while in New York City they pay higher prices for white eggs, although chemical analysis has shown that there is no difference in the quality of white and brown eggs. If white eggs are selling for 72¢ per dozen and brown eggs are selling for 66¢ in the same city, what per cent of the higher price can be saved by buying brown eggs?

2. Bright red apples often bring higher prices than other apples, although they may be much poorer for eating. If red apples sell for $5 per bushel, and equally good yellow apples sell for $3.50 per bushel, what per cent of the higher price can be saved by buying the cheaper apples?

3. In the North yellow corn meal is in demand, while in the South white corn meal is more popular, although the nutritive value of each is the same. When yellow meal is selling at $6\frac{1}{2}$¢ a pound and white meal at 12¢, what is the per cent of saving by buying the cheaper meal?

4. Southern markets prefer yellow to red onions, although, when their skins are removed, the onions are just the same in taste and food value. A saving of 18¢ on a purchase amounting to 98¢ is what per cent of saving?

5. Cheaper cuts of beef, such as the neck, have the same nutritive value as the more expensive cuts, and with proper cooking can be made just as palatable. If a housekeeper serves a chuck roast costing 22¢ a pound in place of a sirloin roast costing 40¢ a pound, what per cent of the higher cost does she save by so doing?

6. If possible, obtain at local stores prices of different cuts of meat. Make up and bring to class five problems about saving money in purchasing meats.

7. The high-school enrollment in the United States in 1890 was 297,894; in 1900, 630,048; in 1910, 1,032,461; and in 1920, 2,041,308. Find the per cent of increase in this enrollment from 1890 to 1900; from 1900 to 1910; from 1910 to 1920; from 1890 to 1920.

8. The population of the United States was 62,947,714 in 1890 and 105,710,620 in 1920. Find the per cent of increase. This is how much less than the last result found in Ex. 7?

9. In 1900 the United States imported 11,259,310 lb. of raw silk, while in 1920, 47,127,122 lb. were imported. What was the per cent of increase over 1900?

10. Last year the Garden Club of a city school received $237.50 for the vegetables which the members raised and sold. John's share was 23.3% of the total receipts; Peter's, 21.5%; Mary's, 16.2%; Robert's, 14.8%; Alice's, 12.7%; Helen's, 8.4%; and Ruth's, 3.1%. How much money did each member receive?

11. Mr. Wilson bought a shoe store on June 1. At the end of the year he found that his monthly sales had been as follows: June, $3827; July, $2631.50; Aug., $2152.75; Sept., $2767; Oct., $3547.50; Nov., $3925; Dec., $4620.25. Find the per cent that each month's receipts were of the total sales since June 1.

12. The total number of automobiles of all kinds exported from the United States in a recent year was 178,732, and in the following year it was 302,924. Show that, to the nearest 0.1%, the rate of increase was 69.5%.

13. The total value of our imports of wood and articles manufactured from wood in a recent year was approximately $120,000,000, which was an increase of 8% over the preceding year. Show that the value of imports in the preceding year was approximately $111,000,000.

Meat and Health. Our school nurse says that the American people eat too much meat and not enough vegetables. The following table shows the average amount of meat consumed by each person in seven different countries in a recent year:

Country	Pounds per Person per Year
United States	187
Germany	115
England	105
Belgium and Holland	75
France	74
Russia	59
Italy	23

PROBLEMS ABOUT MEAT-EATING

1. Find the average consumption for all the countries.

2. What per cent more meat did the average person in the United States eat than the average person in Italy? than the average for all the countries?

3. What per cent of the consumption of meat in the United States was the consumption in Italy? in Russia?

4. What per cent more meat did we use than England? than Germany? than France?

5. In a year when the average consumption of meat in this country was 194 lb. per person per year, the consumption was what per cent greater than in the year for which the figures are given in the table above?

6. If the same food value can be had from 14¢ worth of milk as from 38¢ worth of sirloin steak, what per cent of saving could be made by using milk instead of steak?

Using Vegetables. Reliable authorities say that in the United States we eat only about one third as many green vegetables as we need to keep in good health. In food value, the leading green vegetable is spinach, which is not sufficiently used in this country. The following table shows the number of units of food value in a standard amount of several common vegetables:

Vegetables	Values
Spinach	810
Green peas	475
Green beans	472
Potatoes	414
Sweet potatoes	374
Asparagus	368

Vegetables	Values
Cabbage	367
Celery	350
Carrots	338
Lettuce	299
Onions	295
Tomatoes	192

FOOD VALUE OF VEGETABLES

1. Spinach has what per cent greater food value than asparagus? than onions?

2. Green peas have what per cent greater food value than carrots? than celery?

3. As in Exs. 1 and 2, compare the food value of potatoes with that of cabbage; with that of lettuce; with that of asparagus.

4. Compare the food value of green beans with that of sweet potatoes; with that of carrots; with that of onions.

5. The food value of asparagus is what per cent greater than that of onions? The food value of onions is what per cent less than that of asparagus?

6. The food value of lettuce is what per cent less than that of green peas? The food value of green peas is what per cent greater than that of lettuce?

Food in Packages. There are many foods that can be purchased either in packages or in bulk. The package increases the cost, because the boxes, or containers, the printing, the paper, and the labor must all be paid for by the consumer. The following table shows the prices per pound of several foods which can be bought either in bulk or in packages:

	Bulk Price per Pound	Package Price per Pound	Extra Cost in Packages	Per Cent of Extra Cost
Bacon	42.0¢	58.2¢	16.2¢	38.6%
Cocoa	25.0¢	45.0¢		
Coffee	42.0¢	48.0¢		
Corn meal . .	6.5¢	10.0¢		
Graham flour .	9.5¢	10.0¢		
Macaroni . . .	6.25¢	16.0¢		
Oatmeal . . .	10.0¢	10.4¢		
Oyster crackers	20.0¢	30.0¢		
Rye flour . . .	7.5¢	9.0¢		
Soda crackers .	22.0¢	30.0¢		
Tapioca . . .	15.0¢	25.0¢		

PAYING FOR THE PACKING

1. Copy the above table and fill in the blank columns.

2. Find the average cost per pound in bulk and in packages of the articles in the table. Find also the average extra cost per pound and the average per cent of extra cost.

3. Get similar prices from local stores and find the per cent of saving by buying in bulk. State whether or not you think that the cleanliness of the package offsets the extra cost.

4. Make up five problems to show how much can be saved by a family in making purchases in bulk of various amounts of some of the foods in the table.

How We Use Milk. In a recent year the milk production in the United States was 89,842,000,000 lb. This table, in which the numbers represent millions, shows how it was used:

How Used	Million Pounds	Per Cent of Total
Household purposes . . .	38,619	
Creamery butter	18,375	
Dairy butter	14,385	
Condensed milk	4,813	
Cheese	4,200	
Feeding calves	3,500	
Ice cream	3,450	
Wasted	2,500	
Totals	89,842	100%

USES OF MILK

1. Copy the above table and then complete it by finding the per cent of the total amount used for each purpose.

2. What per cent more milk was used for household purposes than for creamery and dairy butter combined?

3. Compare the amount wasted with that used for making cheese by finding what per cent the former is of the latter.

4. What per cent of the total amount used for making butter was used for dairy butter?

5. What per cent of the ice cream that we use could be made with the wasted milk?

6. What per cent of the calves could be fed with the amount of milk that is wasted?

7. What per cent less milk is used for making dairy butter than is used for creamery butter?

Butter Fat in Milk. In 1890 Dr. Babcock invented the Babcock milk tester, which makes it possible to determine easily the per cent of butter fat (the chief constituent in butter) in milk. He donated his invention to the world, although he could have sold it for millions of dollars.

The composition of standard butter, as determined by the United States Department of Agriculture, is as follows:

Butter fat . .	82.41%	Salt	2.51%
Water	13.90%	Curd	1.18%

PROBLEMS ABOUT BUTTER FAT

1. Using the above table, find the weight of each of the constituents in 720 lb. of standard butter.

2. The dairymen who sell milk to a large factory in central New York are paid in proportion to the per cent of butter fat in the milk which they sell. This table shows the price recently paid per quart for milk containing the per cent of butter fat stated. Find the amount received for 50 qt. of each of the grades of milk given in the table.

Per cent	Price
5.6%	10.6¢
4.7%	9.7¢
4.6%	9.6¢
4.1%	9.1¢
4.0%	9.0¢
3.9%	8.9¢
3.8%	8.8¢

3. From this table find what per cent more the milk that tests 5.6% butter fat is worth than that which tests 3.8%.

4. Farmers who sell cream are paid for the butter fat the cream contains and not for the weight of the cream. How many pounds of cream testing 25% butter fat will it take to equal in value 2500 lb. testing 35% butter fat? How many pounds testing 35% will equal in value 2500 lb. testing 25%?

MISCELLANEOUS PROBLEMS

1. A farmer was offered $1400 for his potato crop in October, but by waiting until spring he received $1620 for it. Deducting $42 to offset the interest lost on the $1400, find the per cent gained by holding the crop.

2. Last year the total registration in a school was 1830, and the average daily attendance was 1710. The attendance was what per cent of the total registration?

3. Mr. Park bought his winter supply of potatoes last fall at $1.40 a bushel. His neighbors, who bought their potatoes during the winter in small quantities, had to pay an average price of 4¢ a pound. Allowing 60 lb. to the bushel, what per cent of the higher price did he save by this method?

4. Mr. Smith and his son bought a motor truck for $5000. Mr. Smith paid $3700 and his son paid the balance. What per cent did each pay?

5. Which gives the better per cent of profit, an investment of $12,000 which yields a profit of $900 or an investment of $10,000 which yields a profit of $800?

6. The estimated population of New York State in 1925 was 11,105,625, and of this number 6,015,496 lived in New York City. What per cent lived in New York City?

7. In 1925 the estimated population of the United States was 113,493,720, and of this number 4,021,320 lived in California. What per cent lived in California?

8. There were 720 pupils in School No. 5 in a small city, and of this number 288 were in the first three grades. What per cent were in these three grades?

9. Ben's scores on eight tests were 84%, 88%, 90%, 90%, 92%, 85%, 87%, and 96%. Find his average score.

PROBLEMS ABOUT INSURANCE

1. The owner of a row of eight dwellings insures each one of them against loss by fire for \$5800. If the annual rate for fire insurance on this class of property is 34¢ per \$100, how much does he pay in premiums if the policies are written for 1 yr.?

2. If, in Ex. 1, the rate for a 3-year policy is $2\frac{1}{2}$ times the annual rate, what will the owner pay in premiums if he takes the insurance for 3 yr.?

3. If, in Ex. 1, the rate for a 5-year policy is four times the annual rate, what will the owner pay in premiums if he insures his property for a 5-year period?

4. A mill owner insures his building against loss by fire for \$134,500, his machinery for \$75,000, and his stock of materials for \$65,500. The annual rate on the building is \$1.25 per \$100, and on the machinery and materials it is \$1.45 per \$100. What premium does he pay for a policy for 1 yr.?

5. A department-store building is valued at \$125,000, and the contents are valued at \$225,000. The owner takes out fire insurance for 80% of the value at an annual rate of 89¢ on the building and \$1.19 on the contents. What premium does he pay for a policy for 1 yr.?

6. The owner of a dwelling insured it for \$5000 for 3 yr. at an annual rate of 36¢ (see Ex. 2). At the end of 1 yr. 4 mo. he sold the property to a man who agreed to pay for the unexpired insurance. How much did the purchaser pay to the owner on this account?

7. A doctor insured his dwelling for \$6500, his furniture for \$3250, and his office fixtures and medical supplies for \$1200. The annual rate on the dwelling was 35¢, on the furniture 39¢, and the rate on the fixtures was the furniture rate plus 10%. What premium did he pay for a 3-year policy (see Ex. 2)?

8. The fire-insurance premiums in a town which had no public fire protection amounted, during a certain year, to \$3675.75. When a fire department and a water system were installed, the insurance companies reduced the rates 15%. For the same amount of insurance, what were the premiums the year after the reduction?

9. The total premiums received by a fire-insurance company in a certain year were \$1,341,225. During the year the fire losses paid were $52\frac{1}{2}\%$ of the premiums, and the expenses were $45\frac{3}{4}\%$. How much did the company pay that year to meet the losses and expenses?

10. A merchant paid a premium of \$929.50 annually on a fire-insurance policy for \$65,000. What was the rate?

11. At the age of 27 yr., when the rate was \$23.14 per \$1000, a man insured his life for \$3500. If he lived to be 47 yr. old, how much did he pay the company in premiums?

12. A man takes out a 20-payment life-insurance policy for \$8000 at an annual rate of \$35.26 per \$1000. During the 20 yr. the dividends paid to the policy holders averaged \$6.80 per \$1000 per year. What was the actual cost to him of the policy by the time it was paid up?

13. As in Ex. 12, find the actual cost of a 30-payment life-insurance policy for \$15,000, when the annual rate is \$25.34, and the dividends average \$4.20 per \$1000 of insurance.

14. A man had an accident-insurance policy that provided weekly payments of \$50 for total disability and \$18.75 for partial disability. An accident disabled him totally for 6 wk. and partially for 12 wk. How much did he receive?

15. A vessel was insured for \$750,000, and the cargo for \$356,750. On account of damage by a storm the insurance company paid 6% of the face of the policy on the vessel and 32% on the cargo. How much was paid?

PROBLEMS ABOUT TAXES

At a tax rate of $1.36 per $100 find the taxes paid on property which is assessed at

1. $600.	6. $8000.	11. $10,000.	16. $55,000.
2. $250.	7. $9200.	12. $12,000.	17. $75,500.
3. $375.	8. $5850.	13. $13,500.	18. $125,000.
4. $840.	9. $6250.	14. $16,250.	19. $225,000.
5. $1250.	10. $8375.	15. $20,500.	20. $675,000.

At a tax rate of $14.70 per $1000 find the taxes paid on property which is assessed at

21. $350.	24. $9300.	27. $12,500.	30. $60,000.
22. $780.	25. $7560.	28. $16,250.	31. $72,500.
23. $1220.	26. $8525.	29. $15,500.	32. $128,500.

33. A city with a total assessed valuation of $75,000,000 has to raise $1,845,000 to meet the year's budget of expenses. What will be the tax rate on each $1000 of assessed valuation?

34. Solve Ex. 33 for a town which has to raise $56,250 and has a total valuation of $3,500,000.

35. After deducting the exemptions to which he is entitled, a man must pay an income tax on $3750.68 at a rate of 2%. Find the amount of his income tax.

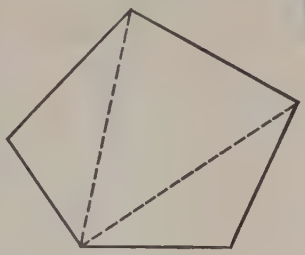
36. In a year in which the receipts of the national government totaled $4,012,000,000 the amount raised by taxes on incomes was $1,842,000,000. Income taxes yielded what per cent of the total receipts?

37. Of the total income-tax receipts mentioned in Ex. 36, $161,100,000 were collected in Illinois. What per cent of the total was paid by this state?

III. MEASUREMENTS

A SILENT READING LESSON

We already know how to measure lengths, time, and weight and how to find the areas of rectangles, triangles, parallelograms, and circles. Thus, we know how to find the number of square yards of linoleum needed to cover a kitchen floor, or the number of square feet of surface to be covered in painting the triangular gable end of a house, or the cost of resilvering the back of a circular mirror at so much per square inch, and so on for similar cases.



We can also find the area of any figure that we can divide into triangles; for example, we can find in this way the area of an irregular field such as is represented by the figure above.

We have not yet learned, however, how to find the volumes of solid figures, a thing that it is often necessary to do. For example, on the farm we might need to know the volume of an oat bin or of a silo used for storing feed for the cattle, or in the city we might need to know how much earth must be removed in excavating for the foundation of a building, or we might wish to know the capacity of a water tank on the top of one of the apartment houses.

Likewise, we have not yet learned how people find the distances across wide rivers, or how they measure the heights of trees, flagpoles, or mountains without climbing them. We may never have to measure the width of a river or the height of a mountain, but we should like to know how such things are done, because they are matters of general information among educated people.

REVIEW OF SURFACE MEASUREMENTS

Numbers 1 to 32, oral

1. How do you find the area of a square?

State the areas of squares with sides of

2. 2 in.	4. 6 ft.	6. $\frac{1}{2}$ in.	8. 0.5 in.	10. 11 in.
3. 5 in.	5. 9 ft.	7. $\frac{1}{4}$ in.	9. 0.7 in.	11. 12 in.

12. How do you find the area of a rectangle?

State the areas of rectangles with sides of

13. 2 in., 4 in.	15. 1 yd., 3 yd.	17. 10 rd., 16 rd.
14. 3 ft., 6 ft.	16. 2 yd., 5 yd.	18. 4 rd., 12 rd.

19. How do you find the area of a parallelogram?

State the areas of parallelograms with bases and heights respectively as follows:

20. 5 in., 2 in.	22. 6 ft., 3 ft.	24. 14 yd., 2 yd.
21. 2 in., 5 in.	23. 3 ft., 6 ft.	25. 20 yd., 8 yd.

26. How do you find the area of a triangle?

State the areas of triangles with bases and heights respectively as follows:

27. 1 ft., 1 ft.	29. 3 ft., 3 ft.	31. 2 in., 5 in.
28. 2 ft., 2 ft.	30. 1 yd., 1 yd.	32. 12 in., 11 in.

33. If we double the sides of a square, how does this affect the area? Draw a figure to illustrate your answer.

34. Find the area of a rectangle $2\frac{1}{4}$ ft. long and $1\frac{1}{2}$ ft. wide.

35. Find the area of a parallelogram whose base is 6 ft. 8 in. and whose height is half as much.

Area of a Trapezoid. There is another four-sided figure, the *trapezoid*, of which we have not yet learned how to find the area. To know how to do this is important, because many fields and building lots have the shape of a trapezoid.

Suppose that we have the trapezoid T shown in this figure. We may draw and cut out a trapezoid D exactly the same size and shape as T, and then fit the two together to form a parallelogram, as shown. That is, the area of a trapezoid is half the area of a parallelogram with the same height and with a base equal to the total length of the two parallel sides of the trapezoid, or, briefly stated,

$$Area = \tfrac{1}{2} \times height \times (lower\ base + upper\ base),$$

the parentheses meaning that we add the lengths of the two parallel sides before we multiply.

For example, in finding the area of a trapezoid with a height of 4 in. and parallel sides of 8 in. and 10 in., we see that the area is $\frac{1}{2} \times 4 \times (8 + 10)$ sq. in.; that is, it is $\frac{1}{2} \times 4 \times 18$ sq. in. Arranging the work as here shown, the result is found to be 36; that is, the area of the trapezoid is 36 sq. in.

$$\frac{\overset{9}{\cancel{4}} \times \cancel{18}}{\underset{1}{\cancel{2}}} = 36$$

AREA OF A TRAPEZOID

1. A farmer has a field in the form of a trapezoid. The height of the trapezoid is 10 rd., and the parallel sides are $31\frac{1}{4}$ rd. and $16\frac{3}{4}$ rd. What is the area of the field?

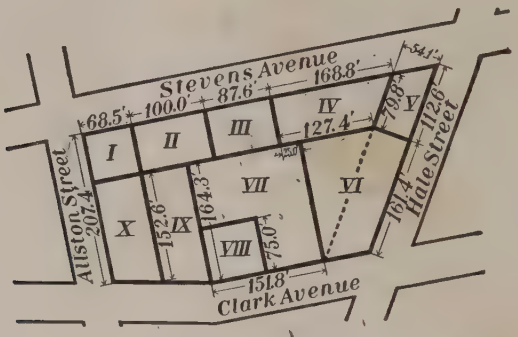
2. A building lot in a village is in the form of a trapezoid. The height of the trapezoid is 66 ft., and the parallel sides are 78 ft. 6 in. and 86 ft. 8 in. What is the area of the lot?

Find the areas of the following trapezoids, the height in each case being given first and then the two parallel sides:

3. $16\frac{1}{2}$ in.; $8\frac{1}{4}$ in. and 12 in. **5.** $4\frac{1}{2}$ ft.; 3 ft. and $4\frac{1}{2}$ ft.

4. 3.5 in.; 3.5 in. and 5.4 in. **6.** 16 rd.; 18 rd. and 24 rd.

7. An irregular city block is divided into ten lots as shown in the figure below. Each measurement is given to the nearest 0.1 ft. (0.1′), and the areas of the lots are to be found to the nearest 0.1 sq. ft. Find the area of the square lot marked I.



8. Find the area of each of lots II, III, IV, and V.

9. Find the area of each of lots IX and X, finding the length of any side that you need to use by adding or subtracting the measurements given on the plan.

10. By dividing lot VI into two parts, as suggested by the dotted line, find the area of the lot.

11. Find the combined area of lots VII and VIII.

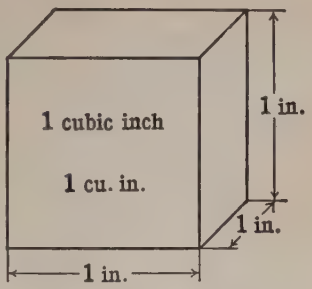
12. Find the area of lot VIII and then subtract it from the result of Ex. 11, thus finding the area of lot VII.

13. By dividing lot VII into two parts, find the area directly, thus checking the result found in Ex. 12.

Table of Cubic Measure. Before learning about volumes, we shall need to review the following table of cubic measure:

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

If a block in the form of a cube is 1 in. on an edge, it is a a 1-inch cube, and its *volume* is 1 cu. in. If the block is 1 ft. on an edge, it is a 1-foot cube, and its volume is 1 cu. ft., and similarly for yards.



Another measure of volume is the *cord*. The standard cord of wood is a pile 8 ft. long, 4 ft. wide, and 4 ft. high, which is equivalent to 128 cu. ft. In practice, however, a pile of wood 8 ft. long, 4 ft. high, and 2 ft. wide is often called "a cord of 2-foot wood," and similarly for other lengths into which the sticks may be sawed.

CUBIC MEASURE

1. In excavating a cellar, a contractor removed 8100 cu. ft. of earth. Express the amount as cubic yards.

2. On one section of a new state road 8000 cu. yd. of crushed rock were used. How many cubic feet were used?

3. A box has a volume of $2\frac{1}{2}$ cu. ft. Express the volume of the box in cubic inches.

Copy these statements, supplying the missing numbers:

4. 2 cu. yd. = —— cu. ft.

5. 81 cu. ft. = —— cu. yd.

6. 2 cu. ft. = —— cu. in.

7. $\frac{1}{2}$ cu. ft. = —— cu. in.

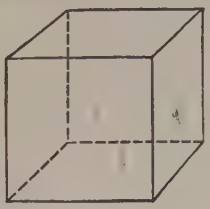
8. $\frac{2}{3}$ cu. yd. = —— cu. in.

9. $3\frac{1}{2}$ cu. yd. = —— cu. ft.

STUDYING THE CUBE

Copy and complete the following statements:

1. A cube has —— square faces. These faces are the base, the top, and the —— sides.



2. A cube has —— edges, which are the straight lines where any two faces meet.

3. A cube has —— corners or vertices.

4. If the edge of a cube is 1 in., the area of each face is —— sq. in., and the area of the entire surface of the cube is therefore —— sq. in.

5. If the edge of a cube is 2 in., the area of each face is —— sq. in., and the area of the entire surface is —— sq. in.

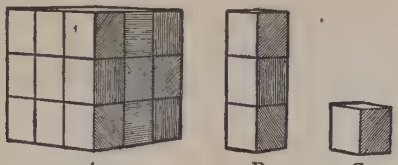
6. If the edge of a cube is 1 in., the volume is —— cu. in. If the edge is 2 in., the volume is —— cu. in.

7. The number which multiplied by itself gives 100 is ——. If the area of the entire surface of a cube is 600 sq. in., the edge is —— in.

Copy the following table and fill the blank spaces:

	Edge of Cube	Total Length of Edges	Area of a Face	Area of Entire Surface	Volume of Cube
8.	1 in.	12 in.	1 sq. in.	6 sq. in.	1 cu. in.
9.	2 in.		4 sq. in.		8 cu. in.
10.	3 in.			54 sq. in.	
11.		60 in.			125 cu. in.
12.		84 in.			
13.			36 sq. in.		
14.			144 sq. in.		

Volume of a Rectangular Solid. A solid with six faces, each of which is a rectangle, is called a *rectangular solid*. Just as a square is a special kind of rectangle (one with four equal sides), the cube is a special kind of rectangular solid. These figures are all rectangular solids, and A and C are cubes.



If the cube C contains 1 cu. in., how many cubic inches does B contain? If A has 9 cu. in. in the bottom layer of cubes, how many cubic inches does A contain?

If A were a box 4 in. long and 3 in. wide, how many cubic inches would there be in the bottom layer of cubes? If it were 5 in. high, how many cubic inches would it contain?

If a box is 5 in. long, 4 in. wide, and 3 in. high, how many cubic inches does it contain?

In finding the amount of earth removed from an excavation 20 yd. long, 10 yd. wide, and 2 yd. deep, we have

$$20 \times 10 \times 2 = 400, \text{ the number of cubic yards.}$$

Instead of this we may use the same plan that we do in finding areas; that is, we may write

$$20 \times 10 \times 2 \text{ cu. yd.} = 400 \text{ cu. yd.}$$

Thus, to find the volume of a rectangular solid, we multiply together the length, width, and height, or, briefly stated,

$$\textit{Volume} = \textit{length} \times \textit{width} \times \textit{height}.$$

A rectangular solid was formerly called a *rectangular parallelepiped*, a name that we still hear occasionally.

VOLUME OF A RECTANGULAR SOLID

1. In our school there is an aquarium 14 in. long, $9\frac{1}{2}$ in. wide, and $8\frac{1}{2}$ in. deep. How many square inches are there in the bottom? How many cubic inches of water does the aquarium contain when level full?

2. The interior of a freight car is 34 ft. long, 8 ft. wide, and 7 ft. high. How many square feet are there in the floor of the car? How many cubic feet of space are there in the car?

3. Near our school they have excavated a cellar 34 ft. long, 16 ft. wide, and 9 ft. deep. How many cubic feet of earth were removed? How many cubic yards?

4. At \$1.40 a cubic yard, how much will it cost to excavate a cellar 28 ft. by 38 ft. by 6 ft.?

5. Near our school they are building a wall which is to be 60 ft. long, 6 ft. high, and $1\frac{1}{2}$ ft. thick. How many cubic feet will the wall contain?

6. How much will it cost to excavate a ditch 240 ft. by 2 ft. by 5 ft. at \$1.20 per cubic yard? at \$1.50 per cubic yard?

7. How many cords of 4-foot wood (128 cu. ft.) are there in a pile 68 ft. long, 4 ft. wide, and 6 ft. high?

8. Suppose that your schoolroom is 36 ft. 8 in. long, 24 ft. wide, and 12 ft. high. What is the number of cubic feet of space in the room? the number of cubic yards?

9. A tunnel 940 ft. long has a cross-section area of 180 sq. ft. How much earth and rock were excavated?

10. How many cubic feet of storage space are there in a storeroom 40 ft. long, 32 ft. wide, and 10 ft. high?

11. In preparing a flower bed in a park it was necessary to fill in a space 80 ft. by 120 ft. to an average depth of $1\frac{1}{2}$ ft. At 75¢ a cubic yard, what was the cost of the earth used?

12. A box is 4 ft. long, 3 ft. wide, and $2\frac{1}{2}$ ft. deep. How many cubic feet does it contain?

13. If the box in Ex. 12 were half as long, but the width and depth were the same, what would be its volume?

14. If each measurement in Ex. 12 were doubled, what would be the volume of the box?

15. From Ex. 14 we may infer that doubling each edge of a rectangular box multiplies the volume by what number?

Copy and complete the following statements:

16. If we double the edge of a cube, we multiply the area of each face by ——, and we multiply the area of the entire surface of the cube by ——.

17. If we double the edge of a cube, we multiply the volume of the cube by ——.

18. Doubling each edge of a rectangular box multiplies the volume of the box by ——.

19. Doubling each edge of a rectangular block, such as a brick, multiplies the area of each face by ——.

20. Dividing each edge of a rectangular box by 2 divides the volume of the box by ——.

21. At $62\frac{1}{2}$ lb. per cubic foot, the weight of water that will just fill a cubic box 6 in. on an edge is —— lb.

22. If a block of stone 2 ft. long, 1 ft. wide, and 6 in. thick weighs 190 lb., a block of the same kind of stone 4 ft. long, 2 ft. wide, and 1 ft. thick weighs —— lb.

23. A block of granite 3 ft. long, 2 ft. wide, and $1\frac{1}{2}$ ft. thick is to be polished on all the faces except the one on which it stands. If the base is 3 ft. long and 2 ft. wide, the number of square feet to be polished is ——.

24. Find the number of cubic inches in a box 3 ft. 6 in. long, 18 in. wide, and 1 ft. deep.

25. The bottom of an ordinary farm-wagon box is usually 3 ft. by 10 ft., and the depth of the box varies from 24 in. to 26 in. Find the capacity in cubic feet for a depth of 24 in.; of 25 in.; of 26 in.

26. If a wagon box 3 ft. by 10 ft. by 24 in. will hold 50 bu. of shelled corn, how many bushels of shelled corn can be put into a wagon box 3 ft. by 10 ft. by 26 in.?

27. Some farmers estimate that $2\frac{2}{5}$ cu. ft. of corn in the ear will produce 1 bu. of shelled corn. How many bushels of shelled corn would there be in a crib 20 ft. long, 12 ft. wide, and 8 ft. high, filled with corn in the ear?

28. How many cubic yards of earth must be removed in excavating a tunnel that is 492 ft. long and has a rectangular cross section 39 ft. wide and 19 ft. 6 in. high?

29. An arithmetic class measured the excavation for the cellar of a new house near the school and found it to be 34 ft. 6 in. long, 24 ft. 6 in. wide, and 7 ft. deep. How many cubic yards of earth were taken out? How much did the excavation cost at $1.50 per cubic yard?

30. The ice chamber of a refrigerator is 16 in. by 24 in. by 18 in. Ice weighs 8% less than water, and 1 cu. ft. of water weighs $62\frac{1}{2}$ lb. If the ice comes in pieces weighing 10 lb., 25 lb., 50 lb., 100 lb., 150 lb., and 200 lb., which one or which two of these pieces is nearest to the amount of ice required to fill the chamber?

31. In building a section of a state highway 4 mi. long and 45 ft. wide, a layer of crushed rock was spread over the road to an average depth of 6 in. At $1.50 per cubic yard, what was the cost of the crushed rock?

REVIEW OF MEASUREMENTS

Copy the following tables, which show measurements of squares, and fill the blank spaces:

	Side	Perimeter	Area
1.	2 in.		
2.		20 in.	
3.			9 sq. in.
4.		60 in.	

	Side	Perimeter	Area
5.			16 sq. in.
6.		22 in.	
7.	2.4 in.		
8.		16.4 in.	

Copy the following table, which shows measurements of rectangles, and fill the blank columns:

	Length	Width	Perimeter	Area
9.	2 ft. 6 in.	1 ft. 8 in.		
10.	3 ft. 9 in.	2 ft. 4 in.		
11.	4 ft. 2 in.	3 ft. 10 in.		
12.	1 ft. 10 in.	18 in.		

Copy the following table, which shows measurements of rectangular boxes, and fill the blank column:

	Length	Width	Depth	Volume
13.	$2\frac{1}{2}$ ft.	2 ft.	1 ft.	
14.	3 ft.	$2\frac{1}{2}$ ft.	$2\frac{1}{2}$ ft.	
15.	$3\frac{1}{2}$ ft.	3 ft.	2 ft.	
16.	4 ft.	$3\frac{1}{2}$ ft.	$2\frac{1}{4}$ ft.	
17.	6 in.	$2\frac{1}{2}$ in.	$2\frac{1}{4}$ in.	
18.	$8\frac{1}{2}$ in.	4 in.	3 in.	
19.	$4\frac{1}{2}$ in.	$2\frac{1}{2}$ in.	$1\frac{1}{4}$ in.	

READING THE GAS METER

1. Here is a picture of three dials on a gas meter. If the left-hand dial indicates ten thousands, the middle dial thousands, and the right-hand dial hundreds, the hands on the dials show that 64,300 cu. ft. of gas has passed through the meter. While more than 64,300 cu. ft. of gas has been used, we shall consider only the hundred last passed by the hand, although some gas companies read meters only to the thousand last passed. Notice that the middle dial is read in the opposite direction from the others. Suppose that this is the meter reading for May 1, that the reading for April 1 was 62,300, and that the gas company charges its customers \$1.20 per M (that is, \$1.20 per 1000 cu. ft.) for gas. How much is the bill for the gas that was used during the month of April?

2. If the reading of this gas meter was 52,700 a month ago, how much is the bill for the month at \$1.10 per M?

3. If the reading in Ex. 2 was 51,500 a month ago, how much is the bill?

4. If 3700 cu. ft. of gas was used in January and 3300 cu. ft. in February, how much was the gas bill each of these months at \$1.35 per M?

5. If 4500 cu. ft. of gas was used in January, and 20% less in June, how much was the June gas bill at \$1.20 per M?

6. If Mrs. Jones were more careful to turn down the burners on her gas stove, she could reduce her gas bill about 8%. If she used 5600 cu. ft. of gas last month at \$1.10 per M, how much could she have saved?

7. Some gas companies allow a cash discount of 5% on bills that are paid before the 10th of a month. If the meter readings are 83,400 and 85,200 on the first days of two consecutive months, and the rate is $1.25 per M, how much will be saved by paying the bill before the 10th?

Copy the following table and fill in the blank spaces:

	Present Reading of Meter	Previous Reading of Meter	Cubic Feet Consumed	Rate per M before Discount	Amount of Bill Rendered	Cash Discount 5%	Net Amount Due
8.	32,400	31,600		$1.10			
9.	26,700	23,900		.90			
10.	48,700	42,600		1.20			
11.	38,500	33,900		1.00			
12.	67,100	63,800		1.40			
13.	72,300	68,400		.85			
14.	52,200	48,600		1.35			
15.	74,300	69,900		1.05			

16. In factories where large amounts of gas are used, or in places where there is natural gas, the meters are generally read only to the thousand last passed. In a factory where the meter readings are 671,000 today and 479,000 a month ago, how much is the gas bill at 90¢ per M?

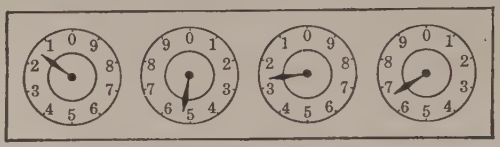
17. In a factory which used natural gas, the meter read 225,000 on March 1 and 412,000 on April 1. How much was the gas bill for March at 45¢ per M?

18. Our school gas meter read 685,000 today and 637,000 a month ago. How much is the bill at 55¢ per M?

19. If possible, read the gas meter in your school or home and find the cost of the gas used in a month.

READING THE ELECTRIC METER

1. Mr. Rogers lights his store by electricity. The meter is read in a way similar to that in which a gas meter is read, and shows thousands, hundreds, tens, and units of *kilowatt hours* (K. W. H.), which is the unit by which electricity is measured. Notice that the second and fourth dials from the left are read one way and that the first and third are read the other way. Read the meter.



2. If the reading in Ex. 1 was 1208 K. W. H. a month ago, how much electricity was used during the month?

3. If a meter reads 3704 K. W. H. on Jan. 1 and 3820 K. W. H. on Feb. 1, how much is the electric-light bill for January at $10\frac{1}{2}¢$ per K. W. H.?

4. Companies usually make lower rates to large users than to small users, and also make lower rates where the electricity is used for power or cooking instead of for lighting. Separate meters record the electricity used for the different purposes. If, in a month, a machine shop uses 375 K. W. H. at 8¢ per K. W. H. for lighting and 6285 K. W. H. at $2\frac{1}{2}¢$ for running the machines, how much is the bill for the month?

5. Mr. Brown used 22 K. W. H. at 12¢ for lighting his house last May and 240 K. W. H. at 3¢ for cooking. If he took advantage of a discount of 5% for payment of the bill within 10 da., how much did he pay?

6. Find the amount paid in settling a bill for 32 K. W. H. at $14\frac{1}{2}¢$ and 119 K. W. H. at $4\frac{1}{2}¢$, less a discount of 5%.

7. If possible, read the electric meter in the school or at home and compute the amount of the bill for a month.

Practical Equivalents. In solving problems, we often need to know the relations between certain measures. For example, if we know the number of cubic feet in a tank, we may wish to know how many gallons the tank will hold. The following tables of equivalents need not be memorized:

Exact Values	Approximate Values
2150.42 cu. in. = 1 bu.	$1\frac{1}{4}$ cu. ft. = 1 bu.
231 cu. in. = 1 gal.	$7\frac{1}{2}$ gal. = 1 cu. ft.
$31\frac{1}{2}$ gal. = 1 bbl.	$4\frac{1}{5}$ cu. ft. = 1 bbl.

The values on the left are exact equivalents, while those on the right are the approximations that are often used. Thus, while 1 bu. = 2150.42 cu in., the value frequently taken in practical cases is $1\frac{1}{4}$ cu. ft. or, less often, 2150 cu. in.

Since 1 bu. = $1\frac{1}{4}$ cu. ft., we can find the number of bushels in a bin by dividing the number of cubic feet by $1\frac{1}{4}$, or by $\frac{5}{4}$; still more conveniently, we may multiply by $\frac{4}{5}$. That is, expressing the measurements in feet, we have

$$\textit{Number of bushels} = \frac{4 \times \textit{length} \times \textit{width} \times \textit{height}}{5}.$$

For example, suppose that we wish to find the capacity in bushels of a bin which is 20 ft. long, 16 ft. wide, and 7 ft. deep.

Using the relation given above, and canceling, we find, as shown at the right, that the number of bushels is 1792; that is, the capacity of the bin is 1792 bu.

$$\frac{\overset{4}{\cancel{4}} \times \overset{\emptyset}{\cancel{20}} \times 16 \times 7}{\underset{5}{\cancel{5}}} = 1792$$

Similarly, to find the capacity in gallons of a rectangular tank which is 10 ft. × 8 ft. × 6 ft., we have

$$10 \times 8 \times 6 \times 7\tfrac{1}{2} \text{ gal.} = 3600 \text{ gal.}$$

Other Equivalents. The following relations pertaining to volumes need not be memorized:

1. A ton of hay varies in volume from about 450 cu. ft. to 550 cu. ft., depending on the quality and on how much the hay has settled. In our work 500 cu. ft. may be taken as the volume of a ton.

2. The volume of a ton of hard coal may be taken as 35 cu. ft. and that of a ton of soft coal as 42 cu. ft.

3. In brickwork, 22 bricks with the mortar may be considered to occupy a volume of 1 cu. ft.

The following examples show how we may apply the facts given in the above statements:

1. A coal bin 20 ft. long and 16 ft. wide is filled to a depth of 7 ft. with hard coal. How many tons of coal are there in the bin?

Referring to the second statement above, as we always do in such cases, we proceed as here shown and find that the number of tons is 64.

$$\frac{\overset{4}{\cancel{20}} \times 16 \times 7}{\underset{5}{\cancel{35}}} = 64$$

2. A mason is to build a wall 25 ft. long, 8 ft. high, and 1 ft. thick. How many thousand bricks will be needed?

Referring to the third statement, we find, as here shown, that 4.4 is the number of thousand bricks needed for the wall.

$$\frac{\underset{40}{\cancel{25}} \times 8 \times 1 \times 22}{\underset{5}{\cancel{1000}}} = \frac{22}{5} = 4.4$$

The mason would probably say that he needed $4\frac{1}{2}$ M to allow for breakage and because the estimate of 22 bricks to 1 cu. ft. is only approximate. Problems in real life do not often "come out even," and we should learn in school always to use common sense in interpreting a result.

PRACTICAL EQUIVALENTS

1. What is the capacity in bushels of a bin which is 8 ft. by 6 ft. by 4 ft.? which is 6 ft. by 9 ft. by 3 ft.?

2. Using the exact equivalent, what is the capacity in bushels of a box which is 18 in. by 3 ft. by 2 ft.?

3. Using the approximate equivalent, what is the capacity in gallons of a tank which is 5 ft. by 6 ft. by 4 ft.?

4. What is the capacity in gallons of a tank which is $3\frac{1}{2}$ ft. by $6\frac{1}{4}$ ft. by $2\frac{2}{3}$ ft.?

5. A cistern is 6 ft. square and $4\frac{2}{3}$ ft. deep. How many barrels of water will it hold?

6. An iron tank is $7\frac{1}{2}$ ft. long, $6\frac{3}{4}$ ft. wide, and $4\frac{2}{3}$ ft. deep. How many barrels of water will it hold?

Find the number of bushels of grain in a rectangular bin

7. 12 ft. by 16 ft., filled to a depth of 6 ft.

8. 14 ft. by 18 ft., filled to a depth of 9 ft.

9. 16 ft. by 24 ft., filled to a depth of 10 ft.

Find the number of bushels of grain in a freight car

10. $7\frac{1}{2}$ ft. by 28 ft., filled to a depth of 7 ft.

11. $7\frac{1}{2}$ ft. by 30 ft., filled to a depth of $5\frac{1}{2}$ ft.

12. A freight car 8 ft. by $26\frac{3}{4}$ ft. is filled with corn to a depth of 6 ft. At 75¢ per bushel, find the value of the corn.

13. How many tons of hay are there in a stack containing 4750 cu. ft.? in a mow containing 8250 cu. ft.?

14. Find the number of tons of hard coal that can be put into a bin which is 21 ft. long, and 15 ft. wide, and 6 ft. high.

15. A coal bin is 21 ft. long, 13 ft. wide, and 8 ft. deep. How many tons of soft coal will it hold?

16. A coal bin is 14 ft. long, $8\frac{1}{2}$ ft. wide, and $6\frac{1}{4}$ ft. deep. How many tons of soft coal will it hold?

17. A haymow is 22 ft. long, 14 ft. wide, and 9 ft. high. How many tons of hay will it hold?

18. A barn has two haymows, each 24 ft. long, 15 ft. 6 in. wide, and 10 ft. high. How many tons of hay will each mow hold? How many tons will the barn hold?

19. How many cords of 4-foot wood (128 cu. ft.) are there in a pile 4 ft. wide, 8 ft. high, and $32\frac{1}{2}$ ft. long?

20. How many cords of 4-foot wood are there in a pile 4 ft. wide, $6\frac{1}{2}$ ft. high, and 44 ft. long?

21. A schoolroom is 30 ft. long, 23 ft. wide, and 12 ft. high. If an average of 200 cu. ft. is allowed for each pupil, as is often done, what is the greatest number of pupils that should be assigned to this room?

22. A gasoline storage tank holds 82,500 gal. What is the capacity of the tank in cubic feet?

23. An aquarium contains 3.8 cu. ft. How many gallons of water will it hold when full?

24. A tank is 18 in. long, $9\frac{1}{4}$ in. wide, and $10\frac{1}{8}$ in. deep. How many gallons of water can be put into the tank?

25. The volume of a can is found to be 1732.5 cu. in. If the can is filled with milk, what is the value, at 12¢ a quart, of the contents of the can?

26. A wall is $32\frac{1}{2}$ ft. long, 2 ft. thick, and $32\frac{1}{2}$ ft. high. How many cubic feet of masonry does it contain?

27. A brick wall is to be built 42 ft. long, $1\frac{1}{2}$ ft. thick, and $6\frac{1}{4}$ ft. high. How many bricks will be needed?

28. If possible, measure your school lot and find the number of bricks needed for a 6-foot wall around it.

SHIPPING FRUIT

1. The standard box for shipping Florida oranges is 12 in. by 12 in. by 33 in. How many cubic inches does it contain?

2. In finding the volume of a New Jersey bushel cranberry box, which is $8\frac{3}{4}$ in. by 12 in. by 22 in., a pupil wrote:

$$8\tfrac{3}{4} \times 12 \times 22 = \frac{35 \times \overset{3}{\cancel{12}} \times 22}{\cancel{4}} = 2310 \text{ (cu. in.)}.$$

How does this box differ from the standard bushel?

3. In shipping Georgia peaches a crate 8 in. by $12\frac{1}{2}$ in. by 22 in. is used. This crate contains how much more than 1 bu.?

4. California apples and pears for local use are shipped in boxes which are 10 in. by 22 in. by 12 in. By how much does one of these boxes differ from the standard bushel?

5. In California the orange box is sometimes $11\frac{1}{4}$ in. by $26\frac{1}{2}$ in. by $11\frac{1}{4}$ in. and sometimes $7\frac{3}{4}$ in. by 22 in. by $17\frac{1}{2}$ in. Which box holds the more, and how much more does it hold?

6. Dried fruits are shipped in 50-pound or 25-pound boxes. The 50-pound box is 9 in. by $15\frac{1}{4}$ in. by 9 in. Find the volume.

7. The 25-pound box mentioned in Ex. 6 is sometimes $9\frac{3}{4}$ in. by $13\frac{3}{4}$ in. by $5\frac{3}{4}$ in., and sometimes 9 in. by 16 in. by 5 in. Which box holds the more, and how much more does it hold?

8. For shipping fruit out of the state the sizes of some of the boxes used in California are as follows:

Plums	$11\frac{3}{4}$ in. by $18\frac{1}{2}$ in. by 4 in.
Cherries	$10\frac{3}{8}$ in. by $16\frac{1}{2}$ in. by $2\frac{1}{2}$ in.
Pears	$11\frac{3}{4}$ in. by $18\frac{1}{2}$ in. by $8\frac{1}{2}$ in.
Peaches	$11\frac{3}{4}$ in. by $18\frac{1}{2}$ in. by $4\frac{7}{8}$ in.

Find the capacity in cubic inches of each of these boxes.

Other Problems Relating to Volumes. Suppose that a storage bin for potatoes is built into the ground. We can measure the length and width; but if the bin is full of potatoes, we cannot conveniently measure the depth. If we know the capacity of the bin, however, we can compute the depth.

That is, in such problems we are given the volume and certain measurements and have to find other measurements. Although not so common as those relating to the finding of volumes, such problems have many uses. The method of solving these problems is illustrated by the following cases:

1. If a box 5 ft. long and 4 ft. wide contains 60 cu. ft., how deep is it?

Since the box is 5 ft. long and 4 ft. wide, the area of the base is 5×4 sq. ft., or 20 sq. ft.

If the box were 1 ft. deep, the volume would be 20 cu. ft. Therefore the required depth must be as many times 1 ft. as 20 cu. ft. is contained in 60 cu. ft., or 3. That is, the box is 3 ft. deep.

$$\frac{\overset{\overset{3}{\cancel{12}}}{\cancel{60}}}{\cancel{5} \times \cancel{4}} = 3$$

2. If a block $2\frac{1}{2}$ in. high contains 70 cu. in., what is the area of the base?

If the block were 1 in. high, the base would contain 70 sq. in., and so the required area of the base is 70 sq. in. $\div 2\frac{1}{2}$; that is, as here shown, it is 28 sq. in.

$$70 \div 2\tfrac{1}{2} = \frac{\overset{14}{\cancel{70}} \times 2}{\cancel{5}} = 28$$

3. How thick is a block which has a volume of 100 cu. in. and a base with an area of 20 sq. in.?

If the block were 1 in. thick, it would contain 20 cu. in. Hence the required thickness is as many times 1 in. as 20 cu. in. is contained in 100 cu. in., or 5; that is, it is 5 in.

$$\begin{array}{r} 5 \\ 2\cancel{0}\overline{)10\cancel{0}} \end{array}$$

PROBLEMS RELATING TO VOLUMES

1. In planning to place a rectangular tank, which is to hold 10 bbl., in the attic of his house, a man estimates that the tank must contain 42 cu. ft. If he can make the tank only 3 ft. 6 in. high on account of the roof, what is the smallest area that the base of the tank will cover?

2. A rectangular tank with base 9 ft. by $16\frac{2}{3}$ ft. contains 900 cu. ft. How deep is the tank?

3. A box with base 13 in. by 20 in. contains 3900 cu. in. How deep is the box?

4. A rectangular tank 4 ft. deep contains 450 cu. ft. What is the area of the base of the tank?

5. A hall 22 ft. high and 30 ft. wide contains 28,600 cu. ft. Find the length of the hall.

6. The area of the base of a block containing 700 cu. ft. is 56 sq. ft. How high is the block?

7. A hall which contains 41,341.3 cu. ft. is 41.3 ft. long and 36.4 ft. wide. How high is the hall?

8. The area of the bottom of a box containing 42,159 cu. in. is 1081 sq. in. How deep is the box?

9. A block of granite is 14 in. long and 8.3 in. wide, and contains 836.64 cu. in. How thick is the block?

10. The area of the floor of an assembly hall which contains 15,246 cu. ft. is 1386 sq. ft. How high is the hall?

11. A storeroom 14.3 ft. long and 13 ft. 6 in. wide has a capacity of 2316.6 cu. ft. How high is the room?

12. The floor of a square bedroom contains 144 sq. ft., and the room contains 1368 cu. ft. Find the length, width, and height of the room, first finding the number which multiplied by itself gives 144.

USING WHAT YOU HAVE LEARNED

1. How many thousand bricks, each 4 in. by 8 in. by 2 in., can be packed in a kiln 25 ft. by 60 ft. by 9 ft.?

2. A rectangular pile of cut stones is 27 ft. long, 5 ft. wide, and 6 ft. high. Find the value of the pile at \$6.40 a cord, a cord of stone having the same volume (128 cu. ft.) as a cord of 4-foot wood.

3. Express in gallons the capacity of a tank which is 6 ft. long, 4 ft. wide, and 4 ft. deep.

4. If it takes 550 cu. ft. of clover hay to make a ton, how many tons of clover hay can be put into a mow 32 ft. long, half as wide as it is long, and half as deep as it is wide?

5. How many gallons of water are there in a watering trough that is 6 ft. long, 16 in. wide, and filled with water to a depth of three fourths of the width?

6. How many bars of soap, each 4 in. long, $2\frac{3}{4}$ in. wide, and $1\frac{1}{2}$ in. thick, can be packed in a box 16 in. long, $13\frac{3}{4}$ in. wide, and 1 ft. deep? Will the bars completely fill the box?

7. When Arthur visited his uncle, who owned a large farm, he told his uncle about the arithmetic he was studying. His uncle told him that in the barn there was an oat bin 8 ft. long and 5 ft. wide, and asked Arthur to find how many bushels of oats there were in the bin. Arthur took a stick and measured the depth of the oats, finding it to be 3 ft. 8 in. If he allowed $1\frac{1}{4}$ cu. ft. to the bushel, what answer should he get?

8. Express in bushels the capacity of a bin which is 4 ft. by 6 ft. by 3 ft. $6\frac{1}{2}$ in.

9. A bin 21 ft. 6 in. long and 10 ft. wide is filled with wheat to the depth of 5 ft. Allowing 2150 cu. in. to the bushel, what is the value of the wheat at \$1.20 a bushel?

Capacity and Weight. The laws of most states specify that a bushel of each of various grains and vegetables shall weigh a certain amount. The following table of typical weights need not be memorized, but should be used for reference:

Apples	48 lb.	Onions	57 lb.
Barley	48 lb.	Peas	60 lb.
Beans	60 lb.	Potatoes, sweet	54 lb.
Buckwheat	48 lb.	Potatoes, white	60 lb.
Corn, shelled	56 lb.	Rye	56 lb.
Oats	32 lb.	Wheat	60 lb.

CAPACITY AND WEIGHT

1. How many bushels of beans are there to a ton?

2. How many bushels of onions are there to a ton?

3. How many bushels are there in a ton of shelled corn?

Find the number of bushels to a ton of

4. Oats.
5. Barley.
6. Buckwheat.
7. Peas.
8. White potatoes.
9. Sweet potatoes.

10. A bushel of rye weighs how many times as much as a bushel of oats?

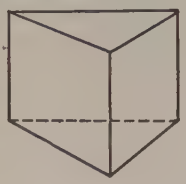
11. A bushel of sweet potatoes is what part as heavy as a bushel of white potatoes?

12. A bushel of buckwheat is what part as heavy as a bushel of shelled corn?

13. If a farmer sells 15 T. of white potatoes to a starch factory, how many bushels does he sell?

14. Find the weight in tons of 1000 bu. of wheat; 1000 bu. of oats; 1000 bu. of rye; 1000 bu. of shelled corn.

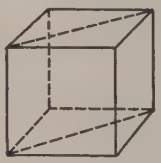
Prism. In the rectangular solids which we have been studying, the base was either a square or a rectangle. In the solid here shown the base and the face opposite and parallel to it are exactly alike, and the other faces are rectangles. A solid like this is called a *prism*, and this particular one is called a *triangular prism*, because the two parallel faces are triangles. The distance between these two parallel faces is called the *height*, or *altitude*, of the prism.



As long as they are exactly alike, the base and opposite face of a prism may have any number of sides greater than two. Thus, a rectangular solid, such as a crayon box, may be called a *rectangular prism*.

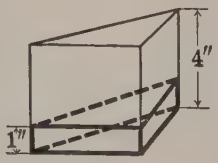
Volume of a Prism. Contractors, builders, and mechanics frequently have to find the volumes of prisms of various kinds.

In learning how to find the volume of a prism, it will help us first to consider the following questions:



If the kind of prism here shown, which, as we know, is a cube, is 4 in. on an edge, what is the area of the base?

The figure below represents a triangular prism which is half of the cube above. What is the area of the base of this half? What is the volume of the section, 1 in. high, which is indicated by the heavy lines? What is the volume of the entire prism?



We have thus found the volume of a triangular prism. The volume of any other kind of prism is found in the same way; that is, the volume of a prism is the product of the area of the base and the height, or, briefly stated,

$$Volume = area\ of\ base \times height.$$

VOLUME OF A PRISM

1. A part for a machine is cast in the form of a prism whose cross section is a trapezoid. The cross-section area is 26 sq. in., and the length of the part is 3′ 2″. Sketch the figure roughly and then find the volume of the part.

2. At 490 lb. per cubic foot, find the weight of the machine part described in Ex. 1.

Find the volume of a prism when the area of the base and the height are as follows:

3. 375 sq. in., 29 in.

4. 48 sq. in., 3 in.

5. 62 sq. ft., 3 ft.

6. $1\frac{1}{2}$ sq. ft., 4 in.

7. $64\frac{1}{2}$ sq. in., 8 in.

8. 62.8 sq. in., 3.1 in.

Find the height of a prism when the volume and the area of the base are as follows:

9. 243 cu. in., 29 sq. in.

10. 140 cu. in., $17\frac{1}{2}$ sq. in.

11. 783 cu. in., 87 sq. in.

12. $178\frac{1}{4}$ cu. in., $19\frac{1}{4}$ sq. in.

Find the volumes of rectangular prisms (rectangular solids) with dimensions as follows:

13. 42 in. long by $6\frac{1}{2}$ in. wide by $3\frac{1}{2}$ in. high.

14. $3\frac{1}{2}$ in. wide by $3\frac{3}{4}$ in. long by 8 in. high.

15. $6\frac{1}{2}$ ft. long by $4\frac{1}{2}$ ft. wide by 4 ft. high.

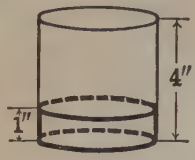
16. 21.3 in. wide by 29.2 in. long by 3.7 in. high.

17. 33.7 in. wide by 36.3 in. long by 12.4 in. high.

18. A rectangular solid with a base $12\frac{1}{2}$ in. square is twice as high as it is wide. Find its volume.

19. A prism has a square base, a volume of 1100 cu. in., and a height of 11 in. Find the lengths of the edges of the base.

Volume of a Cylinder. We all know what a *cylinder* is. An ordinary tin can is a good illustration. For example, suppose that we have a tomato can in which the area of the base is 12 sq. in. If, as here shown, we first consider a section 1 in. high, we have a cylinder 1 in. high with a volume of 12 cu. in. Then if the can is 4 in. high, its volume is 4×12 cu. in., or 48 cu. in. That is, just as in the case of the prism, the volume of a cylinder is the product of the area of the base and the height.



Since the common cylinder has a circular base, the area of the base is $\frac{22}{7}$ times the square of the radius. Therefore, in such a cylinder we have

$$\textit{Volume} = \tfrac{22}{7} \times \textit{square of the radius} \times \textit{height}.$$

VOLUME OF A CYLINDER

1. Find the volume of a cylinder whose base has an area of 600 sq. ft. and whose height is 3 ft. 6 in.

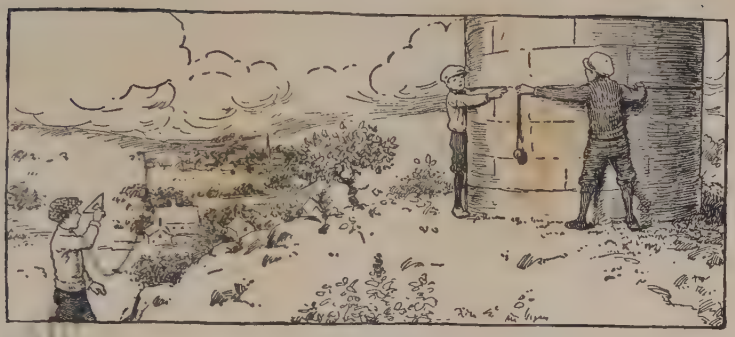
2. A cylindric water tank is 40 ft. high and 40 ft. in diameter. What is the radius of the tank (the radius of the base)? What is the capacity of the tank in cubic feet?

3. How many cubic feet are there in a cylindric stone shaft whose height is 16 ft. and whose diameter is 1 ft. 11 in.?

4. How many gallons (1 cu. ft. $= 7\frac{1}{2}$ gal.) does a cylindric tank hold if the height is 32 ft. and the diameter is 30 ft.?

5. Find the volume of a boiler 15 ft. long, with an inside diameter of 3 ft. 6 in.

6. Find the number of cubic inches in a water pipe of which the length is 18 ft., the outside diameter is 4 in., and the thickness of the metal is $\frac{1}{4}$ in.



7. The boys in the picture are measuring the circumference and the height of the water tank. They find them to be 44 ft. and 20 ft. respectively. Find the diameter of the tank in feet, the volume in cubic feet, and the capacity in gallons.

Find the volume of a cylinder when the area of the base and the height are respectively

8. 9 sq. in. and 4 in.

9. 8 sq. ft. and 9 ft.

10. 27 sq. in. and 13 in.

11. $6\frac{1}{2}$ sq. in. and 3 in.

12. 7 sq. in. and $2\frac{1}{2}$ in.

13. 88 sq. in. and 15 in.

14. $16\frac{1}{2}$ sq. in. and $3\frac{1}{2}$ in.

15. $18\frac{2}{3}$ sq. ft. and 27 in.

16. $48\frac{1}{2}$ sq. in. and 43 in.

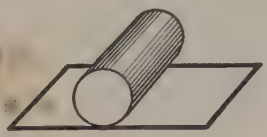
17. 62.2 sq. in. and 7.8 in.

18. Find the height of a cylinder when the volume and the area of the base are respectively $22\frac{1}{2}$ cu. ft. and $7\frac{1}{2}$ sq. ft.; when they are 197.6 cu. ft. and 30.4 sq. ft.

19. Find the area of the base of a cylinder when the volume and the height are respectively 324 cu. ft. and 18 ft.; when they are 33.15 cu. ft. and 2 ft. 2 in.

20. Find the volume of a cylinder when the height and the radius of the base are respectively 27 in. and 4 in.; when they are 32 in. and 8 in.; when they are 2 ft. 6 in. and 2 in.

Curve Surface of a Cylinder. If we should slit the curve surface of a cylinder and spread it out flat, what would the surface become? Try this yourself with a paper cylinder.



How can we find the area of this surface? What is the area of a rectangle 8 in. long and 6 in. wide? What is the area of the curve surface of a cylinder with a circumference of 8 in. and a height of 6 in.? We thus see that the area of the curve surface of a cylinder is the product of the circumference and the height.

Since the circumference of the common cylinder is $\frac{22}{7}$ times the diameter, in such a cylinder we have

Area of curve surface $= \frac{22}{7} \times$ *diameter* $\times$ *height.*

SURFACE OF A CYLINDER

Numbers 1 to 12, oral

State the areas of the curve surfaces of cylinders with heights and circumferences as follows:

1. 6 in., 10 in.	5. 40 in., 15 in.	9. 66 in., 48 in.
2. 60 in., 70 in.	6. 20 in., 20 in.	10. $12\frac{1}{2}$ in., 24 in.
3. 25 in., 40 in.	7. 25 in., 30 in.	11. $16\frac{2}{3}$ in., 36 in.
4. 50 in., 100 in.	8. 60 in., 50 in.	12. $33\frac{1}{3}$ in., 33 in.

13. How much sheet iron is needed for a stovepipe 8 ft. long and 6 in. in diameter, allowing 1 sq. ft. for overlapping?

14. Solve Ex. 13, supposing that the length is 6 ft. and the diameter is 8 in.

15. A tin cup is to be 3 in. in diameter and 3 in. high. Allowing 1 sq. in. for overlapping, how many square inches of tin are needed for the curve surface?

MEASURING CYLINDERS

1. Write a statement telling how you find the circumference and the area of a circle when you know the radius.

2. How many square inches are there in the curve surface of a wire 60 ft. long and $\frac{7}{8}$ in. in circumference?

3. If the height and diameter of a solid cylinder are each 8 in., what is the area of the entire surface; that is, what is the area of the curve surface plus the areas of the two circles which form the bases?

4. Allowing 7 sq. in. for waste, how many square inches of tin are needed to make a cylindric cup which shall be 4 in. high and 4 in. in diameter?

5. How many square feet are there in the curve surface of a water tank that is 40 ft. high and 127.6 ft. in circumference?

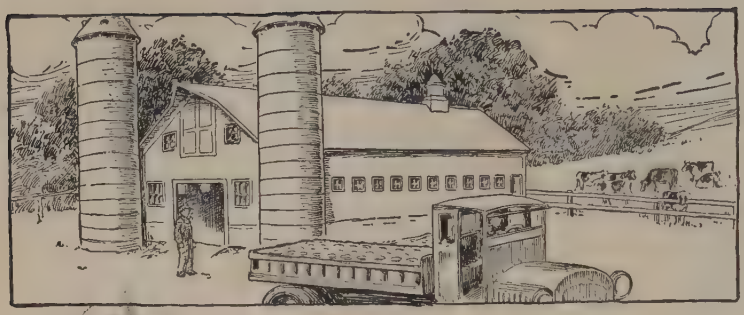
6. How large a surface is covered in painting a smokestack 30 ft. high and $2\frac{1}{2}$ ft. in diameter, outside measurements?

7. In polishing the curve surface of a granite cylindric shaft 3 ft. 6 in. in diameter and 22 ft. 3 in. long, how many square feet of surface are polished?

8. One of the rooms in a factory is heated by 210 ft. of steam pipe 2 in. in diameter. Find the radiating surface; that is, find the area of the curve surface which gives out the heat.

9. A suspension bridge has four cables, each 1872 ft. long and 1 ft. 2 in. in diameter. In order to bid on a contract for painting the bridge, a contractor had to know the surface area of these cables. Find this area.

10. In making metal bedsteads a factory uses iron rods 0.7 in. in diameter covered with thin rolled brass. In filling an order for 6000 ft. of these rods, how many square feet of rolled brass will be needed, not allowing for waste?



PROBLEMS ABOUT SILOS

1. A farmer finds that to feed a herd of ten cows for 6 mo. he needs a cylindric silo 10 ft. in diameter and 25 ft. high. How many cubic feet of silage does such a silo hold?

2. For a herd twice as large as in Ex. 1 a cylindric silo 12 ft. in diameter and 32 ft. high is needed. Has this twice the volume of the other silo? If not, how much does it lack?

3. What would the height of the first silo mentioned in Ex. 2 have to be in order to have twice the volume of the other? What effect would the packing down of the silage, due to the increased height, have upon the amount that could be put into the silo? Can you see why a capacity twice that in Ex. 1 is not needed?

4. The internal diameter of a silo is 12 ft. and the height is 25 ft. How many cubic feet does the silo hold?

5. The outside diameter of a silo is 14 ft. 7 in. Allowing 8 in. on each hoop for joining, how many feet of band iron are needed to make 12 hoops for the silo?

6. How many feet, board measure, of 1-inch lumber will it take to line the inner wall of a silo that is 21 ft. in diameter and 28 ft. high?

7. Ten years ago it cost about $3.10 per ton of capacity to build concrete silos that would hold about 75 T. How much did it cost then to build a concrete silo holding 71 T.? If the cost today has increased 80%, what is now the cost of building such a silo?

8. At $1.90 per square yard, how much will it cost to put a cement floor in a silo that is 21 ft. in diameter?

9. How many cubic feet of silage can be put into a silo that is 17 ft. 6 in. in diameter and 20 ft. high?

10. At 85¢ per square yard, how much will it cost to paint the curve surface of a silo that is 25 ft. high and 18 ft. 6 in. in diameter, outside measurements?

11. While corn silage weighs from 25 lb. to 50 lb. per cubic foot, depending upon the place in the silo from which it is taken and upon the amount of moisture it contains, a fair average is 40 lb. Allowing 40 lb. to the cubic foot, how many tons of silage can be put into a silo that is 18 ft. 8 in. in diameter and 32 ft. high?

12. Allowing 40 lb. daily per cow, how many tons of silage will be needed for a herd of 32 cows for 188 da.?

13. Find the total number of tons of silage that are needed, during a period of 180 da., in order to feed 32 cows 50 lb. daily per cow, 16 cows 40 lb. daily per cow, and 22 calves 16 lb. daily per calf.

14. A silo that is 40 ft. high and 10 ft. in diameter will hold approximately 70 T. of silage. How many tons can be put into a silo 40 ft. high and 20 ft. in diameter?

15. A silo 40 ft. high and 10 ft. in diameter will hold silage enough for 15 dairy cows for 240 da. How many dairy cows can be fed for 240 da. from a silo of the same height and of twice the diameter?

Pyramid. Another solid, about which it is interesting to know, has the shape shown in this figure. The base of this solid, which is called a *pyramid*, may have three or more sides, and the other faces are triangles which meet in a point. This point is called the *vertex* of the pyramid, and the distance from the vertex to the base is called the *height*, or *altitude*, of the pyramid.

Volume of a Pyramid. Suppose that we have a hollow prism and a hollow pyramid of the same base and the same height. If we fill the prism with water and then pour the water into the pyramid, we shall find that the pyramid can be exactly filled three times with the water that fills the prism, — a relationship that is suggested by the figure at the right. That is, from our study of the prism we see that the volume of a pyramid is one third the product of the area of the base and the height, or, briefly stated,

$$Volume = \tfrac{1}{3} \times area\ of\ base \times height.$$

VOLUME OF A PYRAMID

Find the volume of a pyramid when the area of the base and the height are respectively

1. 36 sq. in. and 7 in.
2. 187 sq. ft. and 10 ft.
3. 202 sq. ft. and 8 ft.
4. 89.6 sq. in. and 12.8 in.
5. 400 sq. in. and 30 in.
6. 129.3 sq. in. and 3.7 in.
7. 702.3 sq. in. and 16.8 in.
8. 140.4 sq. in. and 23.4 in.

9. The Great Pyramid in Egypt was originally 482 ft. high and had a square base 768 ft. on a side. What was the original volume? the weight in tons at 168 lb. per cubic foot?

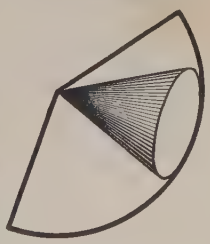
Lateral Area of a Pyramid. The problem of finding the volume of a pyramid is one that arises more often in practical work than that of finding the area of its surface. For our purposes we shall consider only the kind of pyramid in which the vertex is exactly over the center of an equal-sided and equal-angled base, the so-called *regular pyramid*, illustrated here and on page 256.

The altitude of any triangular face of this kind of pyramid is called the *slant height*, and the sum of the areas of the triangular faces is called the *lateral area* of the pyramid.

Since the area of each triangular face is half the product of its base and height, we see that the lateral area of the pyramid is half the product of the perimeter of the base and the slant height, or, briefly stated,

Lateral area $= \frac{1}{2} \times$ *perimeter of base* $\times$ *slant height.*

Cone. The solid called the *cone* is so familiar to every one of us as to need no definition. As in the case of the pyramid, we shall consider only the kind of cone illustrated here and on page 258.



If we slit the curve surface of such a cone and flatten it out, as here shown, it is evident that we have part of a circle. From our study of the pyramid, we may infer that the area of this curve surface is half the product of the circumference of the base and the slant height, or, briefly stated,

Area of curve surface $= \frac{1}{2} \times$ *circumference* $\times$ *slant height.*

Thus, in a cone in which the circumference of the base is 15 in. and the slant height is 4 in., we have

$\frac{1}{2} \times 15 \times 4$ sq. in. $= 30$ sq. in., the area of the curve surface.

Volume of a Cone. The cone has the same relation to the cylinder that the pyramid has to the prism (page 256); that is, the volume of a cone is one third the product of the area of the base and the height.

As in the case of the cylinder, we shall consider only the kind of cone which has a circular base. Then, since the area of this base is $\frac{22}{7}\times$ the square of the radius, in finding the volume of a cone we have

$$\textit{Volume} = \tfrac{1}{3} \times \tfrac{22}{7} \times \textit{square of the radius} \times \textit{height}.$$

Thus, for a cone whose height is 8 in. and whose base has a radius of 2 in., we have

$$\tfrac{1}{3} \times \tfrac{22}{7} \times 2 \times 2 \times 8 \text{ cu. in.} = 33\tfrac{11}{21} \text{ cu. in., the volume.}$$

Such a result is better given decimally as 33.5 cu. in.

PYRAMIDS AND CONES

In each case find the lateral area of a pyramid when the perimeter of the base and the slant height are as follows:

1. 24 in., 16 in. **2.** 4 ft. 3 in., 9 in. **3.** 4 ft. 9 in., 12 in.

In each case find the area of the curve surface of a cone when the circumference of the base and the slant height are as follows:

4. $15\frac{3}{4}$ in., $4\frac{3}{4}$ in. **5.** 7 ft. 6 in., 4 ft. **6.** 3 ft. 6 in., 15 in.

In each case find the volume of a cone when the radius of the base and the height are as follows:

7. 15.4 in., 6 in. **8.** 6.3 in., 15 in. **9.** 42 in., 18 in.

10. In the picture at the top of the page, suppose that the cylinder is 14 in. high and 14 in. in diameter. How many cubic inches are there inside the cylinder that are not occupied by the cone?

11. How much slating is used on a pyramidal steeple whose slant height is 40 ft. and the perimeter of whose base is 48 ft.?

12. A dome is surmounted by a gilded cone whose slant height is 16 ft. 8 in. and the radius of whose base is 3 ft. 6 in. How many square feet must be gilded?

13. A pyramid with a lateral area of 400 sq. in. has a slant height of 10 in. Find the perimeter of the base.

14. A conic spire on a church has a slant height of 68 ft., and the circumference of the base is 60 ft. What is the area of the curve surface?

15. What is the area of the entire surface of a cone whose slant height is 7 ft. and whose base is 7 ft. in diameter?

16. A circular cornucopia is $3\frac{1}{8}$ in. across and $8\frac{3}{4}$ in. deep. How many cubic inches does it contain?

17. A strainer in the shape of a cone is 5 in. across the top and 6 in. deep. Find the area of the curve surface.

18. What is the lateral area of a pyramid with a slant height of 10 in. and a square base 2 in. on a side?

19. What is the volume of a pyramid with a height of 12 in. and a square base 4 in. on a side?

20. The diameter of the base of a cone is 14 in. and the slant height is 20 in. Find the area of the curve surface.

21. Find the volume of a cone whose base has an area of 16 sq. in. and whose height is 8 in.

22. Find the volume of a cone whose base has a diameter of 28 in. and whose height is 12 in.

23. In Ex. 22 find the volume of a cone of twice the diameter.

24. The Great Pyramid originally had a slant height of 616 ft. and a square base which was 768 ft. on a side. Find the original lateral area.

Area and Volume of a Sphere. The last solid that we shall consider is the *sphere*. If we wind half the surface of a sphere with a cord, and then wind this length of cord around a cylinder whose height and diameter are each equal to the diameter of the sphere, we shall find that the cord covers half the curve surface of the cylinder.

As we already know, the area of the curve surface of the cylinder is $\frac{22}{7} \times$ diameter $\times$ height, and the height of the cylinder is twice the radius of the sphere. Hence the area of the surface of the sphere is $\frac{22}{7} \times 2 \times$ radius $\times 2 \times$ radius, or, writing "square of the radius" for "radius $\times$ radius,"

$$Area = 4 \times \tfrac{22}{7} \times \textit{square of the radius.}$$

For the volume of a sphere it is proved in geometry that

$$Volume = \tfrac{4}{3} \times \tfrac{22}{7} \times \textit{the cube of the radius,}$$

where "cube of the radius" means "radius $\times$ radius $\times$ radius."

Thus, in finding the area of the surface and the volume of a ball which has a radius of 4 in., we have

$$4 \times \tfrac{22}{7} \times 4 \times 4 \text{ sq. in.} = 201\tfrac{1}{7} \text{ sq. in., the area;}$$

and $\frac{4}{3} \times \frac{22}{7} \times 4 \times 4 \times 4$ cu. in. $= 268\frac{4}{21}$ cu. in., the volume.

That is, the area of the surface is about 201.1 sq. in., and the volume is about 268.2 cu. in.

AREAS AND VOLUMES OF SPHERES

1. If a sphere has a radius of 7 in., what is the area of the surface of the sphere?

2. Find the area of the surface of a hemispherical dome which has a diameter of 49 ft.

3. The sun's diameter is 866,500 mi. Find the area of the sun's surface to the nearest 1000 sq. mi.

4. Given that 1 cu. ft. of granite weighs 165 lb., find the weight of a sphere of granite 1 ft. in diameter.

5. A bowl is in the form of a hemisphere which is 4.2 in. in diameter. How many cubic inches does it contain?

6. Marble is 2.7 times as heavy as water, and 1 cu. ft. of water weighs 1000 oz. Find the weight of a sphere of marble 3 ft. in circumference.

7. Lead is 11.35 times as heavy as water, and 1 cu. ft. of water weighs 1000 oz. Find, to the nearest ounce, the weight of a lead ball $1\frac{1}{2}$ in. in diameter.

8. Taking the earth as an exact sphere with a radius of 4000 mi., find the volume to the nearest 1000 cu. mi.

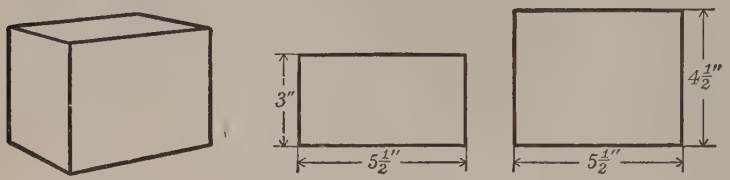
9. Considering the moon as a sphere 2160 mi. in diameter, find the area of its surface to the nearest 1000 sq. mi. and the volume to the nearest 1000 cu. mi.

10. A ball with a diameter of 6 in. is placed in a cubic box which is 6 in. on an edge, inside measurements. How many cubic inches inside the box are not occupied by the ball?

11. Find the volume of a sphere 1 ft. in diameter, of a cylinder of the same diameter and height, and of a cone of the same diameter and height. The volume of the sphere is how many times the volume of the cone? The volume of the cylinder is how many times the volume of the cone?

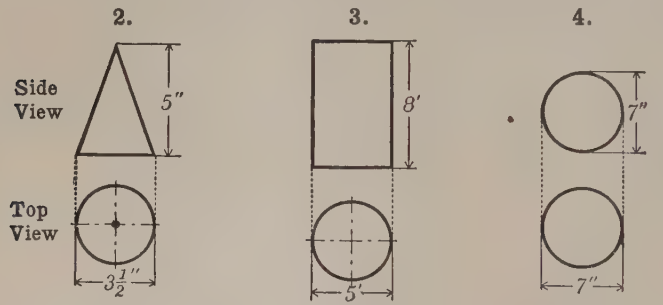
MEASURING SOLIDS FROM WORKING DRAWINGS

1. Below is shown the kind of drawing which a workman might use in proceeding to cut out a rectangular block of the size given. Find the volume of the solid represented.



The Block — Top View — Side View

Find the volume of each of the solids represented below:

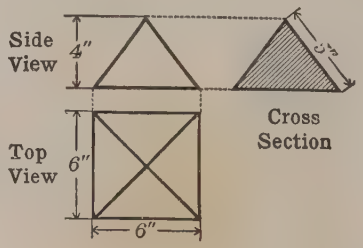


Cone — Cylinder — Sphere

5. This figure represents a pyramid with a square base. Find the area of the base.

6. In Ex. 5 find the volume of the pyramid.

7. In Ex. 5 find the area of each of the four triangular faces and the area of the entire surface of the pyramid.



REVIEW OF THE RULES OF MEASUREMENT

Copy and complete the following statements:

1. The fraction $\frac{22}{7}$ is the number by which we multiply the —— of a circle to obtain the ——.

2. The diameter of a circle is $\frac{7}{22}$ times the ——.

3. The area of a circle is —— times the square of the radius.

4. The area of a —— is half the base times the height.

5. The area of a rectangle is the —— times the ——.

6. The volume of a rectangular solid is the —— times the —— times the ——.

7. The volume of a prism is the —— of the —— times the ——.

8. The —— of a circular cylinder is $\frac{22}{7}$ times the —— of the —— times the height.

9. The area of the curve surface of a circular cylinder is $\frac{22}{7}$ times the —— times the height.

10. The —— of a pyramid or a cone is $\frac{1}{3}$ the —— of the —— times the ——.

11. The area of a sphere is 4 times $\frac{22}{7}$ times the —— of the ——.

12. The volume of a sphere is $\frac{4}{3}$ times —— times the —— of the radius.

13. If we multiply the diameter of a circle by $\frac{22}{7}$, the result is the ——.

14. If we multiply the square of the radius of a circle by ——, the result is the ——.

15. The volume of a cube whose edge is 7 in. is ——.

16. The volume of a sphere whose radius is 7 in. is ——.

REVIEW OF MEASUREMENTS

1. A steamfitter cuts off 68 in. of pipe. Express this as feet and inches; as feet and a fraction; as yards and inches.

2. A milk dealer sells 465 qt. of milk. Express this as gallons and quarts; as gallons and a fraction.

3. A milk dealer sells 85 pt. of cream. Express this as quarts and pints; as quarts and a fraction; as gallons and pints; as gallons and a fraction.

4. If you cut off $\frac{7}{8}$ of a piece of tin that contains 5 sq. ft., how many square inches do you cut off?

5. Counting 64 sheets to a pound, what is the weight of 5000 sheets of paper?

6. If evaporation from a tank is estimated at $8\frac{1}{3}\%$ in a week, how many gallons will be lost in a week from an open tank which contains 4800 gal.?

7. What is the reduction in length of a 240-yard roll of cloth that loses $2\frac{1}{2}\%$ of its length by shrinkage?

8. A sign in a loft says that the maximum weight should not exceed 60 lb. per square foot. If the loft floor is $48' \times 62'$, and if there are already $35\frac{1}{4}$ T. of merchandise on the floor, how much more weight may be safely added?

9. While sailing out to the fishing grounds, Albert heard the skipper say that the water at that point was 36 fathoms deep. Taking the fathom as 6 ft., how many feet is this? How many yards?

10. How many feet are there in 42 fathoms?

11. How many fathoms are there in 42 ft.?

12. Which of the years 1975, 1988, 2000, and 2004 will be leap years? Write the reason for each of your answers.

13. Find to the nearest 100 cu. yd. the amount of earth which must be removed in digging a canal $2\frac{3}{8}$ mi. long, 200 ft. wide, and 16 ft. deep.

14. Taking the weight of 1 cu. ft. of marble as 173 lb., find the weight of a cylindric marble column that is 12 ft. high and 14 in. in diameter.

15. A prism 12 in. high has a base whose area is 154 sq. in. How does the volume compare with that of a cone which is 12 in. high and whose base has a radius of 7 in.?

16. The amount of water that flows over Niagara Falls averages 264,000 cu. ft. per second. Find the average number of gallons carried over the falls per minute.

17. To the nearest 100 cu. yd., how many cubic yards of gravel are required to grade $2\frac{3}{4}$ mi. of road if the gravel is laid 6 in. deep over a width of 24 ft.?

18. How many boards, each 12 ft. long, are required to build a four-board fence about a rectangular field which is 40 rd. wide and contains 20 A.?

19. A steel rod 6 ft. 10 in. long has a triangular cross section whose area is 3 sq. in. Given that 1 cu. ft. of water weighs 62.5 lb., and that steel is 7.83 times as heavy as water, find to the nearest pound the weight of the rod.

20. Find the circumference of a circle whose diameter is 22.6 in. by multiplying the diameter by 3.1416; by $3\frac{1}{7}$; by $\frac{355}{113}$. Compare the results, stating which is the largest and how much larger it is than each of the others.

21. Find the diameter of a circle whose circumference is 471.24 in. by dividing the circumference by 3.1416; by $3\frac{1}{7}$; by $\frac{355}{113}$. Compare the results.

22. What effect does doubling both the diameter and the height have upon the volume of a cylinder?

A SILENT READING LESSON

In place of our common system of measures many foreign countries use what is known as the *metric system*. We often use this system in this country, as when we speak of 450-meter wave lengths in connection with radio.

The practical part of the system is very easily learned, and since it is used in all kinds of scientific work, everyone ought to know something about it.

The principal units of this system are the *meter*, which is equivalent to about $3\frac{1}{4}$ ft. or 1.1 yd.; the *kilogram*, which is equivalent to about 2.2 lb.; and the *liter*, which is equivalent to about 1 qt. In measuring long distances the unit used is the *kilometer*, which is 1000 meters and is equivalent to about 1100 yd., or nearly $\frac{5}{8}$ mi.

Just as 1 mill is $\frac{1}{1000}$ of a dollar,

so 1 millimeter is $\frac{1}{1000}$ of a meter,

and just as 1 cent is $\frac{1}{100}$ of a dollar,

so 1 centimeter is $\frac{1}{100}$ of a meter.

If we know these facts, we know about all of the metric system that we shall really need for our ordinary reading. Then, when we read about a 400-meter wave length, we shall think of a wave that is about 440 yd. long, and when we read about a 10-kilometer race, we shall think of it as a 6-mile race.

One great advantage of the metric system is that, like our system of money, it runs by 10's, 100's, 1000's, and so on.

Metric Length. The part of the table of metric length that we shall be most likely to need is as follows:

1 millimeter (mm.) = 0.001 meter (m.)
1 centimeter (cm.) = 0.01 meter
1 decimeter (dm.) = 0.1 meter
1 kilometer (km.) = 1000 meters

The meter is equivalent to 39.37 in., or about $3\frac{1}{4}$ ft., or 1.1 yd., and the kilometer is equivalent to about 0.6 mi.

MEASURING LENGTH

Taking 1 m. as 39.37 in., express as inches:

1. 47 m.	3. 52 m.	5. 324 cm.	7. 4680 mm.
2. 34.5 m.	4. 67.5 m.	6. 2.83 cm.	8. 3000 mm.

9. Express in inches the diameter of a 75-millimeter gun.

10. A hill is 203 m. high. Express this height in feet.

11. A tower is 37.5 m. high. Express this height in feet.

12. A wheel is 1.3 m. in diameter. Express this in inches.

13. The distance between two European cities is 78 km. Express this distance in miles.

14. When Calvin was in France, he took a trip of 120 km. on the river Seine. Express this distance in miles.

15. If it takes a European train 7 hr. to make a trip of 326 km., what is the average rate in miles per hour?

16. How many inches are there in 1 rd.? How many centimeters are there in 1 km.? It may take you 3 min. to find the first answer, but you can find the other one mentally, for you have simply to think of 100×1000.

17. How many thirty-seconds of an inch are there in a yard? A similar question in the metric system would ask for the number of millimeters in a meter. What is it?

Metric Square Measure. The table of square measure, in which the square meter (m.^{2}) is the unit, is as follows:

1 square millimeter (mm.^{2}) = 0.000001 square meter (m.^{2})
1 square centimeter (cm.^{2}) = 0.0001 square meter
1 square decimeter (dm.^{2}) = 0.01 square meter
1 square kilometer (km.^{2}) = 1,000,000 square meters

Metric Cubic Measure. The table of cubic measure, in which the unit is the cubic meter (m.^{3}), is as follows:

1 cubic millimeter (mm.^{3}) = 0.000000001 cubic meter (m.^{3})
1 cubic centimeter (cm.^{3}) = 0.000001 cubic meter
1 cubic decimeter (dm.^{3}) = 0.001 cubic meter

These tables need not be memorized as they are easily derived from the table of length. That is, 1 cm. = 0.01 m., and so $1\text{ cm.}^2 = 0.01 \times 0.01\text{ m.}^2 = 0.0001\text{ m.}^2$

We often find 1 cm.^{3} written as 1 cc. or 1 ccm.

METRIC SQUARE AND CUBIC MEASURES

Express the following as square centimeters:

1. 750 km.^{2} 2. 37 m.^{2} 3. 4296 mm.^{2} 4. 6.25 m.^{2}

Express the following as cubic meters:

5. 19,750,000 cm.^{3} 7. 7000 cm.^{3} 9. 937 cm.^{3}
6. 826,427,653.84 mm.^{3} 8. 3500 dm.^{3} 10. 486 dm.^{3}

11. How many square inches are there in 3 sq. ft.? How many square centimeters are there in 3 m.^{2}? Which is the easier to find?

12. How many cubic sixteenths of an inch are there in one cubic half inch? A similar question in the metric system would ask for the number of cubic millimeters in 1 cm.^{3} What is it? Which is the easier to find?

Metric Weight. The important part of the table of weight, in which the unit is the gram (g.), is as follows:

1 centigram (cg.) = 0.01 gram (g.)
1 decigram (dg.) = 0.1 gram
1 kilogram (kg.) = 1000 grams
1 metric ton (t.) = 1000 kilograms

A kilogram, commonly called a *kilo*, is approximately equivalent to 2.2 lb., and a metric ton is equivalent to about 2205 lb.

Metric Capacity. The important part of the table of capacity, in which the unit is the liter (l.), is as follows:

1 centiliter (cl.) = 0.01 liter (l.)
1 hectoliter (hl.) = 100 liters

The liter is the capacity of a cube which is 10 cm., or 0.1 m., on an edge (that is, the cube contains 1000 $cm.^3$, or 0.001 $m.^3$) and is practically equivalent to 1 qt.

METRIC WEIGHT AND CAPACITY

Express the following as kilograms:

1. 374 lb.	**4.** 352 oz.	**7.** 3270 g.	**10.** 7380 g.
2. 428 lb.	**5.** 48.4 lb.	**8.** 7290 g.	**11.** 4900 g.
3. 204 lb.	**6.** 300 T.	**9.** 173.8 lb.	**12.** 6350 g.

Express the following as pounds:

13. 2 kg.	**14.** 20 kg.	**15.** 300 kg.	**16.** 3000 kg.

Express the following as liters:

17. 7.5 hl.	**19.** 2.25 hl.	**21.** 2500 cl.	**23.** 15 hl.
18. 8 pt.	**20.** $12\frac{1}{2}$ gal.	**22.** 500 gal.	**24.** 6.83 hl.

25. A man weighs 82 kg. How many pounds is this?

26. A girl weighs 110 lb. How many kilos does she weigh?

USING THE METRIC SYSTEM

1. An importer wishes to order 1200 yd. of silk from a French silk manufacturer. How many meters should he order?

2. The distance from Paris to a certain other French city is 209 km. Express this distance in miles.

3. A cylindric gasoline tank on a foreign-made automobile is 80 cm. long and 35 cm. in diameter. Taking 1000 $cm.^3$, or 1 l., as equivalent to one quart, how many gallons of gasoline will the tank hold?

4. A cubic centimeter of water weighs a gram. What is the metric weight of a cubic meter of water? What is the weight in pounds?

5. In computing the duty on a shipment of 8 t. of merchandise from abroad, an importing house had to know the weight of the shipment in pounds. What was this weight?

6. Find the capacity in liters of a rectangular tank 130 cm. long, 85 cm. wide, and 95 cm. deep.

7. Using the facts given in Ex. 4, find the weight of 1 l. of water, and then find the weight of water needed to fill the tank in Ex. 6.

8. Taking 231 cu. in. to the gallon, find the capacity in gallons of a tank 51 in. long, 34 in. wide, and 37 in. deep, which is practically the same size as the tank in Ex. 6. Taking 8.3 lb. as the weight of 1 gal. of water, find the weight of water needed to fill the tank.

9. Compare the work in Exs. 6–8, and write a statement about which system you found the easiest to use.

10. In repairing a foreign-made automobile a mechanic needed some bolts that had 5 threads per centimeter of length. This was equivalent to about how many threads per inch?

IV. RATIO AND PROPORTION

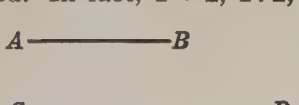
A SILENT READING LESSON

We often have occasion to compare the size of objects. For example, we frequently say that one building is twice as wide as another; that one field has half the area of another; that one hill is one and a half times as high as another; or that one tree is three times as large round as another, and so on.

In the same way we can compare numbers. For example, we know that 4 is twice as large as 2, half as large as 8, and two thirds as large as 6. Division is simply a comparison of numbers, for when we say that $12 \div 2 = 6$, we say in effect that 12 is six times as large as 2.

A fraction is an expressed division. For example, the fraction $\frac{2}{5}$ means that 2 is to be divided by 5, and the fraction $\frac{6}{3}$ means that 6 is to be divided by 3.

A fraction like $\frac{2}{5}$ or $\frac{6}{3}$ is also called a *ratio*, and we say that $\frac{2}{5}$ shows the ratio of 2 to 5 and that $\frac{6}{3}$ shows the ratio of 6 to 3. Sometimes we express the ratio of 2 to 5 by writing 2 : 5, which we read as "the ratio of 2 to 5," but the fractional form is more familiar and so is generally used. In fact, $1 \div 2$, 1 : 2, and $\frac{1}{2}$ are merely three different ways of indicating the division of 1 by 2.



We shall use ratios chiefly in comparing lines. Here are two lines, AB and CD. If we measure the lengths of the lines, we shall find that AB is $\frac{3}{4}$ in. long and CD is $1\frac{1}{4}$ in., or $\frac{5}{4}$ in., long. Hence to express the ratio of AB to CD, we have

$$\frac{3}{4} \div \frac{5}{4} = \frac{3}{4} \times \frac{4}{5} = \frac{3}{5};$$

that is, in simplest form the ratio of AB to CD is $\frac{3}{5}$. Similarly, if we wish to express the ratio of CD to AB, we have

$$\frac{5}{4} \div \frac{3}{4} = \frac{5}{4} \times \frac{4}{3} = \frac{5}{3}.$$

Terms of a Ratio. Just as in division we do not divide 5 ft. by 5 yd., so in expressing ratios, the two numbers, or *terms*, must be of the same kind.

For example, the ratio of \$2 to \$4 is $\frac{2}{4}$, or $\frac{1}{2}$, but the ratio of 10¢ to \$1 is $\frac{10}{100}$, or $\frac{1}{10}$.

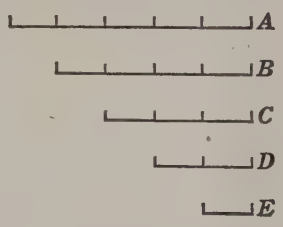
Similarly, the ratio of 6 ft. to 5 ft. is $\frac{6}{5}$, or $1\frac{1}{5}$, but the ratio of 6 yd. to 5 ft. is $\frac{18}{5}$, or $3\frac{3}{5}$.

In a ratio the first term is sometimes called the *antecedent*, and the second term, the *consequent*.

EXPRESSING RATIOS

All work oral

1. Looking at the lines below, state the ratio of E to D; of E to C; of E to B.



Referring to these lines, state the values of the following ratios:

2. E to A.
3. D to A.
4. $2E$ to D.
5. $2D$ to B.
6. D to C.
7. C to A.
8. $4E$ to A.
9. $5E$ to A.

10. What is the ratio of any number to itself? to half itself?

Referring to the above lines, state the values of these ratios:

11. A to $B + E$.
12. A to $B + C$.
13. B to $C + E$.
14. B to $A + E$.
15. A to E.
16. C to B.
17. $3E$ to C.
18. $2D$ to A.
19. B to $A + D + E$.
20. A to $B + C + D$.
21. $B + E$ to $C + D$.
22. $A + E$ to $B + D$.

23. What is the simplest fraction that expresses the ratio of 2 to 4? of 4 to 8? of 4 to 16? of 6 to 18? of 10 to 8? of 12 to 10? of 25 to 15?

Writing Ratios. If David is 4 ft. tall and his brother Will is 5 ft. tall, David is $\frac{4}{5}$ as tall as Will; that is, the ratio of David's height to Will's height is $\frac{4}{5}$.

The ratio of 4 ft. to 5 ft. may be written in any one of the following forms:

$\frac{4 \text{ ft.}}{5 \text{ ft.}}$ $\quad \frac{4}{5}$ $\quad 4 \text{ ft.} : 5 \text{ ft.}$ $\quad 4 : 5$ $\quad 4 \text{ ft.} \div 5 \text{ ft.}$ $\quad 4 \div 5$

We usually, however, write simply the fraction $\frac{4}{5}$, because this form is the one with which we are most familiar.

READING AND WRITING RATIOS

Numbers 1 to 23, oral

1. The ratio of 1 to 2 is $\frac{1}{2}$. What is the ratio of 3 to 4?

2. The ratio of 3 to 9 is $\frac{3}{9}$, but we express it as simply as possible and say that it is $\frac{1}{3}$. What is the ratio of 4 to 20?

3. The ratio of 24 to 4 is $\frac{24}{4}$, but we express it as 6. What is the ratio of 30 to 10?

State as simply as you can the following ratios:

4. 3 to 6.	**9.** 1 to 1.	**14.** 3 to 15.	**19.** 10 to 20.
5. 6 to 3.	**10.** 3 to 3.	**15.** 15 to 3.	**20.** 10 to 5.
6. 4 to 8.	**11.** 9 to 9.	**16.** 20 to 4.	**21.** 20 to 10.
7. 8 to 4.	**12.** 9 to 1.	**17.** 4 to 20.	**22.** 5 to 10.
8. 8 to 1.	**13.** 1 to 9.	**18.** 5 to 25.	**23.** 17 to 34.

24. Of the ratios 3 to 18, 5 to 30, and 8 to 42, which is the largest? Which two are equal?

25. Write two ratios, each equal to the ratio of 7 to 42; to the ratio of $3\frac{1}{2}$ to 21.

26. State which of the following ratios is equal to $\frac{5}{8}$:

$\frac{15}{24}$ $\quad \frac{25}{80}$ $\quad \frac{30}{48}$ $\quad \frac{49}{56}$ $\quad \frac{40}{62}$ $\quad \frac{42}{64}$ $\quad \frac{40}{64}$ $\quad \frac{45}{81}$

27. State which of these ratios is equal to the ratio of 7 to 9:

14 to 18 21 to 30 21 to 27 63 to 81

28. When divided by 2, what number becomes 3? What is the missing numerator in the statement that $\frac{\quad}{2} = 3$?

29. In $\frac{x}{5} = 2$, what number is represented by x?

30. If the ratio of x to 7 is 3, what is the value of x?

31. If Fred is half as old as his brother Jack, what is the ratio of Fred's age to Jack's? of Jack's age to Fred's?

Copy and complete the following statements:

32. 27 in. is —— times 9 in., and so the ratio of 27 to 9 is ——.

33. 12 in. is —— of 36 in., and so the ratio of 12 to 36 is ——.

34. 64 lb. is —— times as heavy as 4 lb. and —— as heavy as 128 lb.; and so the ratio of 64 lb. to 4 lb. is ——, and the ratio of 64 lb. to —— is $\frac{1}{2}$.

Express the following ratios as per cents:

35. 4 to 100. **36.** 4 to 16. **37.** 2 to 50. **38.** 75 to 200.

Find the number represented by x in each of the following cases:

39. $\frac{x}{2} = 5$. **40.** $\frac{x}{4} = 3$. **41.** $\frac{x}{4} = \frac{1}{2}$. **42.** $\frac{2}{x} = \frac{1}{3}$.

43. The ratio of the height of Rob's father to Rob's height is $\frac{3}{2}$. Rob's father is 6 ft. tall. How tall is Rob?

44. If, out of 12 games of basket ball, your team has won 6 and lost 4, and the other 2 were tied, what is the ratio of games won to games lost? of games won to games played? of games tied to games lost? of games tied to the sum of games won and lost?

PROBLEMS ABOUT MIXTURES

1. Suppose that we add a cup of pure vinegar to a cup of water. Find the ratio of the vinegar to the mixture, the phrase "ratio of the vinegar to the mixture" meaning, of course, the ratio with respect to the given unit, the cupful. What is the ratio of the water to the mixture? of the vinegar to the water?

2. If a farmer mixes 1 lb. of timothy seed with 1 lb. of redtop and $\frac{1}{2}$ lb. of clover, what is the ratio of the timothy to the mixture? The timothy is what per cent of the mixture?

3. In Ex. 2, what is the ratio of the clover to the mixture? The clover is what per cent of the mixture?

4. In Ex. 2, what is the ratio of the weight of the clover to the combined weight of the timothy and the redtop?

Copy and complete the following statements:

5. If we mix 1 lb. of clover and 3 lb. of timothy, the ratio of the clover to the mixture is ——.

6. In Ex. 5 we have 4 lb. of mixture. If we wish 32 lb., we should take —— lb. of clover and —— lb. of timothy.

7. In making molasses candy Nora uses $\frac{1}{2}$ cup of butter, 2 cups of sugar, 1 cup of molasses, and $1\frac{1}{2}$ cups of water. The ratio of butter to sugar is ——; of sugar to molasses is ——; of butter to water is ——. If Nora doubled the recipe, she would use —— cup of butter, —— cups of sugar, —— cups of molasses, and 3 cups of ——, and the three ratios would then be ——, ——, and ——.

8. In making fudge Emily uses 5 oz. of chocolate, $1\frac{1}{2}$ cups of milk, and 6 cups of sugar. If she doubles the recipe, she takes —— oz. of chocolate, —— cups of milk, and —— cups of sugar. In each case the ratio of milk to sugar is ——.

Separating a Number in a Given Ratio. Suppose that we wish to separate 25 marbles into two groups having the ratio 2 to 3.

Since one group has the value 2, and the other has the value 3, the whole has the value 5.

Then, since one group is $\frac{2}{5}$ of the whole, it contains $\frac{2}{5}$ of 25, or 10, and the other contains $\frac{3}{5}$ of 25, or 15.

Check. $10 + 15 = 25$, and $\frac{10}{15} = \frac{2}{3}$.

Take the sum of the terms of the ratio as the denominator of each of two fractions and the terms of the ratios as the numerators, and multiply by these fractions the number to be separated.

That is, applying this rule to the above case, we have

$$2 + 3 = 5; \quad \frac{2}{5} \text{ of } 25 = 10; \quad \frac{3}{5} \text{ of } 25 = 15.$$

SEPARATING IN A GIVEN RATIO

Numbers 1 to 6, oral

1. Separate 9 in the ratio 2 to 1; 1 to 1.

2. Separate 8 hr. in the ratio 1 to 3; 7 to 1; 1 to 1.

3. Separate 44 in the ratio 5 to 6; 1 to 10; 9 to 2.

4. The ratio of the length of summer to the length of the year is 1 to 4. How many months are there in the summer?

5. The ratio of clear to cloudy days one April was $\frac{1}{5}$. How many days were clear and how many were cloudy?

6. In playing 33 games of ringtoss the ratio of Will's score to Mollie's was $\frac{5}{6}$. How many games did each win?

7. If in November the ratio of clear to cloudy days was 3 : 2, how many days were there of each kind?

8. Four boys arrange a three-weeks' camping trip. The first boy stays 18 da.; the second comes late and stays 12 da.; the third comes for the last 2 wk.; the fourth stays 3 wk. If the total expense is $32.50, what is each boy's share?

Proportion. An expression of equality between two ratios is called a *proportion*. For example, we see that the ratio \$2 : \$3 is equal to the ratio 10 ft. : 15 ft., and so we may write

$$\$2 : \$3 = 10 \text{ ft.} : 15 \text{ ft.},$$

which is a proportion that is read "\$2 is to \$3 as 10 ft. is to 15 ft." It may be written also as

$$2 : 3 = 10 : 15, \quad \text{or as} \quad 2 \div 3 = 10 \div 15,$$

or, still better, as $\frac{2}{3} = \frac{10}{15}$.

The first and fourth terms of a proportion are called the *extremes*, and the second and third are called the *means*. For example, in the above case, 2 and 15 are the extremes, and 3 and 10 are the means. In any proportion

The product of the means equals the product of the extremes.

For example, in the above proportion we have

$$3 \times 10 = 30, \quad \text{and} \quad 2 \times 15 = 30.$$

Using Proportion. 1. What number bears the same ratio to 7 that 10 does to 35?

Writing x in place of the number to be found, we have

$$\frac{x}{7} = \frac{10}{35}.$$

Since the product of the extremes equals the product of the means, in this case we have

$$35 \times x = 7 \times 10,$$

or, what is the same thing, we may write

$$x = \frac{7 \times 10}{35} = 2.$$

Check. Writing 2 for x, $\frac{2}{7} = \frac{10}{35}$, and $2 \times 35 = 7 \times 10$; that is, since these two products are equal, our result is correct.

2. If a miller can make 4 bbl. of flour from 18 bu. of wheat, how many barrels of flour can he make from 207 bu.?

The amounts of flour produced depend upon (have the same ratio as) the amounts of wheat used; that is,

$$\frac{x \text{ bbl. of flour}}{4 \text{ bbl. of flour}} = \frac{207 \text{ bu. of wheat}}{18 \text{ bu. of wheat}},$$

or

$$\frac{x}{4} = \frac{207}{18}.$$

By what shall we multiply these equals in order to leave x by itself? Multiplying by this number, we find that

$$x = \frac{4 \times 207}{18} = 46, \text{ the number of barrels.}$$

USING PROPORTION

1. Suppose that a boy can ride 2 mi. in 20 min. on his bicycle. In finding how long it will take the boy to go $\frac{3}{4}$ mi. at the same rate, a pupil writes

$$\frac{x}{20} = \frac{\frac{3}{4}}{2}.$$

He then multiplies by 20, and finds that

$$x = \frac{20 \times \frac{3}{4}}{2} = (?).$$

2. At the rate of $55\frac{1}{2}$ mi. in 1 hr. 30 min., how long will it take a train to go 148 mi.?

3. If a man can skate 75 yd. in $8\frac{1}{5}$ sec., how long will it take him to skate 200 yd. at this rate?

4. If a man can skate 220 yd. in 19 sec., how long will it take him to skate 5 mi. if he can maintain this rate?

5. A map is drawn so that 1 in. represents 16 mi. on the surface of the earth. What length represents 40 mi.?

6. A map is drawn so that 1 in. represents 8 mi. How many square miles of land are represented by a portion of the map 3 in. square?

7. If a man can swim 45 yd. in 23 sec., how long will it take him to swim 2 mi. if he can maintain this rate?

8. If a farmer needs 26 lb. of seed for $\frac{1}{2}$ A., how many pounds does he need for 19.9 A.?

9. Find the circumference of a circle whose radius is to the radius of a circle 366.66 in. in circumference as 3.40 : 5.82; that is, find x in the proportion

$$\frac{x}{366.66} = \frac{3.40}{5.82}.$$

10. If a shipment weighing 750 lb. can be transported 100 mi. for a certain amount, would a heavier shipment be transported a shorter or a longer distance for the same amount? How far should a shipment weighing $\frac{1}{2}$ T. be transported for the same amount as 750 lb. for 100 mi.?

11. One man can do a piece of work in 10 da., and another can do the same work in 9 da. If the men are paid in proportion to the work they do, and if the wages of the first are \$5.40 a day, what are the wages of the second?

12. Walter is thinking of a number which has the same ratio to 17 that 35 has to 119. What is the number?

Find the number represented by x in each of these proportions:

13. $\frac{x}{51} = \frac{12}{17}$.

14. $\frac{x}{17} = \frac{54}{51}$.

15. $\frac{41}{11} = \frac{x}{22}$.

16. $\frac{x}{250} = \frac{35}{50}$.

17. $\frac{x}{18.2} = \frac{9.1}{7.3}$.

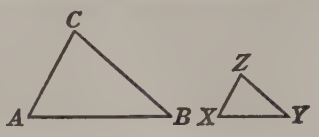
18. $\frac{82.7}{82} = \frac{x}{4.1}$.

19. $\frac{x}{21.7} = \frac{2.4}{3.3}$.

20. $\frac{x}{1.37} = \frac{2.4}{13.7}$.

21. $\frac{x}{0.5} = \frac{12.25}{1.75}$.

Similar Figures. Figures that have exactly the same shape, like the two triangles here shown, are called *similar figures.*



The corresponding sides of similar figures are proportional.

In such a statement we, of course, mean that it is the numbers which represent the lengths of the sides that are proportional; that is, the numbers form a proportion.

Thus, in the two similar triangles here shown, we can form proportions with the sides AB, BC, XY, and YZ; for example, we may write

$$\frac{AB}{BC}=\frac{XY}{YZ}, \quad \text{or} \quad \frac{AB}{XY}=\frac{BC}{YZ}.$$

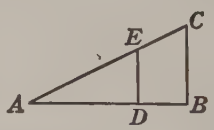
PROPORTION IN TRIANGLES

All work oral

1. In the two similar triangles shown above, if AB is twice XY, how does AC compare with XZ?

2. In the triangles shown above, if the ratio $AC:AB$ is equal to $\frac{3}{4}$, what is the value of the ratio $XZ:XY$? $XY:YZ$?

3. Using this figure, write a proportion involving AD, AB, AE, and AC. If $AD=\frac{2}{3}\,AB$, what is the ratio of AE to AC? of DE to BC? of $\frac{1}{3}\,BC$ to $2\,DE$?



4. In the figure of Ex. 3, if $AB=1\frac{3}{16}$ in., $AD=\frac{9}{16}$ in., and $AE=\frac{5}{8}$ in. $=\frac{10}{16}$ in., what is the length of AC?

5. In the figure of Ex. 3, suppose that BC represents the height of a man 6 ft. tall, DE the height of a boy, AB the man's shadow, and AD the boy's shadow. Show how you would proceed to find the height of the boy after you have measured the shadows.

Measuring Heights and Distances. The height of a tree, of a high building, or of a tower can be found by using proportion.

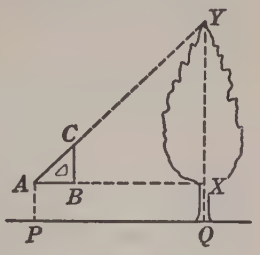
This figure shows how a man can measure the height of a tree by using a right triangle in which $AB = BC$. Holding the triangle so that AB is level, he finds a point P from which he can just see the top of the tree along AC when the triangle is at the height of his eye (the distance AP) above the ground. Then, because

$$AB = BC$$

and

$$\frac{AB}{BC} = \frac{AX}{XY},$$

the height of the tree above X is equal to the distance AX, or PQ, and the height QY is then $QX + XY$.



The figure is not drawn to scale, as the size of the triangle is exaggerated, and so is its height above the ground.

MEASURING HEIGHTS AND DISTANCES

1. What is the height of the tree in the above case, if the triangle ABC is held at a height of 5 ft. 4 in. above the ground and $PQ = 23$ ft. 8 in.?

2. In Ex. 1, suppose that the man uses a triangle in which $AB = 2\,BC$, that $PQ = 75$ ft., and that the point A is 5 ft. 8 in. above the ground. Find the height of the tree.

3. In Ex. 1, suppose that the man uses a triangle in which $AB = 8$ in. and $BC = 6$ in. If $PQ = 66$ ft. and $AP = 5$ ft. 6 in., what is the height of the tree?

Solve Ex. 3 when

4. $PQ = 40$ ft., $AP = 5$ ft.

5. $PQ = 42$ ft., $AP = 5$ ft.

6. $PQ = 45$ ft., $AP = 6$ ft.

7. $PQ = 39$ ft., $AP = 5$ ft. 4 in.

8. $PQ = 70$ ft., $AP = 5$ ft. 2 in.

9. $PQ = 61$ ft., $AP = 5$ ft. 3 in.

SHADOW RECKONING

1. The boys in the picture above find that the fence post is 3 ft. 2 in. high and casts a shadow 3 ft. 6 in. long. They find that the shadow of the flagpole is 50 ft. 8 in. long at the same time. What is the height of the flagpole?

2. If a tree casts a shadow 60 ft. 6 in. long when a post 3 ft. 3 in. high casts one 5 ft. 6 in. long, how high is the tree?

3. How high is a flagpole which casts a shadow 100 ft. long when a gatepost 9 ft. high casts a shadow 11 ft. 3 in. long?

4. How high is a building which casts a shadow 75 ft. long when a post 6 ft. 2 in. high casts a shadow 12 ft. 6 in. long?

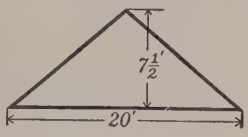
5. How high is a tree which casts a shadow 30 ft. long when a post 6 ft. 3 in. high casts a shadow 3 ft. 9 in. long?

6. Martin and George wished to find the height of the school building. Martin stood a yardstick upright and measured its shadow, which he found to be 4 ft. long. George measured the shadow of the building and found it to be 36 ft. long. How high was the building? Draw a figure to illustrate.

7. Using either the method given above or the one given on page 281, find the approximate height of your school building or of some object or building near the school.

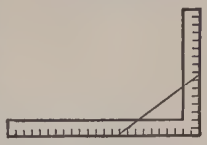
OTHER USES OF PROPORTION

1. To a scale of 8 in. to 20 ft., a pupil drew a plan for the gable end of a roof here shown. What length represented the $7\frac{1}{2}$-foot "rise" of the roof? By using proportion, how could the pupil find from the plan the real length of the "slope" of the roof?



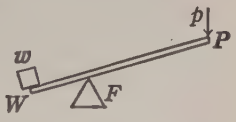
2. The teacher said that the plan in Ex. 1 was not as convenient as one showing only half the gable, with the 10-foot "run" represented by 10 in. Draw such a plan and measure the slope. How long is the slope of the roof?

3. In finding the slope in a case like Ex. 1 a carpenter would use his square. On the tongue, or short arm, he would take a point $7\frac{1}{2}$ in. from the corner; then on the blade, or long arm, he would take a point 10 in. from the corner. He would then take the number of inches between these points as the number of feet in the slope of the roof. Is this correct? Draw the figure and write the reasons involved.



4. In a lever supported at a fulcrum F, if sufficient power (p) is applied downward at P, a weight (w) can be lifted upward at W. In problems involving levers, we have the relation

$$\frac{p}{w} = \frac{FW}{FP}.$$



If $w = 90$ lb., $FW = 1\frac{1}{2}$ ft., and $FP = 4\frac{1}{2}$ ft., what power (p) is needed to lift the weight?

5. Paul and Carl just balance a seesaw. Paul, who weighs 100 lb., sits 4 ft. from the fulcrum which supports the plank. If the plank is 9 ft. long, how much does Carl, who sits at the other end, weigh?

Proportion in Business. While proportion is chiefly valuable in connection with measurements, it can also be used in solving business problems, although there are often easier ways. Simple ratio, however, is sometimes used, as in dividing a sum of money into two parts having a given ratio.

In solving by proportion a problem which asks, "If 3 books cost \$4.20, how much will 5 books cost?" we, of course, assume that the books are all sold at the same price, and write

$$\frac{x}{\$4.20} = \frac{5}{3}.$$

Then, as in the case on page 277, we may write

$$x = \frac{5 \times \$4.2\cancel{0}}{\cancel{3}} = \$7.00, \text{ the required cost.}$$

1.40

PROPORTION IN BUSINESS

1. If 7 lb. of meat cost \$2.24, how much will 5 lb. cost?
2. If 6 yd. of cloth cost \$8.28, how much will 7 yd. cost?
3. If 5 books cost \$8.25, how much will 9 books cost?

Copy this table and fill the blank spaces:

	Amount	Cost	Amount	Cost
4.	5 yd.	\$9	14 yd.	
5.		\$21	3 yd.	\$9
6.	9 books	\$18		\$10
7.	6 hats		7 hats	\$31.50
8.		42¢	17 pencils	\$1.02

9. Make up and solve three problems like Exs. 1–3.

V. REVIEW AND DRILL

MINIMUM ESSENTIALS

Add and also subtract as follows:

1.	2.	3.	4.
\$427.32	\$932.01	\$870.23	\$900.01
198.56	687.56	592.68	48.75

Multiply as follows:

5.	6.	7.	8.
\$423.78	\$573.81	\$609.98	23 ft. 8 in.
0.65	0.74	127	15

Divide, in turn, as follows:

9. 864 by 16, 32, and $\frac{4}{5}$.

10. 9.72 by 0.36 and 36.

11. 58 ft. 8 in. by 8 and 4.

12. 64 lb. 2 oz. by 9 and 12.

Express the following as decimals and also as per cents:

13. $\frac{1}{2}$. 14. $\frac{3}{4}$. 15. $\frac{5}{8}$. 16. $\frac{2}{3}$. 17. $\frac{5}{6}$. 18. $\frac{5}{12}$. 19. $\frac{11}{12}$.

20. At 35¢ per square foot, find the value of a rectangular building lot with a frontage of 83.2 ft. and a depth of 102.5 ft.

21. Find the volume of a rectangular stone block which is 28 in. long, 14 in. wide, and 18 in. high.

22. At 190 lb. per cubic foot, find the weight in Ex. 21.

23. Find the area of a circle with a radius of 4.9 in.

24. Find the area of the curve surface and the volume of a cylinder 18.9 in. high and 1 ft. in diameter.

Find the number represented by x in each of these proportions:

25. $\frac{x}{24} = \frac{13}{26}$.

26. $\frac{x}{57} = \frac{14}{42}$.

27. $\frac{x}{1.4} = \frac{3.5}{17.5}$.

28. $\frac{x}{3.2} = \frac{6.7}{53.6}$.

29. $\frac{14.5}{4.35} = \frac{x}{0.9}$.

30. $\frac{6.75}{1.35} = \frac{x}{25}$.

PROBLEMS WITHOUT NUMBERS

All work oral

1. If you know the number of cups of flour needed for a certain number of loaves of bread, how do you find the number of cups of flour needed if you wish to make a certain other number of loaves?

2. If you know how much water flows through a pipe in a minute, how do you find the amount of water that will flow through the pipe in a whole day, at the same rate?

3. If you know the capacity of a water tank in cubic feet, how do you find its capacity in gallons?

4. If you know what per cent of its original length an iron rod expands when heated from an ordinary temperature to red heat, how do you find the expansion in heating to red heat an iron rod which is a given number of inches long?

5. If you know the number of hours that a man works each day of the week, and the rate at which he is paid per hour, how do you find the amount of money which is due him at the end of the week?

6. At a given price per square yard, how do you find the cost of laying a concrete walk in front of the schoolhouse?

7. At a given price per cubic yard, how do you find the cost of making a rectangular excavation for the cellar of a house?

8. If you know how much a cubic foot of granite weighs, how do you find the weight of a granite cylinder when you know the radius and the height?

9. How can you find the height of your school building by means of shadows and the height of a post?

10. How can you find the height of a tree by sighting over the top of a stick of known length?

PROBLEMS FOR COMPLETION

1. A man bought 5 loads of coal which weighed 2475 lb., 2830 lb., 1970 lb., 2150 lb., and 1950 lb. The coal cost him \$11.45 a ton. Complete the problem in any reasonable way and then solve it.

Complete and solve the following problems:

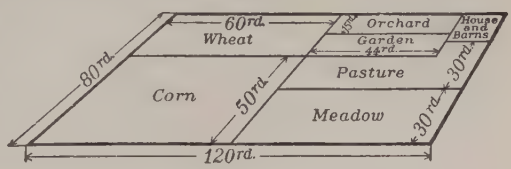
2. A grocer sold Mrs. Towne $4\frac{1}{4}$ lb. of butter @ 52¢, 3 bunches of celery @ 18¢, 12 lb. of beans @ 16¢, 8 cans of corn @ 18¢, and 2 bottles of olives @ 60¢.

3. An electric meter indicated 198 K. W. H. on May 1 and 251 K. W. H. on June 1. The company which supplies the electricity charges $11\frac{1}{2}$¢ per K. W. H.

4. On a bill of goods amounting to \$170, a dealer is allowed a discount of 10%, 8%.

5. A cow gave 675 lb. of milk in a month, and the milk tested 4.2% butter fat.

6. A rectangular farm 120 rd. long and 80 rd. wide is laid out as shown in this plan. Some of the dimensions are not given, but are easily found from the plan.



7. A cylindric water tank is 10 ft. in diameter and 8 ft. high. A cubic foot is equivalent to $7\frac{1}{2}$ gal.

8. A grain bin is 18 ft. long, 14 ft. wide, and 6 ft. deep. A bushel is approximately equivalent to $1\frac{1}{4}$ cu. ft., and the exact equivalent of the bushel is 2150.42 cu. in.

9. A city lot in the form of a rectangle is 22.5 ft. along the front, and the area of the lot is 1080 sq. ft.

FARM-CLUB PROBLEMS

1. In a calf-feeding contest held by a farm club, a boy fed a calf during a period of 120 da. The calf gained, on an average, $1\frac{3}{4}$ lb. per day. If the calf weighed 70 lb. when the contest began, how much did it weigh when the contest ended?

2. A boy bought a calf for \$35 and entered a calf-feeding contest. His expense account showed the following items:

600 lb. whole milk @ \$2.50 per hundredweight;
1200 lb. skim milk @ 50¢ per hundredweight;
2 bu. whole oats @ 65¢ per bushel;
50 lb. oil meal @ \$3 per hundredweight;
120 lb. clover hay @ \$1 per hundredweight.

If he sold the calf for \$72, how much did he receive, above these expenses, for his labor?

3. If a boy feeds some calves 150 lb. of hay at \$20 per ton, how much should he charge to his expense account?

4. A boy fed a calf for a certain period, during which the calf gained 325 lb. in weight. The expenses were as follows:

Whole milk	\$11.00	Hay	\$2.00
Skim milk	5.00	Oil meal	.50
Oats	1.65	Labor	6.50
Bran	1.20	Medicine	.75

Find the total cost per hundredweight of gain.

5. How long will $2\frac{1}{2}$ bu. of oats, weighing 32 lb. per bushel, last if a boy feeds 1 lb. twice each day to each of two calves?

6. In a certain year the corn clubs in this country reported that they raised 523,000 bu. of corn at an expense of \$143,000. If the corn was sold at an average price of 61¢ a bushel, how much, above expenses, was received?

7. Each of two boys in a farm club planted an acre of potatoes. One boy treated his seed potatoes chemically (so that they would grow better) at an expense of $1.10, but the other did not. The first boy raised 200 bu. of potatoes, which he sold at an average price of $1.48 per bushel. The other boy raised only 140 bu., which he had to sell at 98¢ per bushel, because they were of poorer quality. How much did the first boy gain by treating the seed potatoes?

8. A bushel of potatoes weighs 60 lb. If a boy carries 30 lb. and his sister carries 20 lb. on a trip from the field, how many bushels will they both carry together in 24 trips?

9. A boy raised 240 bu. of potatoes. He sold 35 bu. for seed potatoes at $2.25 a bushel, and of the remainder of the crop he sold 120 bu. at $1.35 a bushel and the rest at 90¢ a bushel. How much money did he receive?

10. A girl raised 10,800 lb. of potatoes, but the yield would have been increased one fifth by spraying. At 60 lb. to the bushel, how many bushels would she have gained by spraying?

11. A member of a farm club turned in the following account of his receipts and expenses in a recent year:

Receipts		Expenses	
Value of seed on hand	$4.00	Cost of seed	$1.80
Value of table stock	14.40	Chemical treatment	.20
Sales, selected	6.80	Rental of land	1.50
Sales, common	3.20	Spraying	.50
Prize won	2.00	Labor	12.80

Find the amount of the member's profit.

12. Allowing 60 lb. to the bushel, find the amount reported to a farm club as the receipts from the sale of 16,800 lb. of potatoes at $1.40 a bushel.

CANNING CLUBS

1. In a recent year the canning clubs reported that 7900 members canned, on an average, $25\frac{1}{2}$ qt. of food per member. Find the total number of quarts canned.

2. The clubs in Ex. 1 reported that the total cost of canning averaged 14¢ a quart. If the clubs sold the food at an average of 21¢ a quart, how much did they receive above expenses?

3. The canning club in a school near Dunkirk put up 3304 cans in one season. Sarah Kelley put up 104 of these cans. This was what per cent of the total number?

4. There are eleven girls in a canning club. Six of them put up 35 qt. each and the others put up 29 qt. each. How many quarts did the club put up?

5. A girl put up 4 qt. of fruit in pint cans each week day during the month of July, in a year in which that month had four Sundays. How many pint cans of fruit did she put up during the month?

6. A girl raised tomatoes, spending 60¢ for plants, 85¢ for fertilizer and spraying, $1.20 for lumber for trellises, and 5¢ apiece for 25 cans. She put up 25 quart cans, which she sold at 30¢ each. How much did she receive for her labor?

7. In canning peaches, a woman estimates that for each can the peaches cost 8¢, the sugar 3¢, the fuel 1¢, and the can itself 5¢. If she could buy peaches already put up at 25¢ a can, how much does she save on 50 cans by putting up the peaches herself?

8. Margaret raised some beans and sold 63 lb. at 5¢ a pound. The woman who bought the beans put them up in 30 one-quart cans. How much more would Margaret have received if she had canned the beans herself, at an expense of $8\frac{1}{2}$¢ per can, and sold them at 30¢ per can?

ECONOMY IN THE HOME

1. If each of the 114,000,000 persons in this country wastes on an average $1.10 worth of food per month, how much is wasted by all the population in a year?

2. If a housekeeper throws away, on an average, a slice of bread every day, and if there are 16 slices to a 10-cent loaf, how much money does she waste in a year (365 da.)?

3. If 10,000,000 housekeepers are as wasteful in this respect as the one referred to in Ex. 2, how much money do they all waste in this way in a year?

4. If the average waste of wheat flour in this country is 1 oz. per week for each of the 24,000,000 families, how many pounds does the country waste per year (52 wk.) in this way?

5. There are 196 lb. of flour in a barrel, and $4\frac{1}{2}$ bu. of wheat are required to make a barrel of flour. Find to the nearest hundred bushels the amount wasted annually in Ex. 4.

6. Allowing 16 bu. of wheat to the acre, how many acres of land would be required to supply the waste found in Ex. 5?

7. The average consumption of meat per person in this country was at one time 194 lb. per year. Some experts on diet recommend 4 oz. per day as sufficient for the average person. The average consumption for each person is how much more per year than is recommended?

8. If meat costs, on the average, 28¢ a pound, how much might be saved in a year by our population of 114,000,000 people if we brought the average consumption of meat down to 4 oz. per day for each person, as mentioned in Ex. 7?

9. If each of the 24,000,000 families in this country wastes, on an average, 1 oz. of meat per day, how many pounds are thus wasted in a year? How much is this worth at an average price of 28¢ per pound?

VI. LITTLE EXAMINATIONS

I. 1. $\$1850.75 + \$260.50$.
2. $\$1127.38 - \$999.90$.
3. $125 \times \$72.16$.
4. $\$867.75 \div 0.35$.
5. $37\frac{1}{2} \times \$630.40$.
6. $\frac{1}{2} + \frac{7}{8} - \frac{5}{16}$.
7. $1\frac{3}{4} + 2\frac{7}{16} - 3\frac{3}{8}$.
8. $\frac{3}{8} \times \frac{7}{16} \times 5\frac{1}{3}$.
9. $6\frac{3}{4} \div 3\frac{7}{8}$.
10. $3\frac{7}{8} \div 6\frac{3}{4}$.

II. 1. $0.37\frac{1}{2}\%$ of $560.
2. 125% of $200.
3. $66\frac{2}{3}\%$ of $978.
4. $4\frac{3}{4}\%$ of $1250.
5. $66\frac{2}{3}\% \div 8\frac{1}{3}\%$.
6. \$35 is 6% of \$ ——.
7. \$60 is $2\frac{1}{2}\%$ of \$ ——.
8. \$45 is —— % of \$135.
9. \$6.20 is ——% of \$49.60.
10. \$42.60 is —— % of \$284.

III. 1. Find the commission at $5\frac{1}{4}\%$ on a sale totaling \$5732.
2. \$1491 is what per cent greater than \$1050?
3. At 15% off, find the sale price of a suit marked \$65.
4. At 37¢ per \$100, find the cost of a policy for \$7900.
5. Find the tax at \$2.84 per \$100 on a valuation of \$6850.

IV. 1. Find the area of a rectangle 16.5 in. by 28.4 in.
2. Find the volume of a box 18 in. by 22 in. by 12 in.
3. Find the capacity in bushels of a bin holding 250 cu. ft.
4. Find the area of a circle with a radius of $5\frac{1}{4}$ in.
5. How do you find the volume of a cylinder?

V. 1. Separate 625 into two parts having the ratio 2 to 3.
2. Separate 1728 into two parts having the ratio 7 to 5.

Find the number represented by x in these proportions:

3. $\frac{x}{18} = \frac{7}{9}$.
4. $\frac{x}{15} = \frac{7}{3}$.
5. $\frac{x}{2\frac{1}{2}} = \frac{\frac{3}{4}}{\frac{2}{3}}$.

CHAPTER IV

I. BANKING PRACTICE

A SILENT READING LESSON

Last year we learned that banks exist chiefly to keep money safely and to lend the money needed to carry on business and industry. We learned about *savings banks* and *commercial banks,* the latter including *trust companies* which also manage estates and trust funds, and we learned that all these banks are closely supervised by *bank examiners* appointed by the state or national government.

We also learned about *bank books, deposits,* and *checks,* and about the *monthly statements* that the bank sends to a depositor so that he can compare his own record of his account with that kept by the bank.

If we should visit a bank it would give us a better understanding of modern banking practice. We should then learn the difference between a *savings account* and a *checking account,* and why the former pays a higher rate of interest. We should also see why it is unlawful for anyone to write a check on a bank where he has no account, or why a bank regards as a poor business man one who writes a check for a larger amount than he has on deposit.

While at the bank we can also learn about *certificates of deposit,* which are used when money is to remain on deposit for some time, and about the *safe deposit boxes* that people rent for storing their valuables. All these things would make an interesting subject for class discussion.

Bank Forms. The most important things about banks that the average citizen needs to know are how to make deposits and how to draw checks.

The first form here given shows the *deposit slip* that is filled out when making a deposit in order to show the bank to whose account the amount deposited is to be credited.

THE SECURITY BANK
NEW YORK, N. Y.

Deposited by

Name John T. Adams

Date July 7, 1928

COIN	75	25
BILLS	150	00
CHECK ON First National BANK	37	50
Farmers' "	8	75
West Coast "	11	68
"	283	18
"		

The second form shows a *check* such as Mr. M. R. St. John will give to Mr. A. P. Jones if he wishes to pay Mr. Jones $4.68. In this case Mr. Jones is the *payee* and Mr. St. John is the *maker* of the check.

In cashing or depositing the check Mr. Jones will have to *indorse* it by signing his name across the back of the check.

GRANT NATIONAL BANK

MOBILE, ALA., Feb. 2, 1930

Pay to the order of A. P. Jones $4.68

Four and 68/100 Dollars

No. 136

M. R. St. John

As we learned last year, in order to remove any doubt as to whether or not a man has sufficient funds on deposit to pay a check, the maker may have it *certified* by taking it to his bank and having it stamped across the face to indicate that the bank guarantees its payment.

Promissory Note. Another thing that we learned last year was that when a bank lends money it requires the borrower to sign a *promissory note* in some such form as the following:

$1000.00 LOUISVILLE, KY., May 15, 1929

Sixty days after date I promise to pay to the order of The Exchange National Bank One thousand no/100 Dollars, value received.

No. 547 Due July 14, 1929 William F. Taylor

In order to be sure that the note will be paid, the bank may require the borrower to "put up *collateral*"; that is, to leave with the bank some property of value, such as stocks or bonds, that can be sold by the bank in case the loan is not repaid. This collateral is, of course, returned to the borrower if the note is paid when due.

Bank Discount. As we also learned last year, banks frequently lend money on notes like the one above, and deduct the interest, called *discount*, in advance. The balance, called the *proceeds*, is then paid to the borrower.

In case Mr. Taylor wishes to pay off part of the note (that is, to make a *partial payment*) before it is due, the bank will *rebate* (pay back to him) a proportionate part of the discount that he paid, and will indorse on the back of the note the amount paid off. Since some banks, in such cases, cancel the old note and issue a new note for the balance of the loan, the customs of local banks should be ascertained.

On interest-bearing notes, the general rule is first to apply the payment to the interest due to date and then to apply the balance to reducing the principal.

REVIEW DRILL CHART IN BANK DISCOUNT

Find as rapidly as you can the discount at 6% when the face and the period of discount are as follows:

1. $1500, 30 da.
2. $3600, 36 da.
3. $1960, 28 da.
4. $3750, 72 da.
5. $1690, 18 da.
6. $6300, 24 da.
7. $7100, 48 da.
8. $2950, 54 da.
9. $4200, 12 da.
10. $3700, 30 da.
11. $6200, 42 da.
12. $3200, 24 da.
13. $2750, 18 da.
14. $1590, 42 da.
15. $1850, 72 da.
16. $5100, 24 da.
17. $2300, 30 da.
18. $3000, 18 da.
19. $1890, 13 da.
20. $7550, 30 da.
21. $3650, 12 da.
22. $5700, 48 da.
23. $8375, 72 da.
24. $6450, 24 da.
25. $4500, 54 da.
26. $7200, 18 da.
27. $3680, 42 da.
28. $1570, 38 da.
29. $3500, 12 da.
30. $2500, 30 da.
31. $5800, 15 da.
32. $9400, 36 da.
33. $2000, 60 da.
34. $6000, 15 da.
35. $470, 48 da.
36. $450, 30 da.
37. $870, 20 da.
38. $900, 40 da.
39. $820, 90 da.
40. $550, 15 da.
41. $620, 60 da.
42. $150, 72 da.
43. $300, 90 da.
44. $650, 60 da.
45. $190, 30 da.
46. $840, 10 da.
47. $240, 15 da.
48. $850, 60 da.
49. $700, 80 da.
50. $350, 120 da.
51. $730, 90 da.
52. $320, 45 da.
53. $480, 45 da.
54. $530, 60 da.
55. $530, 75 da.
56. $560, 15 da.
57. $625, 30 da.
58. $750, 90 da.
59. $825, 60 da.
60. $850, 30 da.
61. $800, 15 da.
62. $900, 75 da.
63. $650, 90 da.
64. $140, 90 da.
65. $740, 48 da.
66. $480, 36 da.

REVIEW OF BANK FORMS

1. Suppose that you have an account at the People's Bank in your city or town. Write the deposit slip which you would fill out when making a deposit consisting of $48.50 in cash and checks for $17.25, $6.50, and $10.63.

2. As in Ex. 1, make out a deposit slip for a deposit consisting of $68.73 in cash, a check on the Reed National Bank in payment of a bill for $56.78 from which 2% was deducted for prompt payment, a check on a Detroit bank in payment of a bill for $425.80 less 30%, and a check on the State Trust Co. in payment of a bill for $274.65 less 3%.

3. If Mr. Clark's bank balance is $273.50, and he draws checks for $17.68 and $29.75, how much money does his check book then show that he has in the bank?

4. Write a check, dated today, for $2.50 on the Corner Trust Company, Richmond, Va., payable to the order of John Doe, and sign it with your name. Then indorse the check as John Doe should indorse it in order to cash it.

5. Suppose that Mr. Rogers has $247.50 in the bank on the first day of the month and that during the month he deposits $48.75, $187.60, $268.90, $232.75, $14.68, $142.30, $72.70, and $68.65. If during the month he draws checks for $192.68, $95.43, $110.70, $182.50, $73.20, $15.40, $7.26, $6.93, $3.27, and $28.75, what is his balance on the last day of the month?

6. Suppose that you pay by check the following bills for goods purchased and are allowed the discount specified: $427.60, less 2%; $106.80, less 20%, 2%; $649.50, less 10%, 5%, 2%; and $327.75, less 15%, 5%, 5%. If your bank balance was $1878.93 before you wrote the checks for the bills, how much money does your check book show that you now have in the bank?

7. If Richard Roe borrows $175 at his bank for 30 da. at 6%, how much discount (interest) does he pay?

8. In Ex. 7, if Mr. Roe borrows twice as much for twice as long, how much discount does he pay?

9. Write a promissory note for the amount borrowed in Ex. 7, making it payable to the order of the bank (inserting the name of some bank that you know). The note should not specify any interest, since the discount was paid at the time the loan was made.

10. Mr. Sinclair borrows $750 from his brother and gives a note for 6 mo., bearing interest at 6%. If he pays the principal and interest by check on the date the note is due, what is the amount of the check which he draws?

11. On an invoice amounting to $1782.50, for goods for the holiday trade, a merchant is allowed a discount of 3% for prompt payment. In order to take advantage of the reduction, he will have to discount a 60-day note for $1750 at his bank. If the discount rate is 6%, how much does he save by borrowing the money and paying the invoice promptly?

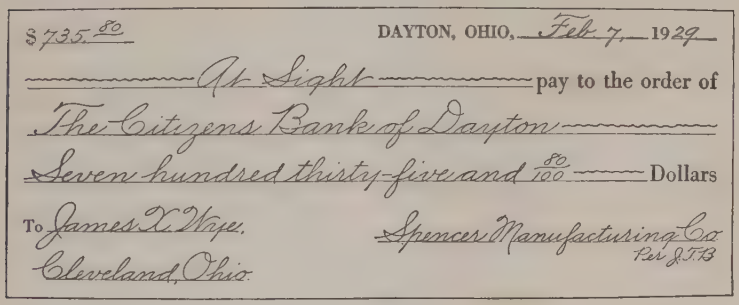
12. A man can buy an automobile for $2275 in cash, or by paying $1000 down and dividing the balance into three notes for equal amounts, due in 30 da., 60 da., and 90 da. respectively, with interest at 6%. If he takes the second plan, how much does he pay for the car? How much could he have saved by paying cash?

13. An automobile dealer borrowed $20,000 on a 90-day note, which his bank discounted at 6%. If at the end of 30 da. he paid off $5000 on the note, how much discount was rebated? If at the end of 60 da. he pays off $8000 more, how much discount is rebated? What is the balance due on the original note at the end of the 90 da.?

A SILENT READING LESSON

Besides rendering the services about which we have already learned, modern banks act as agents for their depositors in collecting accounts, in transmitting funds from one place to another, and in many other ways.

For example, suppose that Mr. James X. Wye, a hardware dealer in Cleveland, Ohio, owes money to the Spencer Manufacturing Co. of Dayton, Ohio, and that the company is unable to collect the amount due. The company may "draw" on Mr. Wye by means of the following *sight draft*, which is so called because payment is demanded upon its presentation:


$735.80 DAYTON, OHIO, Feb 7, 1929

At Sight pay to the order of

The Citizens Bank of Dayton

Seven hundred thirty-five and 80/100 Dollars

To James X. Wye, Cleveland, Ohio

Spencer Manufacturing Co
Pres. J.T.S.

The Spencer Manufacturing Co. (the *drawer*) deposits this draft with its bank, the Citizens Bank (the *payee*) in Dayton. The bank then forwards it to some bank in Cleveland, which presents the draft to Mr. Wye (the *drawee*) and asks him if he desires to "honor" it. If so, Mr. Wye pays it, and the Cleveland bank then notifies the Dayton bank that the draft is paid. The Citizens Bank then places the amount to the credit of the company's account at the bank and notifies the Spencer Manufacturing Co. that it has done so.

Banks usually charge a small commission for collecting drafts, which is deducted from the amount collected.

A SILENT READING LESSON (CONTINUED)

In some kinds of business another sort of draft, called a *trade acceptance*, is sometimes used in making payments for goods. The following form shows a common type of such a draft:

TRADE ACCEPTANCE

No. 612 Atlanta, Ga. Jan. 6, 1929

Sixty days after date pay to the order of ourselves

Eight hundred seventy-six and 25/100 Dollars ($876.25)

The obligation of the acceptor hereof arises out of the purchase of goods from the drawer.

To Towne & Dole Co.
671 Western Avenue,
Jacksonville, Fla.

ACCEPTED
DATE Jan. 9, 1929
PAYABLE AT Merchants Bank
OF Jacksonville, Fla.
SIGNED Towne & Dole Co.
Per J. B. Dole

Brown & Thurlow Co.
Per E. L. Brown

In the above case we may suppose that the Brown & Thurlow Co. (the *drawer*) sends this draft with the invoice covering a shipment of goods. Upon receipt, it is at once accepted by the Towne & Dole Co. (the *drawee*); that is, as shown in the model form, the drawee signs the acceptance across the face. The acceptance, which is now returned to the drawer, has practically the standing of a promissory note that does not bear interest, and may be discounted at a bank. The drawer, if he wishes, can thus get his money immediately, less the discount, even though the goods were sold on credit for a period of 30 da. or 60 da.

Trade acceptances have an advantage over the method of letting accounts run for a long time. By using acceptances a manufacturer, for example, does not have to carry on his books a large number of accounts with individual customers, but may transfer all these details to his bank, which collects the acceptances for him when they are due.

SIGHT DRAFTS AND ACCEPTANCES

1. The Westinghouse Linen Co. of Portland, Ore., draws a sight draft on R. T. Dow of Oregon City for $150. If the bank charges 0.2% for collection, what are the proceeds? Write the draft, dating it today and making it payable to some bank, as in the model form on page 299.

2. Robertson Bros. draw on J. P. Shipley at sight for $37.50. The bank charges 10¢ for collection. The charge for collection is what per cent of the face of the draft?

3. A. F. Reed owes M. S. Coe for 2 doz. suits @ $271.50, $1\frac{1}{2}$ doz. suits @ $234, and $7\frac{1}{2}$ doz. pairs trousers @ $40. Coe draws on Reed for the money, and the bank charges him $1.25 for collecting. What rate is this?

4. Suppose that John Bent, of Philadelphia, Pa., owes you $250, and that you draw on him today for this amount. Write the draft, payable to the order of some bank near your home. If the bank charges you $\frac{1}{10}\%$ for collecting the money, what are the proceeds?

5. The terms on which a miller in St. Paul sells an order of flour amounting to $500 are "30 da. net, covered by trade acceptance." The miller sends the acceptance (which the purchaser has signed and returned) to his bank 20 da. before it is due and has it discounted at $\frac{1}{2}\%$ per month for this period. What are the proceeds of the draft; that is, how much does the miller's bank place to his credit?

6. A 30-day trade acceptance for $975, which was signed by the drawee on Aug. 10, is discounted by the drawer on Aug. 16 at 6%. What is the period of discount; that is, what is the result of subtracting the number of days from Aug. 10 to Aug. 16 from the number of days that the acceptance runs? What are the proceeds?

REVIEW OF COMMERCIAL FORMS

1. Write a bill for four items of groceries purchased from some grocer in your vicinity, giving his name and address. Then add the amounts and receipt the bill.

2. As in Ex. 1, write a bill for four items of dry goods purchased at some store in your vicinity.

3. Write two pages of a cash account such as you might keep for personal receipts and expenses. The left-hand page should show the receipts, and the opposite page the expenses. Give six items on a page and balance the account.

4. Write a deposit slip for four items of money or checks to be deposited at some bank in your vicinity.

5. Write a check for $2.75 to pay for some purchase. If you have $68.93 in the bank before making out the check, what is your bank balance after the check is paid? Show how the check should be indorsed.

6. Write a promissory note for $25, payable to John Doe one year after today, with interest at 6%, and sign it with the name "Richard Roe."

7. How much interest is due on the note in Ex. 6 on the date on which payment is promised?

8. In Ex. 7 suppose that Richard Roe paid the note and interest by giving a check. What is the date of the check? To whom would he make the check payable? Write the check and show how it should be indorsed.

9. Write a check payable to the order of John Doe for $1.50. Date it properly, and sign it with the name "Richard Roe." Show how Mr. Doe should indorse the check.

10. Write a sight draft for $200 such as might be made by a wholesaler on a retail dealer.

II. STOCKS AND BONDS

A SILENT READING LESSON

A group of boys decided to organize a baseball club and found that the cost of uniforms and equipment would be \$112. If 14 boys had been willing to join, each would have had to pay $\frac{1}{14}$ of \$112, or \$8. Some of the boys could not afford to contribute as much as others, and so they divided the \$112 into 224 shares (or parts) of 50¢ each, selling to each boy as many shares as he cared to buy. Fred was one of the chief promoters and took 40 shares, thus making his payment \$20. James took 20 shares, and the others took various amounts.

At the first three games the club made large profits. If the boys had all contributed equally, each would have received $\frac{1}{14}$ of the profits. Since this would not be fair to those who put in the most money, they divided the profits according to the number of shares that each boy owned.

James now tried to buy some of Fred's shares, so as to get more of the profits; but since the profits were so large, Fred would not agree to sell for less than 60¢ a share.

The boys had formed a *corporation* with a *capital* of \$112. There were 224 *shares of stock*, which had a *par value* of 50¢ each. The profit each boy received was a *dividend*, and these dividends were so large that a share was worth more than 50¢; that is, the stock was *above par*.

In practical life men form corporations in much the same way, only they play the game of business instead of playing the game of baseball.

The laws of the different states specify the provisions under which corporations may be formed. There are many advantages, both from a legal and a business point of view, in organizing as a corporation, and a large part of modern business is now done by firms that are incorporated.

FORMING CORPORATIONS

All work oral

1. In organizing a tennis club some boys issue shares at 50¢ each. If they need $150 to start the club, how many shares should they issue?

2. If William buys 30 shares, how much does he pay?

3. If Frank pays in $10, how many shares does he take?

4. If the profits from renting the courts and from the sale of tickets to the match games are so large that each share is worth 55¢, what is the per cent of increase in value?

5. If the stock rises to 20% above the par value of 50¢, how much is each share then worth?

6. A heavy storm injures the courts, and the boys have to raise more money. Frank offers to sell his stock for 45¢ a share, thinking that someone may wish to buy at this price despite the poor prospects of the club. This price is what per cent below par?

7. The boys elect six of their number to direct, or manage, the club. These six members are called "directors," and at a meeting they order an "assessment" of 10% on the par value of each share. How much has to be paid on each share?

8. If you own $5 worth of stock in a baseball club, and the stock rises to 20% above par, how much is it then worth?

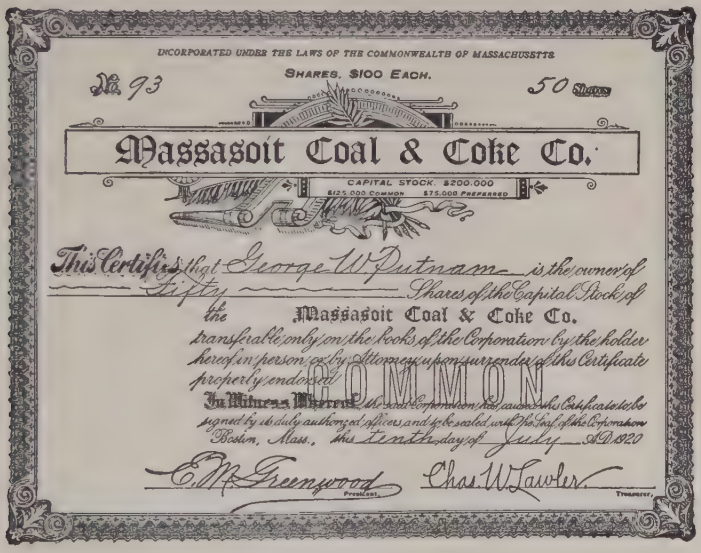
9. If you own $5 worth of stock in a tennis club, and the stock drops to 10% below par, how much is it then worth?

10. Why may stock be above or below par?

11. Why is it necessary in business to have stock companies, or corporations, instead of having one man furnish the capital?

12. Why is it necessary for the stockholders to select a few men to direct the business instead of managing it themselves?

Stockholders. The *capital*, or *capital stock*, which is the amount invested for the purpose of carrying on the business of a corporation, is divided into *shares*. A person who buys shares is called a *stockholder*, and receives a *stock certificate* showing how many shares he owns. A stockholder usually has one vote for each share at the stockholders' meetings.


INCORPORATED UNDER THE LAWS OF THE COMMONWEALTH OF MASSACHUSETTS

No. 93 SHARES, $100 EACH. 50 Shares

Massasoit Coal & Coke Co.

CAPITAL STOCK, $200,000
$125,000 Common $75,000 Preferred

This Certifies that George W Putnam is the owner of Fifty Shares of the Capital Stock of the Massasoit Coal & Coke Co.
transferable only on the Books of the Corporation by the holder hereof in person, or by Attorney, upon surrendering this Certificate properly endorsed.

COMMON

In Witness Whereof the said Corporation has caused this Certificate to be signed by its duly authorized officers and its Seal to be hereunto affixed.
Boston, Mass., this Tenth day of July A.D. 1920

E. M. Greenwood, President. Chas. W. Lawler, Treasurer.

A Stock Certificate

In our work the par value of a share of stock is to be taken as $100 unless otherwise stated, although in many cases the par value may be $50, $25, $5, or even $1. Sometimes stock is issued without a stated par value.

Directors and Officers. The stockholders elect a few of their number, called *directors*, to supervise the business. The directors elect the *officers* of the company, who are generally a president, a vice president, a secretary, and a treasurer.

Dividends. The net profits (the profits after all expenses are paid) are divided among the stockholders in the form of *dividends*. There are frequently two kinds of stock: *preferred stock*, which often pays a fixed rate of dividend, and *common stock*, which pays a dividend depending upon the profits left after the preferred dividend is paid.

For example, if the capital of a corporation is \$100,000, and the par value is \$100, there are 1000 shares outstanding. If in a year the company earns \$6000 above all expenses, the directors may declare a dividend of \$6000 ÷ \$100,000, or 6%. Then, for each share of stock that a man owns, he will receive 6% of \$100, or \$6.

$$\textit{Dividend} = \textit{rate} \times \textit{par value}.$$

CORPORATION PROBLEMS

1. A company capitalized at \$3,000,000 declares a dividend of 5%. How much does the holder of 100 shares receive?

2. A company with a capital of \$500,000 divides \$45,000 in dividends. How much does the holder of 30 shares receive?

3. A company divides \$50,000 in dividends, the stockholders receiving \$4 per share. What is the capital?

4. A company with a capital of \$50,000,000 declares a dividend of $2\frac{3}{4}\%$ every 6 mo. How much money does it distribute annually among the stockholders?

5. If a company with a capital of \$500,000 earned \$11,725.40 above expenses in the last quarter (3 mo.), what rate of dividend can it declare for this quarter and still carry about \$1700 to the next quarter?

6. A company with a capital of \$250,000 has earned \$15,000 this year above all expenses, and decides to divide all but \$2500 of this amount in dividends. What is the dividend rate?

Buying and Selling Stock. Stocks are usually bought and sold through a *broker*, who buys and sells stock in the *stock exchange*, a kind of auction room for such business.

Stock Quotations. A *quotation*, or *quoted price*, of $118\frac{3}{4}$ means that the *market price* per share is \$118.75. If a stock is paying a high rate of dividend, people will pay more than \$100 for a share whose par value is \$100, and the stock is said to be *above par*, or *at a premium*. If a stock with a par value of \$100 is quoted at 100, it is said to be *at par*; and if quoted below 100, it is said to be *below par*, or *at a discount*.

Brokerage. The broker charges *brokerage*, or commission, for his services. A charge of \$15 per 100 shares on stocks selling between \$10 and \$125 per share, and \$20 per 100 shares on stocks selling at \$125 per share or over, is added to the cost of the stock at the market price when the broker buys for a customer. When the broker sells stock, he deducts the brokerage from the amount received and pays the balance to the customer. There is a minimum brokerage charge of \$1 in all purchases or sales of stock.

For example, how much does a man, buying through a broker, pay for 300 shares of stock quoted at $117\frac{1}{8}$?

The cost of the stock at the market price is $300 \times \$117.12\frac{1}{2}$, or \$35,137.50. The brokerage is $3 \times \$15$, or \$45, so that the man pays \$35,137.50 + \$45, or \$35,182.50, for the stock.

Cost = number of shares × quotation + brokerage.

The man who sells the above stock receives from his broker \$35,137.50 − \$45, or \$35,092.50.

Proceeds = number of shares × quotation − brokerage.

The brokerage on 6 shares or less of the above stock would be the minimum charge of \$1; on 10 shares the brokerage would be $\frac{1}{10}$ of \$15, or \$1.50.

BUYING AND SELLING STOCK

Numbers 1 to 6, oral

State the brokerage on the following sales of stock:

1. 100 shares @ $97\frac{7}{8}$.
2. 200 shares @ 137.
3. 500 shares @ $69\frac{3}{4}$.
4. 50 shares @ $87\frac{3}{8}$.
5. 75 shares @ $172\frac{1}{2}$.
6. 3 shares @ 149.

Find the amounts paid in making the following purchases of stock through a broker:

7. 100 shares @ $87\frac{1}{8}$.
8. 300 shares @ $136\frac{1}{2}$.
9. 400 shares @ $54\frac{3}{8}$.
10. 1000 shares @ $47\frac{7}{8}$.
11. 50 shares @ $142\frac{5}{8}$.
12. 75 shares @ $172\frac{5}{8}$.
13. 125 shares @ $97\frac{3}{4}$.
14. 10 shares @ $119\frac{7}{8}$.
15. 5 shares @ $63\frac{3}{8}$.
16. 60 shares @ 147.

Find the amounts received from selling the following stocks through a broker:

17. 75 shares @ 67.
18. 500 shares @ $37\frac{1}{8}$.
19. 600 shares @ $96\frac{3}{8}$.
20. 50 shares @ $99\frac{7}{8}$.
21. 200 shares @ $127\frac{1}{4}$.
22. 4 shares @ $158\frac{3}{8}$.
23. 15 shares @ $56\frac{1}{4}$.
24. 2400 shares @ $80\frac{1}{2}$.
25. 40 shares @ $129\frac{5}{8}$.
26. 3 shares @ $88\frac{3}{4}$.

27. A man bought 30 shares of American Locomotive @ 124, 20 shares of Bethlehem Steel @ $40\frac{3}{4}$, and 5 shares of Union Pacific @ 139. How much did he pay, including brokerage?

28. A man sold 40 shares of General Motors @ $71\frac{1}{2}$, 75 shares of American Sugar @ $66\frac{7}{8}$, and 5 shares of Baldwin Locomotive @ $112\frac{7}{8}$. How much did he receive from his broker?

Bond. A *bond*, like a note, is a written promise to pay money. A corporation that wanted to borrow \$5000 for 2 mo. would do so on its note; but if it wanted to borrow \$500,000 and not pay it back for 10 yr., it would do so by issuing bonds, probably in denominations of \$1000 or \$500 each.

All the conditions under which the bonds are issued are stated in the bond, including the amount for which issued (the *face*), the rate of interest, the date of maturity, and so on. The interest on bonds is usually paid semiannually.

Bonds are often secured by a *mortgage*, an agreement by which the bondholders may sell the property pledged if the bonds are not paid when due.

The stockholders of a corporation are the owners of the property, while the bondholders are owed money by the corporation and so are creditors. A stock certificate is like a deed to a house; a bond is like a mortgage on it. Bonds bear a fixed rate of interest, but the dividends on stock depend upon the net profits of the company.

Income from Stocks and Bonds. Bond interest and stock dividends are always computed on the *face value*; that is, on the par value.

For example, in finding the *amount* of income received on 4% bonds which have a par value of \$5000, we have

$$0.04 \times \$5000 = \$200, \text{ the income.}$$

Similarly, in finding the *rate* of income received from a stock which pays 5% and is bought at $112\frac{1}{4}$, we have

$$\$112.25 + \$0.15 \text{ (brokerage)} = \$112.40, \text{ the cost per share,}$$

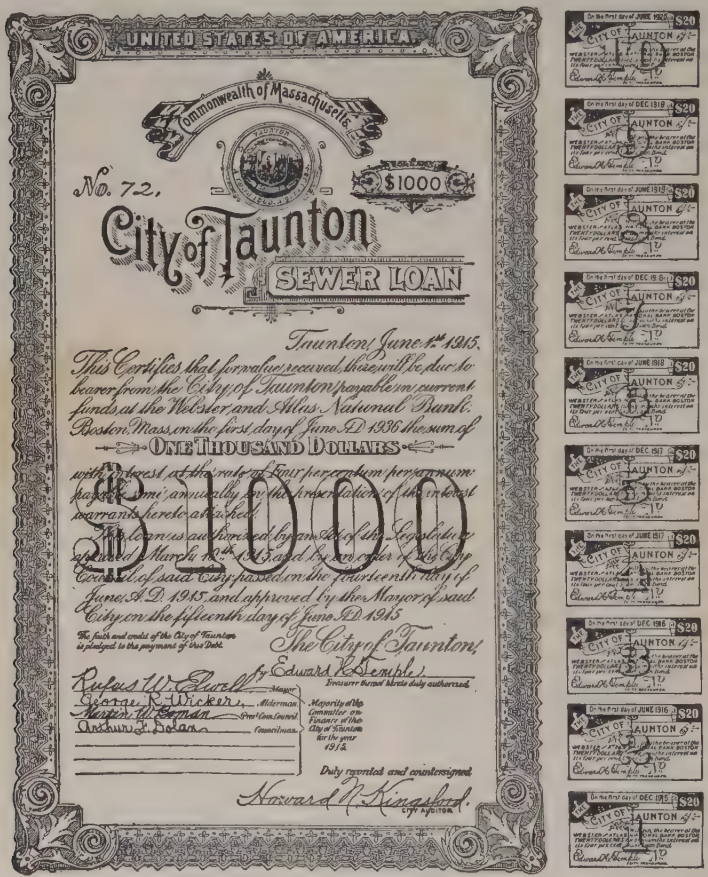
$$0.05 \times \$100 = \$5.00, \text{ the income per share,}$$

and

$$\$5.00 \div \$112.40 = 0.044 \cdots.$$

That is, the rate of income, although 5% on the par value, is approximately 4.4% *on the investment.*

Bond Coupons. Bonds are often issued with small attached notes, which are called *coupons*, as in the specimen here shown:


UNITED STATES OF AMERICA.
Commonwealth of Massachusetts.
No. 72.
$1000
City of Taunton
SEWER LOAN
Taunton, June 1st 1915.
This Certifies that for value received there will be due to bearer from the City of Taunton payable in current funds at the Webster and Atlas National Bank, Boston, Mass. on the first day of June A.D. 1936 the sum of
ONE THOUSAND DOLLARS
with interest at the rate of Four per centum per annum payable semi-annually on the presentation of the interest warrants hereto attached.
This Bond is issued by and with the approval of the Governor and Council of Massachusetts, acting under the authority of Chapter 490 of the Acts of the year 1913, and by virtue of an order of the City Council of said City passed on the fourteenth day of June, A.D. 1915, and approved by the Mayor of said City on the fifteenth day of June A.D. 1915
The faith and credit of the City of Taunton is pledged to the payment of this Debt.
The City of Taunton,
By Edward K. Semple,
Treasurer thereof thereto duly authorized.
Rufus W. Purrell, Mayor
George R. Wicker, Alderman
Harrison W. Seaman, Chairman Committee on Finance
Arthur F. Dolan, Councilman
Majority of the Committee on Finance of the City of Taunton for the year 1915
Duly recorded and countersigned
Howard N. Kingsford,
City Auditor
$1000
CITY OF TAUNTON
$20

A Coupon Bond

When a payment of interest is due on the bond, the bond holder cuts off the proper coupon and presents it for payment. Banks accept and collect bond coupons for their depositors.

Newspaper Quotations. The newspapers give the market prices of the leading stocks and bonds. The problems on the next page may be solved by using the current stock quotations found in the newspapers or by using the following list:

At., Top., & S.F.	$120\frac{7}{8}$	United Drug	$121\frac{1}{2}$
N.Y. Central	$114\frac{1}{2}$	U.S. Rubber	$38\frac{1}{4}$
Union Pacific	$139\frac{1}{2}$	General Electric	269

A par value of \$100 per share and the brokerage rates on page 307 are to be used in all problems about stocks.

Profit or Loss on Stock. In figuring the profit or loss on a stock transaction,

The difference between the total cost (cost + brokerage) and the net selling price (selling price − brokerage) is the gain or loss.

It is often easier, however, to figure the brokerage separately, as in the following examples:

1. If a man buys 100 shares of Western Union Telegraph at $118\frac{1}{4}$ and sells the stock at $117\frac{1}{2}$, before receiving any dividends, how much does he lose?

The man loses \$118.25 − \$117.50, or \$0.75, on each share.

On 100 shares he loses $100 \times \$0.75$, or \$75.

He must, however, pay two brokerage charges, one when he buys the stock and one when he sells it; and hence he pays $2 \times \$15$, or \$30, for brokerage.

His total loss on the transaction is \$75 + \$30, or \$105.

2. If a man buys 1000 shares of American Locomotive at 116 and sells the stock at $119\frac{1}{2}$, before receiving any dividends, how much does he gain or lose?

The man gains on each share \$119.50 − \$116, or \$3.50.

On 1000 shares he gains $1000 \times \$3.50$, or \$3500.

The brokerage is $2 \times 10 \times \$15$, or \$300.

His total gain is therefore \$3500 − \$300, or \$3200.

STOCKS AND BONDS

1. What is the annual income from 50 shares of stock which pays a dividend of $1\frac{3}{4}\%$ quarterly?

2. If a bond which has a par value of \$100 and pays 6% is bought for \$104, including brokerage, what is the rate of income from the investment?

Disregarding dividends, find the gain or loss on the following sales of stocks, bought at the quotations given on page 311:

3. 100 shares of At., Top., & S. F. @ $129\frac{1}{4}$.

4. 50 shares of N. Y. Central @ $105\frac{7}{8}$.

5. 15 shares of Union Pacific @ $137\frac{3}{4}$.

6. 3 shares of United Drug @ $126\frac{7}{8}$.

7. 400 shares of U. S. Rubber @ $45\frac{5}{8}$.

8. 150 shares of General Electric @ $255\frac{5}{8}$.

9. If a Fourth Liberty Loan bond, which has a par value of \$100 and pays $4\frac{1}{4}\%$, is bought for \$108.25, including brokerage, what is the rate of income from the investment?

10. If the bond in Ex. 9 is bought for \$101.80, including brokerage, what is the rate of income from the investment?

11. A man buys 100 shares of stock at $129\frac{3}{4}$. If the stock pays dividends at the rate of $6\frac{1}{2}\%$ annually, find the rate of income from the investment.

12. At the time the man in Ex. 11 bought his stock, he could have invested \$13,000 in a mortgage paying $5\frac{1}{2}\%$, but he preferred the stock. What is the difference in income?

13. If you can consult a daily newspaper which gives the prices of stocks and bonds, keep track of the *fluctuations* (changes in price) of some bond or stock for a week, and report to the class.

Government Bonds. The governments of states, counties, cities, and towns, and our national government as well, often issue bonds when they are financing some public enterprise that is too large to be paid for at one time by the money raised by taxes.

Bonds issued by a national government are called *government bonds*, while those issued by state, city, and other local governments are called *municipal bonds*.

The United States could borrow money before the World War at from 2% to 3%, and states could borrow at about 4%, while cities, counties, and villages paid a little more. While at present the rates are somewhat higher, they vary so much from time to time that no standard rate can be given.

Cities also raise money by special *assessments*. For example, if a street is to be paved, the city will require owners of property fronting on the street to pay a special assessment in addition to their ordinary taxes, because of the increase in the value of their property which is caused by improving the street.

GOVERNMENT AND MUNICIPAL BONDS

1. If a city borrows \$5,000,000 at $4\frac{1}{2}$%, what is the annual interest charge?

2. If a state borrows \$40,000,000 at $4\frac{1}{4}$%, what is the annual interest charge?

3. If a nation wishes to borrow \$6,000,000,000 and must pay $4\frac{1}{4}$%, what will be the annual interest charge?

4. A certain suburb of Chicago, with an assessed valuation of \$3,500,000, decided to borrow \$125,000 in order to erect a new school building. At $5\frac{1}{4}$%, by how much will the annual interest charge on the loan increase the tax rate on each \$1000 of assessed valuation?

5. During the World War the first issue of government bonds amounted to \$2,000,000,000 and bore interest at $3\frac{1}{2}\%$. This was called the First Liberty Loan, and was oversubscribed by a little more than 50%. By how many dollars was the loan oversubscribed? What is the annual interest charge for borrowing this \$2,000,000,000?

6. The second issue of bonds amounted to \$3,808,766,150. This was called the Second Liberty Loan, and bore interest at 4%. How much is the annual interest on this loan?

7. A little later the government borrowed \$4,176,516,850 (the Third Liberty Loan) at $4\frac{1}{4}\%$. Find the amount of interest paid annually by the government on this loan.

8. The Fourth Liberty Loan, amounting to \$6,989,047,000, was issued still later, and bore interest at $4\frac{1}{4}\%$. Find the amount of interest paid annually on this loan.

9. The government borrowed also \$4,500,000,000 at $4\frac{1}{4}\%$. How much was the annual interest charge on this loan?

10. If a man owns \$5000 worth of government bonds which pay $4\frac{1}{4}\%$, how much interest does he receive semiannually from the bonds?

11. In Ex. 10, how much more interest does the man receive semiannually than he would receive if he owned \$5000 worth of bonds paying $3\frac{1}{2}\%$?

12. If a man owns \$8500 worth of bonds of the Fourth Liberty Loan, which pays $4\frac{1}{4}\%$, how much interest does he receive semiannually?

13. If a man owns \$12,000 worth of bonds of the city of Rochester, and the bonds pay interest at the rate of 4%, how much interest does he receive in a year?

14. Solve Ex. 13, supposing that the man owns \$15,500 worth of the bonds.

III. THRIFT AND INVESTMENTS

A SILENT READING LESSON

Formerly people hoarded their savings in hidden places about their homes. Now they usually invest money that is not needed for immediate use, so that it will not only be safer than when kept at home but will also bring in an income. Unfortunately, some fraudulent investments are offered to the public, while others are so risky as to be undesirable. There are, however, many types of investments that are perfectly safe. It is well to remember that, in general, if the rate of income offered by any form of investment is larger than the current rates on sound investments, there is usually a greater risk of losing the money put into it.

We should learn how to distinguish between good investments and bad investments, so that we may invest wisely whatever money we may be able to save. Hundreds of millions of dollars are lost in this country every year through unscrupulous promoters of worthless stock, despite such laws as have been passed by many of our states to prevent the sale of fraudulent stocks.

Buying stocks *on a margin*, which means that the purchaser pays only a small amount down and hopes that the stock will rise so that he can then sell it at a profit, is a very dangerous way to invest money. The stock may go down instead of up, and, if it goes down as many dollars as he paid, he will lose all that he put in. He also, of course, must pay the brokerage charges, whether he wins or loses, and so buying stocks in this way is a gamble. Never gamble in stocks; the chances are against you.

We should never make any kind of investment without asking the opinion of some reliable banker who has no financial interest in the property or stock that we plan to buy.

THRIFT AND INVESTMENTS

1. If you have \$85 to invest and deposit it in a savings bank that pays 4% a year, compounded semiannually, how much interest is due you at the end of 6 mo.?

2. A woman has saved \$2750. She can deposit it in a savings bank which pays 4%, compounded semiannually, or can invest it in an oil company which advertises that it pays 10% on its stock. Her banker, however, advises her that the oil stock is a risky speculation. What is the difference in income for the first year? Which investment should she make?

3. A man has saved \$4500. He can invest it in a well-managed manufacturing business, about which he is thoroughly informed, so as to receive 7% on his investment. Is this a good investment? How much income will he receive a year?

4. A man comes to town and offers investments in a fruit plantation in Mexico. Nobody has heard of the man or of his plantation. He says the business will pay 15%, and he asks Mr. Doe to invest \$5000. How much income does he promise to Mr. Doe? Mr. Doe can invest the money near home at 6%. How much will this bring him per year? Which investment should you advise him to make? Write your reasons.

5. Mrs. St. John has inherited \$4500, and has a chance to invest it at 5% in a first-class security or at 12% in a banana plantation that her banker tells her is a very risky speculation. How much would be the income from each? Which investment should she make?

Find the income from each of the following investments:

6. \$600 at 5%.
7. \$575 at 5%.
8. \$875 at 4%.
9. \$625 at $5\frac{1}{2}$%.
10. \$777 at $4\frac{1}{2}$%.
11. \$1450 at 4%.
12. \$9750 at 6%.
13. \$6225 at 6%.
14. \$3500 at 3%.

INVESTING IN REAL ESTATE

1. Besides depositing money in banks or purchasing stocks and bonds, people who have saved money often buy real estate, which means land and buildings. A man paid $6000 for a house and rents it for $50 a month. He pays $186 a year for taxes, insurance, and repairs. What per cent on his investment does he receive?

2. Mr. Lowe rents to Mr. Ryan a house in Denver for $75 a month. If one year Mr. Lowe pays $165 for taxes, $30 for insurance and repairs, and $130 for a street-paving assessment, how much was his net income from the property that year?

3. A man owns a house which he rents for $80 a month, but in 3 yr. the house is vacant for 2 mo. The house cost him $9000, and his annual expenses for taxes, repairs, and insurance are $187.50. What per cent of income does the man receive from his investment?

4. Mr. Phillips bought a house for $6000 and rented it for $45 a month. He paid annually $150 for taxes, $4.50 for insurance, and $45 for repairs. Three years later he sold the property for $6800. How much more did he gain than he would have gained by investing the $6000 for 3 yr. at 6%?

5. A man bought a house for $4800 and placed on it a mortgage of $2000 on which he has to pay 6%. Allowing $150 a year for taxes, insurance, and repairs, and interest at 5% on his investment of $2800, how much does the house cost him each year? This is how much less than paying $50 a month rent for a place in which to live?

6. Mr. Young paid $8000 for a house. He rents it for $60 a month. In a certain year he pays $166 for taxes, $6 for insurance, and $48 for repairs. What per cent does he receive on his investment that year?

CHOOSING INVESTMENTS

1. If on Jan. 1 you have $1000 to invest, and put it in a savings bank which pays 4%, compounding the interest every 6 mo., how much money do you have at the end of 6 mo.? of 2 yr.? of 3 yr.? of 4 yr.?

2. Instead of depositing in the savings bank the $1000 mentioned in Ex. 1, suppose that with it you had been able to buy 10 shares of stock which paid 3% every 6 mo. How much would you receive in dividends in 3 yr.?

3. After finding the incomes in Exs. 1 and 2, write a statement of the reasons why, under certain circumstances, you might prefer to put your money in the savings bank and why, under other circumstances, you might prefer to buy the stock.

4. If you should receive through the mails a circular letter offering you stock in a moving-picture concern and promising dividends of at least 20%, should you invest your $1000 in this way? If so, what would lead you to do it? If not, why should you hesitate? If in doubt, whom should you consult?

5. If you were advised to do so by a man who possessed good judgment and who was not financially interested in the property, would you invest the $1000 in a building lot in a growing section of a city? What are the arguments in your mind for and against such an investment?

6. One of the best ways to save money is to set aside a fixed sum, even if small, each week. Many banks conduct Thrift Clubs, in which the members deposit each week a fixed amount, such as 50¢, $1, $2, or $5, for 50 wk. The banks pay interest that is equivalent to $1\frac{1}{2}\%$ on the total of the deposits during the year, and at the end of that time send to each member a check for his deposits and interest. How large is the check which a member receives if each week he deposits 50¢? $1? $2? $5?

IV. DRAWING TO SCALE

A SILENT READING LESSON

Most of us have probably seen the workmen on buildings looking at *blue prints*, which are photographic copies, on blue paper, of the architects' drawings. These drawings are made to *scale*. In our work in measurement we have already used drawings that were made to scale; that is, drawings in which 1 in. represented some greater length, such as 1 ft., 1 yd., or 1 mi. If we draw to the scale $\frac{1}{10}$, each line in the drawing is $\frac{1}{10}$ as long as it is in the object represented.

Maps are always drawn to scale. If a map of the state in which we live could be drawn full size, it would be exactly as large as the state itself. In the same way, a plan of a floor, drawn full size, would be as large as the floor. For practical purposes, therefore, scale drawings are necessary.

We must be very careful in drawing to scale, because the computations made from the drawing will be incorrect if the drawing is not made precisely. We should handle our *rulers* carefully so as not to get nicks in them, we should keep our pencils sharp, and we should have a pair of *compasses* and, if possible, a *protractor* for measuring angles.

The common kinds of lines that we shall use are the *straight line* and the *curve*. The *broken line*, made up of straight lines though not itself straight, is sometimes used. The four sides of a square, taken as one line, illustrate a broken line.

Whenever we speak simply of a line, we always mean a straight line and not any one of the other kinds of lines that are mentioned above.

We should also know how to divide a line or an angle into two equal parts, how to draw triangles of different kinds, and how to make other constructions of this kind. We shall soon learn how to do these things.

LINES AND ANGLES

1. Using a ruler, draw on the board a straight line 1 ft. long. Beneath this line draw another line to represent the first one, using the scale $\frac{1}{12}$; that is, make it $\frac{1}{12}$ as long.

2. Measure the length of the teacher's desk and draw a line on the board to represent this length, using the scale $\frac{1}{4}$.

3. Copy this broken line on the board, drawing the angles freehand (that is, making them as nearly like those here shown as you can without measuring them), and making each of the three parts of the broken line ten times the length here shown.

4. Draw three curves of different shapes on the board, letting them represent some curves that you see in the room.

5. Here are five angles. Copy them freehand on the board and under each figure that you draw write the name *acute*, *right*, or *obtuse*, depending upon which kind of angle it is:

6. Tell the class about three places in some building, street, or pattern where you have seen a right angle.

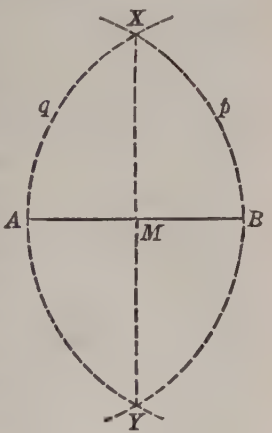
7. As in Ex. 6, tell about three acute angles and three obtuse angles that you have seen.

8. Tell the class about three places in some building or pattern where you have seen curves used.

9. Tell the class what scale you would use if you were making on the board or on your drawing paper a plan of the floor of your schoolroom, and tell why that would be a good scale.

10. As in Ex. 9, tell the class what scale you would use if you were drawing a plan of the top of your desk.

Bisecting a Line. To *bisect* a straight line means to find the mid-point between the ends; that is, to cut the line into two equal parts. We might bisect the line AB here shown by measuring off half of it, but it would not be very accurate.



A much better way is to place the steel point of our compasses at A and the pencil point at B, and then draw part of a circle, as shown by the curve p. Then place the steel point at B and the pencil point at A and draw the curve q. These two curves cross at X and Y, and if we join X and Y, the point M will bisect AB.

In making figures like this on the board, a string tied to a crayon may be used in drawing the circles.

Drawing a Perpendicular. We can draw a perpendicular to the line AB at M by using a method like the one above.

To do so we first place the steel point of the compasses at M, and mark off two points A and B (they do not have to be at the ends of the line) so that MA is equal to MB. Then using A and B as centers, as before, we draw parts of two circles like p and q, which cross at X. Drawing a line between X and M, this line will be perpendicular to AB and the angles at M will be right angles, as must always be the case with a perpendicular.

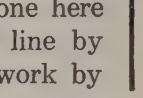
Perpendiculars can also be drawn by using a small square like a carpenter's square or a right triangle made of some stiff material like cardboard or celluloid. This is perhaps the best way when we wish to draw a perpendicular at the end of a line that we do not care to extend (a thing which happens too rarely to be considered further).

BISECTORS AND PERPENDICULARS

1. Draw a line on the board and then bisect it by the method given on page 321. Check your work by measuring the length of each part with a ruler.

2. Draw a line on the board, mark a point P on the line, as here shown, and then draw a perpendicular to the line at the point P.

3. Draw on the board a vertical line like the one here shown, but about 1 ft. long. Bisect this 1-foot line by the method given on page 321, and check your work by measuring the parts with a ruler.



4. Draw a vertical line on your paper, mark on it any point Q, and then at Q draw a line that shall be perpendicular to the vertical line.

5. Draw on the board a slanting line like the one here shown and then bisect it.

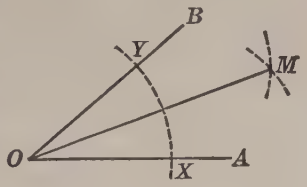
6. Draw on your paper a slanting line, mark on it any point T, and then at T draw a line perpendicular to the slanting line.

7. On the board draw a vertical line, a horizontal line, and a slanting line, each 2 ft. long. Bisect each line and check your results by measuring to see if one part of each line is equal to the other part of the line.

8. Draw on paper a line 3 in. long. At a point 1 in. from one end draw a perpendicular. If you have a protractor, measure the angles to see if they are each 90°.

9. Draw on paper a line 2 in. long and bisect it with ruler and compasses. To check your work, fold the paper along the bisector (the line which bisects the 2-inch line), stick a small pin through one end of the 2-inch line, and see if the pin goes through the other end of the line.

Bisecting an Angle. To *bisect* an angle means to draw a line through the *vertex* (the point where the lines meet to form the angle) so as to cut the angle into two equal angles.

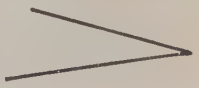


In speaking of the angle formed by the lines *OA* and *OB* in this figure, we call it "the angle *AOB*." The vertex is always designated by the middle letter.

To bisect the angle *AOB* here shown, place the steel point of the compasses at *O* and draw part of a circle which crosses both *OA* and *OB*, thus fixing the points *X* and *Y*. Now place the steel point at *X* and the pencil point at *Y* and draw part of a circle, and then, placing the steel point at *Y* and the pencil point at *X*, draw another part of a circle crossing the first one at *M*. Now draw *OM*, which will bisect the angle.

BISECTING AN ANGLE

1. Draw on the board an angle like the one here shown and then bisect it.



2. Draw on the board any angle that you wish and then bisect it.

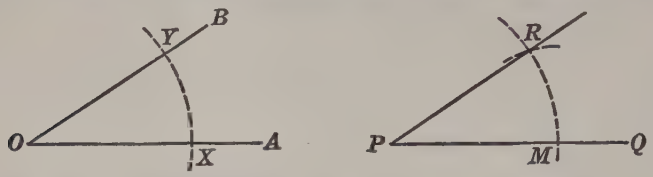
3. Draw on the board any triangle and bisect each of the three angles. If your work is accurate, the three bisectors will all meet at one point.

4. Draw on paper three triangles of different shapes and draw the bisectors of the angles, as in Ex. 3. If in any one of them the bisectors do not all meet at one point, your work has not been carefully done.

5. Draw on paper three triangles of different shapes. At the mid-point of each side draw a perpendicular. As in Ex. 4, the three perpendiculars should all meet in one point.

Making One Angle Equal to Another. When we copy a figure, we often have to make one angle equal to another.

For example, suppose that AOB is an angle of a figure that we wish to copy, and that PQ is a line on which we wish to make an angle that shall be equal to the angle AOB.



First, with the center O, draw part of a circle which cuts off the equal lines OX and OY. Then, keeping the compasses opened just as they were, place the steel point at P and with the pencil point mark M by drawing part of a circle. Then PM will be the same length as OX.

Now place one point of the compasses at X and the other at Y, and then, without changing the opening of the compasses, place the steel point at M and draw part of a circle crossing the first one at R.

Then draw PR, and the angle QPR thus formed will be exactly equal to the angle AOB.

MAKING EQUAL ANGLES

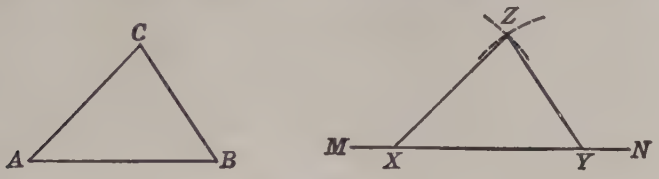
1. Draw any angle that you wish on the board and draw an angle equal to it by the method described above.

2. Repeat Ex. 1 on paper, checking your work by using a protractor, if one is available, to measure the angles.

3. Draw one line perpendicular to another, and then bisect one of the right angles thus formed.

4. Draw two lines which cross each other at any convenient angle, and bisect each of the four angles thus formed.

Making One Triangle Equal to Another. Suppose that we wish to draw a triangle that shall be exactly equal in all respects to the triangle *ABC*, which is shown below at the left.



First draw any line *MN*. Then place the two points of the compasses at *A* and *B* and, without changing the opening of the compasses, transfer the length *AB* to the line *MN*, as shown by the points *X* and *Y*.

Now place the compasses so that the points lie on *A* and *C*, and, without changing the opening, place the steel point at *X* and with the pencil point draw part of a circle. Then doing the same for *B* and *C*, place the steel point at *Y* and draw another part of a circle crossing the first one at *Z*.

Then, drawing the lines *XZ* and *YZ*, the triangle *XYZ* will be exactly equal to the triangle *ABC*.

In the same way we can make a triangle with its sides of any given lengths, provided only that the given lengths are such as will make a triangle possible.

EQUAL TRIANGLES

1. On paper draw a triangle of any shape and then make a triangle exactly equal to it.

2. Draw a triangle with sides $1\frac{1}{2}$ in., 1 in., and $1\frac{3}{4}$ in. long.

Draw triangles with sides of the following lengths:

3. 1 in., 1 in., 1 in.

4. $1\frac{1}{2}$ in., $1\frac{1}{2}$ in., 2 in.

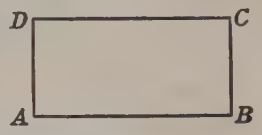
5. 2 in., 3 in., 3 in.

6. $1\frac{1}{4}$ in., $1\frac{1}{4}$ in., $1\frac{3}{4}$ in.

OTHER EASY FIGURES

1. Draw a rectangle with its base 2 in. long and a height of 1 in., being careful to draw accurate right angles by the method given for drawing perpendiculars.

2. If your work in Ex. 1 was accurate, the two diagonals (the lines from one corner to the opposite one, as from A to C and from B to D in this figure) are equal. If this is not the case, do your work over again until, when you apply this test, you find that the diagonals are equal.

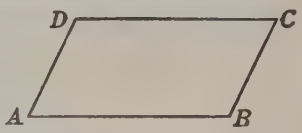


3. Enlarge the figure of Ex. 2 to three times its size; that is, so that each side of the new rectangle is three times the length of the one shown above.

4. The area of the new rectangle in Ex. 3 is how many times the area of the original one?

5. Reduce the figure that you drew in Ex. 3, making each side two thirds as long. The area of this rectangle is how many times the area of the original one in Ex. 2?

6. Enlarge this parallelogram so that each side is four times as long. Be careful to make the angles equal to those that are here shown, using the method given on page 324.



7. Check your work in Ex. 6 by seeing if each diagonal in your figure is four times as long as the corresponding diagonal in the figure shown above.

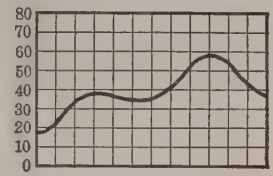
8. Reduce the figure that you drew in Ex. 6, making each side three fourths as long.

9. Draw a plan of your desk top to the scale $\frac{1}{8}$, and then enlarge the plan, making each side twice as long.

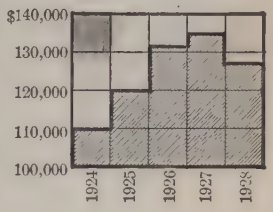
V. GRAPHS

A SILENT READING LESSON

In the newspapers, we have probably seen diagrams like the one here shown. Perhaps they were used in advertisements, or in weather reports, or in setting forth certain important facts about railroads or other business corporations. As we have already seen on page 2, pupils can keep a record of their progress in arithmetic and other studies in this way. The ups and downs in their work are shown by the ups and downs of the irregular curve in a *curve-line graph*, or by the variations in the lengths of the different bars in a *bar graph*.



If a large business house wishes to compare the amounts of its sales for each of the five years from 1924 to 1928, it might do so by means of the bar graph here shown. In this graph the length of each bar shows the sales above $100,000, each vertical space represents $10,000 and each horizontal space a year. As we can easily see, the graph tells the story about the sales much more clearly than columns of figures would tell it.



The bars of this graph might have started at $0 or at any other convenient point, and the spaces might have each represented $1000 or some other amount. If this had been done, the graph would have been larger, but it would not have told the story any more effectively.

Graphs must be accurate, clear, and neat if they are to tell the story better than it could be told by a mass of figures.

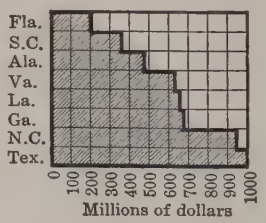
Horizontal Bars. Instead of placing the bars in a graph vertically we may, if it is more convenient, place them horizontally as in the illustration given below.

For example, in some of our states agriculture is the chief source of wealth, in others mining, and in others manufacturing. Taking the value of manufactured goods, to the nearest $1,000,000, as a basis of comparison, the government reports show the following statistics for eight states in a recent year:

Florida . . .	$213,000,000	Louisiana . .	$676,000,000
South Carolina .	381,000,000	Georgia . . .	693,000,000
Alabama . . .	492,000,000	North Carolina .	944,000,000
Virginia . . .	644,000,000	Texas	1,000,000,000

Let us now make a graph and see how much clearer these facts will stand out. The most convenient way is to use a piece of *squared paper*, or *coordinate paper*, which can be purchased already ruled in squares. If squared paper is not available, it is easy to rule some.

Since the numbers run from $200,000,000 to $1,000,000,000, and we wish to make only a general comparison of the amounts, we may let each horizontal space represent $100,000,000.

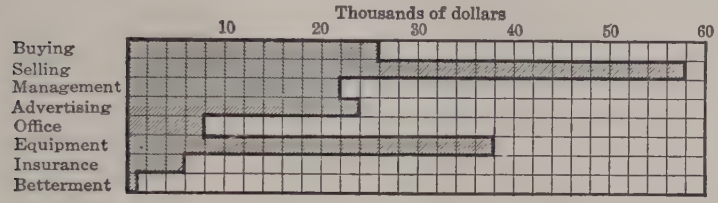


We see at a glance that Texas produced manufactured goods to the value of about $1,000,000,000 in that year, that North Carolina stood next, and that Virginia, Louisiana, and Georgia were very close together, while Florida, whose chief source of wealth is in fruit growing and market gardening, produced the least.

The term "graph" is generally applied to all pictorial representations of this kind, although a bar graph is sometimes called a *bar pictogram*.

BAR GRAPHS

1. The following graph shows the distribution of the various expenses of running a large department store in a recent year:



Write a statement as to the purpose for which the largest amount was spent; the smallest amount. How does the amount spent for advertising compare with that for buying? for selling? for insurance?

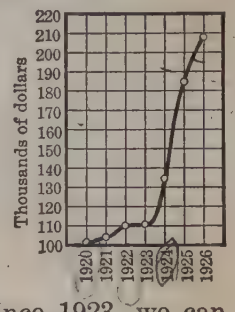
Represent the following statistics by either kind of bar graph:

2. The approximate areas of the twelve largest states, to the nearest 1000 sq. mi., are as follows: Arizona, 114,000; California, 158,000; Colorado, 104,000; Idaho, 84,000; Minnesota, 85,000; Montana, 147,000; Nevada, 111,000; New Mexico, 123,000; Oregon, 97,000; Texas, 267,000; Utah, 85,000; Wyoming, 98,000. Let one space in your graph represent 10,000 sq. mi.

3. The total yearly sales of a company were as follows: 1923, $58,000; 1924, $82,000; 1925, $98,000; 1926, $135,000; 1927, $140,000; 1928, $145,000; 1929, $160,000. After drawing the graph, interpret it; that is, state your conclusions about the business of the company.

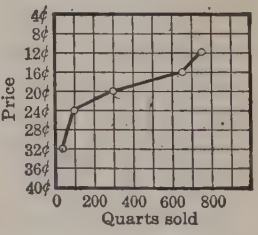
4. The total yearly income of a company for the last ten years was as follows: $275,000, $285,000, $295,000, $325,000, $310,000, $330,000, $360,000, $340,000, $375,000, $400,000. Interpret the graph as in Ex. 3.

Curve-Line Graph. In representing certain kinds of statistics the most convenient graph is the curve-line graph. This kind of graph is particularly useful in telling the story about changes from day to day or from year to year.



For example, suppose that a company whose sales were \$103,000 in 1920, \$105,000 in 1921, \$110,000 in 1922, and \$111,000 in 1923 found that its sales increased to \$135,000 in 1924, \$185,000 in 1925, and \$208,000 in 1926. While these figures show a decided increase in sales since 1923, we can see the sudden change much more easily by glancing at the curve in the graph shown above.

Demand Curve. In some graphs we use a series of short straight lines connected so as to give the effect of a curve, as in the graph, called a *demand curve*, which is here shown.



For example, it was found in a certain market that when strawberries sold at 32¢ a quart, 25 qt. were sold daily; when the price dropped to 24¢, 100 qt. were sold daily; at 20¢, 300 qt. were sold; and so on. It is evident that the average number of quarts sold per day is related to the price, and this relation may be indicated graphically as here shown. The story is thus told much more vividly than by a mere recital of figures. We can also see that it is not always more profitable to sell at the highest prices, because the sales may be so small as to give but a small total profit.

Business houses often find that a study of demand curves, like the one above, helps them in making plans for purchasing stock to meet future requirements.

CURVE-LINE GRAPHS

1. In six consecutive years the average price per pound of lake copper was as follows: 12.7¢, 16.7¢, 16.7¢, 13.5¢, 28¢, 32¢. Represent the changes by a curve-line graph.

2. The total yearly sales of a grocery for a period of ten years were as follows: \$16,000, \$18,000, \$20,000, \$25,000, \$24,000, \$26,000, \$28,000, \$36,000, \$43,000, \$52,000. Draw a graph to show the trend of business.

3. From the graph in Ex. 2 estimate the probable amount of business for the next year. Basing your conclusion upon the general trend of business shown by the graph, do you think it will be advisable to plan to enlarge the store and its stock in this neighborhood?

4. Construct a demand curve from the following facts which were ascertained with respect to a certain article:

Price	12¢	10¢	8¢	6¢	4¢	2¢
Number sold . .	400	500	1800	3000	8000	8200

5. A grocer's records of sales show that he has sold 10 baskets of peaches at \$1.75 a basket, 30 baskets at \$1.50, 70 baskets at \$1.20, 80 baskets at \$1.10, and 90 baskets at \$1. Draw the demand curve and estimate the probable sale of peaches if the price drops to 90¢ a basket.

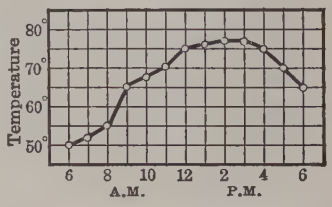
6. If the business of the grocer mentioned in Ex. 5 has increased 20% since last year, when the above facts were recorded, draw a demand curve for this year, assuming that he will sell 20% more at each price mentioned above than he did last year. Using your curve as a guide, estimate how many baskets of peaches he will probably be able to sell this year when the price is \$1.45 a basket.

TEMPERATURE GRAPHS

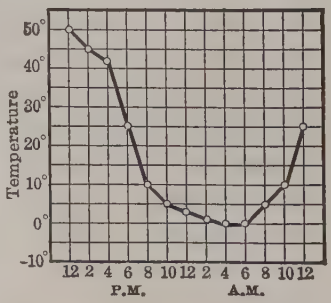
1. The temperature in a certain place in the summer was taken every noon for a week. It was found to be 64° on Sunday, 68° on Monday, 74° on Tuesday, 60° on Wednesday, 62° on Thursday, 70° on Friday, and 75° on Saturday. Represent these statistics by means of a bar graph.

2. Would the night temperatures in Ex. 1 be lower or would they be higher than the noon temperatures? With what we have given in the problem, explain why it would not be proper to use a curve line that might be taken to represent the general range of temperature for the week.

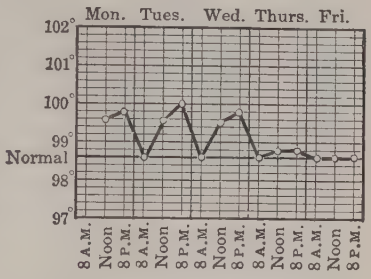
3. On May 1 the temperature between 6 A.M. and 6 P.M. at a certain place was as shown by this graph. Because the hours at which the temperature was taken are so near together, there is probably no decided change between them, and so we use a line graph. Write the approximate temperature for each hour and half hour of the day.



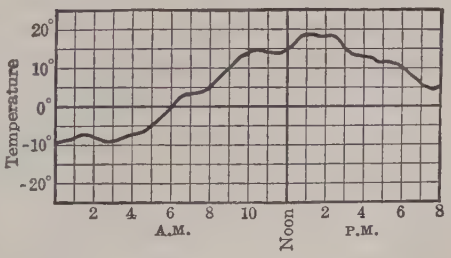
4. When the temperature is below zero, as is often the case in the North, this fact is indicated by a minus sign. For example $-5°$ means 5° below zero. This graph shows the changes in temperature from noon on a day in January to noon on the following day. Write the approximate temperatures for every hour of this period, and a statement telling when the temperature fell most rapidly and when it rose most rapidly.



5. John was out of school one week because he was sick with the mumps. On Monday noon the doctor called, and took John's temperature by placing a small thermometer under his tongue. Instead of being 98.6°, which is the temperature of a healthy person, John's temperature was 99.6°; and so the doctor said it was 1° above normal. He asked John's mother to take the temperature at 8 A.M., noon, and 8 P.M. each day. She did so, and kept the record on a temperature chart, which is usually made on specially ruled paper like that here shown. On Thursday evening, when the doctor called, he looked at the chart, and said that the temperature was so nearly normal that John would be all right the next day. From the chart find John's temperature at 8 A.M. and at 8 P.M. on Tuesday and on Wednesday; at noon and at 8 P.M. on Thursday; and at 8 A.M., noon, and 8 P.M. on Friday.

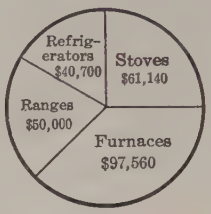


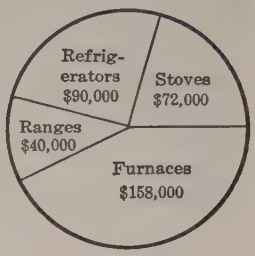
6. The Weather Bureau uses self-recording thermometers that trace graphs on moving strips of paper. In one of our Northern cities on a winter day the graph from midnight to 8 P.M. was as here shown. State the temperature at midnight; at noon; at about 2 30 A.M.; at 6 P.M. How many degrees did the temperature rise between midnight and 1 P.M.?



Circular Graph. In showing certain kinds of statistics, we sometimes use *circular graphs*, like the ones on this page.

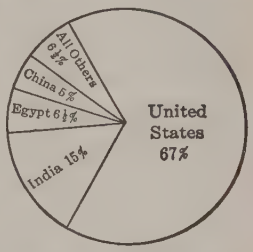
For example, the two circles in this graph make a much more effective comparison than does the table below of the sales, in two consecutive years, of a factory which makes furnaces, ranges, refrigerators, and stoves.





Articles	First Year	Second Year
Furnaces	$97,560	$158,000
Ranges	50,000	40,000
Refrigerators	40,700	90,000
Stoves	61,140	72,000

Since the second circle is considerably larger than the first, the graph shows that the total business has increased. It also shows that although the sale of stoves in the second year was greater, there was a relative shrinkage, and that the sale of ranges fell off both actually and relatively the second year.



A circular graph is particularly convenient when we are dealing with per cents. For example, this graph shows the per cents of the world's mill supply of cotton contributed by each country in a recent year. It tells us at a glance that the United States contributed considerably more than half the supply of the whole world. Although the construction of circular graphs is too difficult for us at this time, we can easily learn how to read the stories that such graphs tell.

TELLING STORIES BY GRAPHS

1. The distances of a train from its starting point in a run of 10 min. were measured and were found to be as follows:

Minutes	1	2	3	4	5	6	7	8	9	10
Miles	0.2	0.5	0.9	1.5	2.2	3	3.8	4.6	5.1	5.5

Draw a curve-line graph and tell what it shows about the speed of the train during this time.

2. The height of a growing plant, which was measured on eight consecutive days in June, was found to be as follows:

Day of the month	1	2	3	4	5	6	7	8
Height in inches	3.1	4.2	5.2	6.1	6.8	7.5	8.2	8.7

Draw a curve-line graph showing the growth of the plant, and from it determine in what part of the period the plant grew most rapidly.

3. Between 6 A.M. and noon on a rainy day in April the amount of rain which had fallen was found to be as follows:

Time	6 A.M.	7 A.M.	8 A.M.	9 A.M.	10 A.M.	11 A.M.	Noon
Inches	0.1	0.4	0.6	0.6	0.8	1.1	1.5

Draw a curve-line graph showing the variation in rainfall.

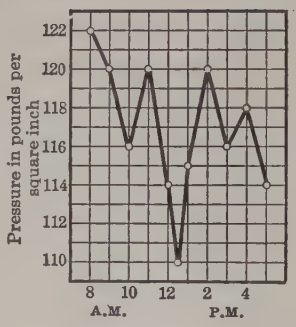
4. The population of the earth in millions, by continents, is approximately as follows: Africa, 143; North America, 140; South America, 56; Asia, 873; Australasia, 16; Europe, 465. Draw a graph, using that kind which you think tells the story most effectively.

5. A book on health gives the following average heights, in inches, for a growing boy between the ages of 5 yr. and 15 yr.:

Age . . .	5	6	7	8	9	10	11	12	13	14	15
Height .	42	44	46	49	52	54	56	58	61	63	68

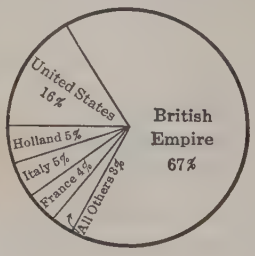
Draw a curve-line graph and from it determine in what part of this period the average boy grows most rapidly.

6. At 8 A.M. in a certain mill the steam gauge on the boiler showed a pressure of 122 lb. to the square inch. The pressure varied through the day as shown by this graph. Write the pressure for every hour from 8 A.M. to 5 P.M. and also for 12 30 P.M. Why was the pressure so high at 8 A.M.? Give a probable reason why the pressure was so low at 12 30 P.M.



7. Tom took a test in arithmetic every week for 10 wk. His scores for the number of "rights" in these tests, beginning with the first week, were as follows: 24, 26, 34, 30, 36, 36, 38, 44, 42, 46. Represent Tom's progress in arithmetic by means of a broken-line graph.

8. In a recent year the relative standing of the countries of the world having ships of over 15,000 T. was as shown in this circular graph. Write a statement of what the graph tells you about the standing of our country as compared with the British Empire; with Holland and Italy; with France; about the standing of all countries as compared with the British Empire.

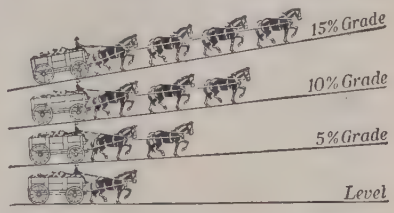


PICTORIAL GRAPHS

1. The following table shows the power required to haul 1 T. on various types and grades of roads, a 5% grade being one that rises 5 ft. in every 100 ft. measured horizontally:

Grade	Macadam Road	Gravel Road	Sand or Earth Road
Level	50 lb.	140 lb.	240 lb.
1%	70 lb.	160 lb.	260 lb.
2%	90 lb.	180 lb.	280 lb.
5%	150 lb.	240 lb.	340 lb.
10%	250 lb.	340 lb.	440 lb.
15%	350 lb.	440 lb.	540 lb.

This illustration, which shows another kind of graph, the *pictorial graph*, represents the increase of power that is necessary to haul the same load on various grades of a gravel road. From the above table find out whether this graph tells a true story, considering, of course, any fraction of a horse as a whole horse in determining the number needed to haul a given load on each of the different grades.



2. The power required to haul 1 T. on level macadam is what part of the power required on a 10% macadam grade? If a horse can haul 5000 lb. on level macadam, how much can it haul with the same effort up a 10% macadam grade?

3. A good horse can haul about 2000 lb. on an improved level gravel road. At 56 lb. to the bushel, how many bushels of corn can two horses haul over this road?

4. An "8–16 horse-power" tractor has about the same power as 8 work horses; but since horses cannot be used continuously, a tractor can do as much in 100 da. as 8 horses can do in 1 yr. The following pictures and table illustrate an effective way of comparing the relative cost of maintenance:

8 Horses for 1 yr.

Tractor for 100 da.

$53.76	Interest on investment	$43.50
80.00	Depreciation	145.00
18.16	Harness depreciation	None
16.32	Shoeing or repairs	36.25
701.76	Feed or fuel	154.00
None	Lubricating oil	35.00
156.56 (Feeding, etc.)	Labor (Oiling, etc.)	25.00
4.24	Miscellaneous	None

Find the cost of maintaining 8 horses for 1 yr. and of maintaining the tractor for 100 da.

5. If an 8–16 horse-power tractor is used, how much is saved on work requiring the use of 8 horses for 1 yr.?

6. When the tractor is not in use, the only expenses are depreciation and interest on investment. For the horses, however, the expenses are practically the same as given in the table. The cost of one day's idleness for the horses is how much greater than that for the tractor?

7. If the tractor is used only 100 da. in the year, what is the daily cost of fuel, lubricating oil, and oiling (labor)? How does this compare with the cost of feed and feeding (labor) per day for the 8 horses?

VI. POWERS AND ROOTS

A SILENT READING LESSON

When we studied the area of a circle, we used the *square* of the radius, and found that the square of 3 means 3×3, or 9. We often write 3^2 (read "3 square") for the square of 3, which is also called the *second power* of 3.

We may also have the *third power* (often called the *cube*) of a number. For example, the third power of 2 is $2 \times 2 \times 2$, or 8, which shows that 2 is taken three times as a *factor*. We often write 2^3 (read "2 cube") for the third power of 2.

In the same way we may have the fourth power of a number, and other powers. The power is always indicated by the *exponent*; for example,

$3^2 = 3 \times 3 = 9$, where the exponent is 2;

$4^3 = 4 \times 4 \times 4 = 64$, where the exponent is 3;

$5^4 = 5 \times 5 \times 5 \times 5 = 625$, where the exponent is 4.

We may also have powers of fractions; for example,

$(\frac{1}{2})^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, the square of $\frac{1}{2}$;

$(\frac{1}{2})^3 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$, the cube of $\frac{1}{2}$.

Similarly, we may have powers of decimals; for example,

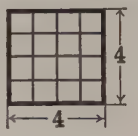
$0.1^2 = 0.1 \times 0.1 = 0.01$, the square of 0.1;

$1.2^3 = 1.2 \times 1.2 \times 1.2 = 1.728$, the cube of 1.2.

From what we have learned, we see that the area of a square may be obtained by taking the second power of its side. For example, if the side of a square is 5 in., the area is 5^2 sq. in., or 25 sq. in.

Similarly, the volume of a cube may be obtained by taking the third power of its edge. For example, if the edge of a cube is 3 in., the volume is 3^3 cu. in., or 27 cu. in.

Square Root. If a square is 4 units on a side, it has an area of 4×4, or 16 square units. Since 16 is the square of 4, we can easily see why 4 is called the *square root* of 16; that is,



The square root of a number is one of its two equal factors.

The square root of 16 is indicated by the symbol $\sqrt{16}$, which is read "the square root of 16."

Approximate Square Roots. A number like 16 is called a *perfect square,* but 10 is not a perfect square. Just as we say, however, that $\frac{2}{3} = 0.667$, to the nearest 0.001, so we may say that the square root of 10 is 3.16, to the nearest 0.01. If we find the value of 3.16^2, we shall see that it is nearly 10.

Square Root of a Perfect Square. Square roots of perfect squares may often be found by factoring.

For example, in finding the square root of 441, we separate 441 into its factors, as here shown, and then separate these factors into the two equal groups, 3×7 and 3×7. Hence we see that

$$\sqrt{441} = \sqrt{21 \times 21} = 21.$$

$3\overline{)441}$
$3\overline{)147}$
$7\overline{)49}$
7

FINDING SQUARES AND SQUARE ROOTS

Numbers 1 to 24, oral

State the squares of the following numbers:

1. 2.	3. 6.	5. 10.	7. 3.	9. $\frac{1}{2}$.	11. 0.2.
2. 4.	4. 8.	6. 11.	8. 7.	10. $\frac{1}{3}$.	12. 1.1.

State the square roots of the following numbers:

13. 9.	15. 16.	17. 121.	19. 169.	21. 1.44.	23. 4900.
14. 64.	16. 81.	18. 144.	20. 225.	22. 0.64.	24. 2500.

Find the squares of the following numbers:

25. 2.3.	27. $2\frac{1}{2}$.	29. 2.4.	31. 0.7.	33. 1200.
26. 4.7.	28. $3\frac{1}{3}$.	30. 3.6.	32. 0.81.	34. 3602.

By factoring, find the square roots of the following numbers:

35. 196.	37. 625.	39. 729.	41. 1296.	43. 12.25.
36. 484.	38. 324.	40. 576.	42. 1089.	44. 40.96.

Find the sides of squares whose areas are as follows:

45. 4.84 sq. in.	47. 4.41 sq. in.	49. 5929 sq. yd.
46. 1.96 sq. ft.	48. 4356 sq. in.	50. 10.89 sq. yd.

Find the perimeters of squares whose areas are as follows:

51. 6561 sq. ft.	53. 19,600 sq. ft.	55. 16,384 sq. in.
52. 12,100 sq. in.	54. 146.41 sq. in.	56. 16,900 sq. ft.

57. At the corner of a square piece of land that contains 57,600 sq. ft., a square lot containing 6400 sq. ft. is laid out. To a convenient scale, draw a plan of the land and find the perimeter of each piece.

58. If a square lot contains 18,225 sq. ft., what is the perimeter? What is the perimeter of a square lot of four times this area? The perimeter of the second lot is how many times the perimeter of the first?

59. The combined area of two square building lots which front on the same street is 8900 sq. ft., and the area of the larger lot is 6400 sq. ft. What is the frontage of each lot?

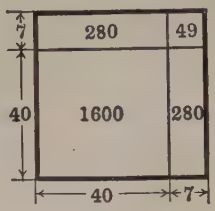
60. If a square lot has an area of 28,900 sq. ft., what is the perimeter? What is the perimeter of a square lot that contains nine times this area? The perimeter of the second lot is how many times the perimeter of the first? The perimeter of the first lot is what part as long as that of the second?

Square of the Sum of Two Numbers. Since $47 = 40 + 7$, the square of 47 may be obtained in the following manner:

$$\begin{array}{rr} & 40 + 7 \\ & 40 + 7 \\ \hline & (40 \times 7) + 7^2 \\ 40^2 + & (40 \times 7) \\ \hline 40^2 + 2 \times (40 \times 7) + 7^2 & \end{array}$$

$$= 1600 + 560 + 49$$

$$= 2209.$$



Every number consisting of two or more figures may be regarded as composed of tens and units. That is, as shown by both the multiplication and the figure,

The square of a number contains the square of the tens, plus twice the tens times the units, plus the square of the units.

This principle, which is very important in finding square roots, should be clearly understood before proceeding further.

Locating the Root. Since $1 = 1^2$, $100 = 10^2$, and $10{,}000 = 100^2$, it is evident that the square root of any number between 1 and 100 lies between 1 and 10, and that the square root of any number between 100 and 10,000 lies between 10 and 100. In other words, considering only whole numbers which are perfect squares, the square root of a number of one or two figures is a number of one figure; the square root of a number of three or four figures is a number of two figures; and so on.

Separating into Periods. We thus see that if a whole number is separated into periods of two figures each, beginning at the right, the number of figures in the integral part (the part that is a whole number) of the square root is the same as the number of periods. The left-hand period may, of course, have one figure or two figures. For example, the square root of 47 61 has two figures, and that of 1 61 29 has three.

Finding a Square Root. In finding the square root of 2209, for example, we first separate the number into periods of two figures each, beginning at the right. In so doing, we see that there are two periods in the number, and hence there are two figures in the integral part of the square root.

The first period, 22, contains the square of the tens' figure of the root. Since the greatest square in 22 is 16, then 4, the square root of 16, is the tens' figure.

Subtracting 16, the square of the tens, as shown, 609 must contain twice the tens times the units, plus the square of the units. We then divide 609 by 2×4 tens, or 80, to obtain the approximate units' figure, which we find to be 7.

$$\begin{array}{r|l} 22\ 09 & 47 \\ 16 & \\ \hline 80 & 6\ 09 \\ 87 & 6\ 09 \\ \hline \end{array}$$

Since in squaring $40 + 7$ we had $2 \times 40 \times 7 + 7^2$, which we may write as $(2 \times 40 + 7) \times 7$, we add 7 to 80 and multiply the result by 7, which gives us 609. Subtracting this result, there is no remainder; that is, $\sqrt{2209} = 47$.

We may check the work by finding that $47^2 = 2209$.

FINDING A SQUARE ROOT

Find the square root of each of the following numbers:

1. 144.	**3.** 625.	**5.** 900.	**7.** 3249.	**9.** 3969.
2. 169.	**4.** 729.	**6.** 3600.	**8.** 3721.	**10.** 5041.

Find the side of a square that has an area of

11. 6724 sq. ft.	**13.** 9025 sq. ft.	**15.** 7921 sq. ft.
12. 7569 sq. ft.	**14.** 9409 sq. ft.	**16.** 6889 sq. yd.

Find the square root of each of the following fractions by taking the square root of each term:

17. $\frac{144}{529}$. **18.** $\frac{169}{225}$. **19.** $\frac{961}{1024}$. **20.** $\frac{1089}{1156}$. **21.** $\frac{1936}{5041}$.

Square Root with Decimals. In finding the square root of a decimal, for example, of 151.29, the first step is to separate the number into periods, beginning at the decimal point. We then proceed as shown in the following solution, which should be compared step by step with the summary given below:

Given number		1 51.29 | 12.3	square root
Square of 1		1	
First trial divisor	20	51	first remainder
First complete divisor	22	44	from 2×22
Second trial divisor	24.0	7.29	second remainder
Second complete divisor	24.3	7.29	from 0.3×24.3

Check. $1.23 \times 1.23 = 151.29$.

If the number whose root we wish to find is not a perfect square, we may annex pairs of zeros at the right and continue until we have the desired number of decimal places.

Summary of Square Root. To find a square root,

Separate the number into periods of two figures each, beginning at the decimal point.

Find the greatest square in the left-hand period and write its square root as the first figure of the required root.

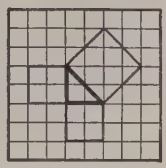
Subtract the square from the left-hand period, and to the remainder annex the next period.

Double the part of the root already found and use it as the trial divisor to find the second figure of the root. Add this result to the trial divisor and multiply by the number represented by the second figure of the root.

Subtract this result, bring down the next period, and proceed as before until all the periods have been used, or until the desired number of decimal places is reached.

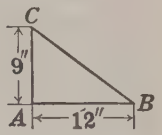
Square of the Hypotenuse. In a right triangle the side opposite the right angle is called the *hypotenuse* of the triangle.

Suppose that on a floor made of square tiles we mark out a right triangle and on each of the sides mark out a square, as here shown. We then see that the square on the hypotenuse contains eight small squares while each square on a side contains four such squares. In geometry it is proved that in any right triangle



The square of the hypotenuse is equal to the sum of the squares of the other two sides.

For example, in finding the length of the hypotenuse BC of the right triangle here shown, we have



$$(BC)^2 = 12^2 + 9^2 = 144 + 81 = 225,$$

and $$BC = \sqrt{225} = 15.$$

That is, the hypotenuse is 15 in. long.

RIGHT TRIANGLES

1. In the triangle ABC, above, find the length of the hypotenuse when $AB = 24$ ft. and $AC = 9$ ft.

2. From the relation $(BC)^2 = (AB)^2 + (AC)^2$ in the triangle ABC, above, we have $(AB)^2 = (BC)^2 - (AC)^2$. Find the length of AB when $BC = 105$ in. and $AC = 63$ in.

3. If the side of a square is 17 in., what is the length of the diagonal to the nearest 0.1 in.?

4. Find, to the nearest 0.1 in., the side of a square whose area is 3 sq. in.; whose area is 6 sq. in.

5. What, to the nearest 0.01 in., is the length of the diagonal of a square whose area is 81 sq. in.? of one that has an area four times as large?

6. Find the length of the diagonal of the floor of a hall that is 51 ft. by 68 ft.

7. Find, to the nearest 0.01 ft., the length of the diagonal of a square that contains 4 sq. ft.

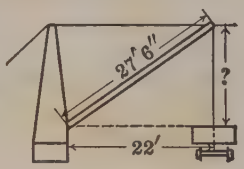
8. The two sides of a right triangle are 20 in. and 30 in. Find the length of the hypotenuse to the nearest 0.01 in.

9. The two sides of a right triangle are 57 in. and 76 in. Find the length of the hypotenuse.

10. What is the length of a wire drawn taut from the top of a 100-foot flagpole to a spot 75 ft. from the base?

11. In order to hold a telegraph pole vertical, a wire is stretched from a point on the pole 18 ft. above the ground to a stake 13 ft. 6 in. from the foot. How long is the wire?

12. The length of the arm of a derrick for unloading coal is 27 ft. 6 in. When the arm swings over a car 22 ft. from its foot, as here shown, how far is the top of the arm above the car?



Find, to the nearest 0.01, the length of the hypotenuse of a right triangle in which the other two sides are

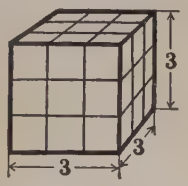
13. 32 ft., 28 ft.
14. 40 ft., 75 ft.
15. 321 ft., 428 ft.
16. 36 in., $67\frac{1}{2}$ in.
17. 72 ft., 135 ft.
18. $56\frac{1}{2}$ in., $72\frac{1}{2}$ in.

19. Find the diagonal of a square that contains 9409 sq. ft.

20. The foot of a 45-foot ladder is placed $13\frac{1}{2}$ ft. away from a building against which the top of the ladder rests. How high does the ladder reach on the building?

21. How far from the wall of a house must the foot of a 25-foot ladder be placed in order that the top of the ladder may just touch a window sill 20 ft. from the ground?

Cube Root. If a cube has an edge of 3 units, it has a volume of $3 \times 3 \times 3$, or 27 cubic units. Since 27 is the cube of 3, we can easily see why 3 is called the *cube root* of 27. That is, we see that



The cube root of a number is one of its three equal factors.

The cube root of 27 is indicated by the symbol $\sqrt[3]{27}$, which is read "the cube root of 27."

A number like 27 is a *perfect cube*, but 28 is not. We use approximate cube roots, however, just as we use approximate square roots; that is, $\sqrt[3]{28} = 3.04$, to the nearest 0.01.

Cube Root of a Perfect Cube. The cube root of a perfect cube may often be found by factoring.

For example, in finding the cube root of 216, we factor as here shown, and see that the factors form the three equal groups 3×2; that is,

$$\sqrt[3]{216} = \sqrt[3]{6 \times 6 \times 6} = 6.$$

2)216
2)108
2)54
3)27
3)9
3

In the same way, we can show that

$$\sqrt[3]{\tfrac{1}{27}} = \sqrt[3]{\tfrac{1}{3} \times \tfrac{1}{3} \times \tfrac{1}{3}} = \tfrac{1}{3}.$$

FINDING CUBE ROOTS

Find each of the following cube roots by factoring:

1. $\sqrt[3]{125}$. 3. $\sqrt[3]{729}$. 5. $\sqrt[3]{512}$. 7. $\sqrt[3]{1728}$. 9. $\sqrt[3]{4096}$.
2. $\sqrt[3]{\frac{1}{125}}$. 4. $\sqrt[3]{\frac{1}{729}}$. 6. $\sqrt[3]{\frac{1}{512}}$. 8. $\sqrt[3]{\frac{1}{1728}}$. 10. $\sqrt[3]{5832}$.

11. Find the edge of a cube whose volume is 13.824 cu. in.

Find the edge of a cube whose volume is

12. 343 cu. in. 14. 2744 cu. in. 16. 15,625 cu. ft.
13. 1000 cu. in. 15. 3375 cu. ft. 17. 21,952 cu. ft.

Using a Table of Roots. There are methods of finding the cube root of any number, just as we have seen that there is a method of finding the square root. These methods, however, are seldom used in ordinary work. When we wish to find square roots or cube roots, we generally use a table like the one given on page 349. The following examples show how this table can be used:

1. Find the square root of 39.

Looking for 39 in the column headed "No." (number), we follow along this line to the column marked "Square Roots." We there find 6.245; that is, $\sqrt{39} = 6.245$, to the nearest 0.001.

2. Find the cube root of 88.

On line 88 in the column marked "Cube Roots," we find 4.448; that is, $\sqrt[3]{88} = 4.448$, to the nearest 0.001.

3. Find the square root of 841.

The column of numbers ("No.") goes only to 100. We therefore look to see if 841 is in the column marked "Squares." We thus find that $841 = 29^2$, and so $\sqrt{841} = 29$.

4. Find the cube root of 884,736.

In the column marked "Cubes" we find that $884{,}736 = 96^3$, and so $\sqrt[3]{884{,}736} = 96$.

5. Find the square root of 2000.

Although 2000 is not given in the column marked "Squares," we may find $\sqrt{2000}$ by observing that $\sqrt{2000} = \sqrt{100 \times 20}$. Then, from the table, $\sqrt{100} = 10$ and $\sqrt{20} = 4.472$, and so $\sqrt{2000} = 10 \times 4.472 = 44.72$.

6. Find the square root of 0.45.

First writing $\sqrt{0.45} = \sqrt{0.01 \times 45}$, we see that $0.01 = 0.1 \times 0.1$, and, from the table, we find that $\sqrt{45} = 6.708$. Hence we have $\sqrt{0.45} = 0.1 \times 6.708 = 0.6708$.

POWERS AND ROOTS

No.	Squares	Cubes	Square Roots	Cube Roots	No.	Squares	Cubes	Square Roots	Cube Roots
1	1	1	1.000	1.000	51	2 601	132 651	7.141	3.708
2	4	8	1.414	1.260	52	2 704	140 608	7.211	3.733
3	9	27	1.732	1.442	53	2 809	148 877	7.280	3.756
4	16	64	2.000	1.587	54	2 916	157 464	7.348	3.780
5	25	125	2.236	1.710	55	3 025	166 375	7.416	3.803
6	36	216	2.449	1.817	56	3 136	175 616	7.483	3.826
7	49	343	2.646	1.913	57	3 249	185 193	7.550	3.849
8	64	512	2.828	2.000	58	3 364	195 112	7.616	3.871
9	81	729	3.000	2.080	59	3 481	205 379	7.681	3.893
10	100	1 000	3.162	2.154	60	3 600	216 000	7.746	3.915
11	121	1 331	3.317	2.224	61	3 721	226 981	7.810	3.936
12	144	1 728	3.464	2.289	62	3 844	238 328	7.874	3.958
13	169	2 197	3.606	2.351	63	3 969	250 047	7.937	3.979
14	196	2 744	3.742	2.410	64	4 096	262 144	8.000	4.000
15	225	3 375	3.873	2.466	65	4 225	274 625	8.062	4.021
16	256	4 096	4.000	2.520	66	4 356	287 496	8.124	4.041
17	289	4 913	4.123	2.571	67	4 489	300 763	8.185	4.062
18	324	5 832	4.243	2.621	68	4 624	314 432	8.246	4.082
19	361	6 859	4.359	2.668	69	4 761	328 509	8.307	4.102
20	400	8 000	4.472	2.714	70	4 900	343 000	8.367	4.121
21	441	9 261	4.583	2.759	71	5 041	357 911	8.426	4.141
22	484	10 648	4.690	2.802	72	5 184	373 248	8.485	4.160
23	529	12 167	4.796	2.844	73	5 329	389 017	8.544	4.179
24	576	13 824	4.899	2.884	74	5 476	405 224	8.602	4.198
25	625	15 625	5.000	2.924	75	5 625	421 875	8.660	4.217
26	676	17 576	5.099	2.962	76	5 776	438 976	8.718	4.236
27	729	19 683	5.196	3.000	77	5 929	456 533	8.775	4.254
28	784	21 952	5.292	3.037	78	6 084	474 552	8.832	4.273
29	841	24 389	5.385	3.072	79	6 241	493 039	8.888	4.291
30	900	27 000	5.477	3.107	80	6 400	512 000	8.944	4.309
31	961	29 791	5.568	3.141	81	6 561	531 441	9.000	4.327
32	1 024	32 768	5.657	3.175	82	6 724	551 368	9.055	4.344
33	1 089	35 937	5.745	3.208	83	6 889	571 787	9.110	4.362
34	1 156	39 304	5.831	3.240	84	7 056	592 704	9.165	4.380
35	1 225	42 875	5.916	3.271	85	7 225	614 125	9.220	4.397
36	1 296	46 656	6.000	3.302	86	7 396	636 056	9.274	4.414
37	1 369	50 653	6.083	3.332	87	7 569	658 503	9.327	4.431
38	1 444	54 872	6.164	3.362	88	7 744	681 472	9.381	4.448
39	1 521	59 319	6.245	3.391	89	7 921	704 969	9.434	4.465
40	1 600	64 000	6.325	3.420	90	8 100	729 000	9.487	4.481
41	1 681	68 921	6.403	3.448	91	8 281	753 571	9.539	4.498
42	1 764	74 088	6.481	3.476	92	8 464	778 688	9.592	4.514
43	1 849	79 507	6.557	3.503	93	8 649	804 357	9.644	4.531
44	1 936	85 184	6.633	3.530	94	8 836	830 584	9.695	4.547
45	2 025	91 125	6.708	3.557	95	9 025	857 375	9.747	4.563
46	2 116	97 336	6.782	3.583	96	9 216	884 736	9.798	4.579
47	2 209	103 823	6.856	3.609	97	9 409	912 673	9.849	4.595
48	2 304	110 592	6.928	3.634	98	9 604	941 192	9.899	4.610
49	2 401	117 649	7.000	3.659	99	9 801	970 299	9.950	4.626
50	2 500	125 000	7.071	3.684	100	10 000	1 000 000	10.000	4.642

USING THE TABLE OF ROOTS ON PAGE 349

Using the table, find the square root of

1. 25.	3. 64.	5. 7.	7. 29.	9. 67.
2. 49.	4. 81.	6. 13.	8. 47.	10. 91.

Using the table, find the cube root of

11. 8.	13. 64.	15. 17.	17. 28.	19. 51.
12. 27.	14. 13.	16. 19.	18. 46.	20. 99.

Using the table, find the square root of

21. 0.97.	23. 961.	25. 5184.	27. 7744.	29. 9025.
22. 0.56.	24. 2116.	26. 6889.	28. 8649.	30. 9604.

Using the table, find the cube root of

31. 5832.	33. 19,683.	35. 85,184.	37. 531,441.
32. 9261.	34. 42,875.	36. 97,336.	38. 804,357.

39. The volume of a cube is 2197 cu. in. Using the table of roots, find the length of an edge.

40. Find the length of an edge of a cube which has a volume of 39 cu. ft.

41. The area of one face of a cube is 78 sq. in. Find the volume of the cube.

42. The total area of all six faces of a cube is 36 sq. in. Find the volume of the cube.

43. From the table on page 349, what is the square root of 1764? of 1849? The square root of 1775, which lies between these numbers, is therefore 42 plus some fraction. Find the figures in the integral part of the square root of 230.

44. Find the integral part of the square root of 509; of 850; of 6730; of 8675.

VII. LETTERS TO REPRESENT NUMBERS

A SILENT READING LESSON

We live in a more rapid age than our great-grandfathers did. They never saw an automobile, an airplane, or a railroad train that could go a mile a minute. They never dreamed of rapid communication by telegraph, telephone, or radio. They never heard of telling a story by means of a motion picture. They traveled about cities in a horse car instead of an electric car; they wrote letters with a pen instead of a typewriter; and their newspapers were printed by hand, from type that was set by hand.

It is the same way in arithmetic. When our great-grandfathers went to school they learned a rule which was stated like this:

To find the number of square units in the area of any rectangle, multiply the number of linear units in the base by the number of linear units in the height.

A little later this rule was shortened to read:

The area of a rectangle is the base times the height.

This form of the rule was then shortened to read:

$$Area = base \times height.$$

It was then seen that it would be easier to write simply

$$A = b \times h.$$

And so it comes about that today we write this rule as

$$A = b \cdot h,$$

or, more often, in the simpler form

$$A = bh,$$

in which it is understood that when two letters appear together, like bh, their values are to be multiplied.

Arithmetic Shorthand. From the reading lesson on page 351 we see that we can make up a kind of shorthand for arithmetic by using letters to represent numbers.

We can also shorten still more the work of making computations in arithmetic. For example, in finding the area of a rectangle 8 in. long and 3 in. wide, people used to write

$$8 \times 3 \text{ sq. in.} = 24 \text{ sq. in.}$$

When we use arithmetic shorthand we do not need labels like *ft.*, *lb.*, or *sq. in.* We know that the rule for the area is

$$A = bh,$$

and we therefore write the work, without using any labels, simply as

$$A = 8 \times 3 = 24.$$

Our common sense then tells us that this means that the area is 24 sq. in. Similarly, if the rectangle had been 8 ft. long and 3 ft. wide, we should have known that the area was 24 sq. ft.

Letters Represent Numbers. The letters used in this kind of shorthand always represent numbers. Thus,

n means some number,

$3\,n$ means three times this number,

and $\frac{1}{2}\,n$ means half the number.

Similarly, $2.5\,n + 7$ means that 7 is to be added to 2.5 times the number, and $0.3\,n - 5$ means that 5 is to be subtracted from 0.3 of the number.

We may also represent one number by a, another by b, another by c, and so on. Then $a + b$ means that the two numbers represented by these letters are to be added, and $\frac{a}{b + c}$ means that the value represented by a is to be divided by the value of $b + c$.

We speak of numbers represented by letters as *literal numbers* or *literal quantities.*

Adding Literal Numbers. Just as we add 2 ft. and 3 ft., so we can add $2\,a$ and $3\,a$; that is, just as

$$2 \text{ ft.} + 3 \text{ ft.} = 5 \text{ ft.},$$

so

$$2\,a + 3\,a = 5\,a.$$

This means that 2 times any number plus 3 times the same number (because it is represented by the same letter) is 5 times that number. We know that this is true because

$$2 \times 7 + 3 \times 7 = 5 \times 7,$$
$$2 \times 1\tfrac{1}{2} + 3 \times 1\tfrac{1}{2} = 5 \times 1\tfrac{1}{2},$$

and so on for whatever number we may let a represent.

Subtracting Literal Numbers. In the same way, just as

$$9 \text{ in.} - 4 \text{ in.} = 5 \text{ in.},$$

so

$$9\,n - 4\,n = 5\,n,$$

and

$$9\,a - 4\,a = 5\,a.$$

If n represents 6 and a represents 2.5, which we shorten to $n = 6$ and $a = 2.5$, these last two statements become

$$9 \times 6 - 4 \times 6 = 5 \times 6,$$

and

$$9 \times 2.5 - 4 \times 2.5 = 5 \times 2.5.$$

We know that this is true, because, as we can easily see by multiplying, the above expressions mean that

$$54 - 24 = 30,$$

and

$$22.5 - 10 = 12.5.$$

Meaning of This Shorthand. When we write $9\,n - 4\,n = 5\,n$, we are simply stating in shorthand the following sentence:

If from nine times any number we take four times the same number, we have five times the number.

Such a long statement might have been good enough for our great-grandfathers, but it is too long for us.

WORKING WITH LITERAL NUMBERS

Perform these additions rapidly:

1.	2.	3.	4.	5.
6 ft.	$6\,a$	$6\,n$	$6\,x$	$6\,w$
4 ft.	$4\,a$	$4\,n$	$4\,x$	$4\,w$

6. How much is $7\,a + 5\,a$? What does the addition become when $a = 3$? when $a = 6$? when $a = 9$?

Perform these additions rapidly:

7. 12 in. + 8 in.

8. 12 yd. + 8 yd.

9. $12\,x + 8\,x$.

10. $12\,y + 8\,y$.

11. $17\,n + 5\,n$.

12. $16\,a + 10\,a$.

Perform these subtractions rapidly:

13.	14.	15.	16.	17.
7 in.	$7\,x$	$7\,a$	$9\,n$	$25\,y$
2 in.	$2\,x$	$2\,a$	$2\,n$	$5\,y$

18. How much is $16\,x - 4\,x$? What does this subtraction become when $x = 1$? when $x = 3$? when $x = \frac{1}{2}$? when $x = \frac{1}{4}$?

Perform these additions rapidly:

19.	21.	23.	25.	27.
3 in.	$3\,b$	$3\,x$	7 ft.	$11\,a$
2 in.	$2\,b$	$2\,x$	2 ft.	$15\,a$
7 in.	$7\,b$	$7\,x$	8 ft.	$2\,a$

20.	22.	24.	26.	28.
$7\,a$	$3\,n$	$8\,x$	$7\,x$	$36\,y$
$3\,a$	$7\,n$	$7\,x$	$2\,x$	$42\,y$
$2\,a$	$2\,n$	$4\,x$	$8\,x$	$81\,y$

29. How much is $2\,a + 3\,a + 5\,a$? What does this become when $a = 3$? when $a = 5$? when $a = 1$?

30. How much is 7 in. + 2 in. − 3 in.? $7\,a + 2\,a - 3\,a$? $7\,x + 2\,x - 3\,x$? $7\,n + 2\,n - 3\,n$?

A SILENT READING LESSON

There are a few new words of which we need to know the meaning when using literal numbers.

For example, in the expression $3\,a$ we shall, for our present purposes, speak of the number 3 as the *coefficient* of a. If we study algebra next year, we shall find that coefficients are sometimes letters; that is, in the expression ax we may speak of a as the coefficient of x. If no coefficient is written, it is understood that the coefficient is 1; that is, a is the same as $1\,a$, and so, in cases like the following, we have

$$a + 2\,a = 3\,a,$$

and

$$5\,n - n = 4\,n.$$

In the expression $a + b$, the letters, which represent numbers as usual, are known as the *terms* of the expression. In the expression $a - b$, the terms are also a and b. Terms are always separated from one another by the + or − sign. Thus, in the expression $2\,a + 3\,b - c$ the terms are $2\,a$, $3\,b$, and c.

In the expression bh, which was given on page 351, b and h are *factors* of the expression; they are not terms, because they are not separated by the + or − sign. Thus, the factors of $2\,a$ are 2 and a, and the factors of $6\,ax$ are 6, a, and x. In such cases, we do not, for ordinary purposes, need to give the factors of a coefficient like 6.

If we know the value of b in the expression bh, but do not know the value of h, we speak of b as a *known quantity* and of h as an *unknown quantity*; that is, it is customary in this work to say known or unknown quantity instead of known or unknown number.

In work with literal expressions we may thus have known and unknown terms, known and unknown factors, known and unknown numbers, and known and unknown quantities.

WRITING LITERAL EXPRESSIONS

Write an expression that shall mean

1. The sum of any two numbers; for example, a and b.

2. The difference between any two numbers; for example, x and y with x the larger.

3. The difference, as in Ex. 2, but taking x as the smaller.

4. The product of any two numbers.

5. The quotient of any number divided by any other number, letting m represent the dividend and x represent the divisor and writing the result in the fractional form customarily used with literal numbers.

6. The quotient, as in Ex. 5, but taking m as the divisor and x as the dividend.

7. The sum of any three numbers.

8. The sum of twice one number and three times another number.

9. Five times one number less three times another number.

10. In Ex. 9, what is the value of your result if the first number is 8 and the second is 2? if the first is 3 and the second is 5? if the first is $\frac{1}{5}$ and the second is $\frac{1}{3}$?

11. What is the value of $2a$ when $a = 17\frac{1}{2}$? when $a = 1$? when $a = 0$? when $a = 32.7$?

12. What is the value of $\frac{1}{2}x$ when $x = 8$? when $x = 17$? when $x = 0$? when $x = 1.37$? when $x = \frac{1}{2}$?

13. What is the value of the expression $\frac{a+b}{c}$ when $a = 9$, $b = 6$, and $c = 3$? when $a = 7$, $b = 15$, and $c = 2$?

14. What is the value of $\frac{2a+3b-c}{3a-b+5c}$ when $a = 5$, $b = 2$, and $c = 1$? when $a = \frac{1}{2}$, $b = \frac{1}{3}$, and $c = 2$?

A SILENT READING LESSON

When we learned about the powers and roots of numbers, we found that the square of 3 means 3×3, or 9, and that we write 3^2 for "the square of 3." Just as in 3^2 the 2 is called the *exponent* of 3, so in a^2 the 2 is the exponent of a, and shows that a is to be raised to the *second power* or is to be *squared*.

Similarly, a^3 means that a is to be raised to the *third power*, or *cubed*. In this case, the exponent of a is 3.

Just as the expression x means the same as $1\,x$, so it also means x^1; that is, we never write the exponent 1.

In our work with numbers we also learned that, because $4^2 = 4 \times 4 = 16$, we call 4 the *square root* of 16, and because $2^3 = 2 \times 2 \times 2 = 8$, we call 2 the *cube root* of 8. Similarly, n^2 means $n \times n$ and x^3 means $x \times x \times x$, and so

n is the *square root* of n^2,

x is the *cube root* of x^3,

y is the *fourth root* of y^4,

and so on. We shall not in our work, however, need to use exponents larger than 3, or any roots except the square root and the cube root.

Thus, if we wish to represent the square of any number, we may write simply n^2; if we wish to represent twice the square of any number, we may write $2\,n^2$; and if we wish to represent half the square root of any number, we may write $\frac{1}{2}\sqrt{n}$; and so on.

Similarly, when studying the measurement of the circle, we found that the area of a circle is approximately $\frac{22}{7}$ times the square of the radius. We can now express this important relation in the more convenient shorthand, as

$$A = \tfrac{22}{7}\, r^2.$$

WRITING LITERAL EXPRESSIONS

1. Write a with the coefficient 3.

2. Write b with the exponent 2. What is the value of the result when $b = 3$? when $b = \frac{1}{2}$? when $b = 1$?

3. Write the sum of twice one number, represented by n, and 7 times the same number. What is the value of the result when $n = 6$? when $n = 1$? when $n = \frac{1}{2}$? when $n = 0$?

4. The letter c comes before the letter e. In $3\,a^2$ the coefficient is written first and the exponent last. Do you see a simple way of remembering that the coefficient comes first and the exponent comes last in an expression like this? If so, what is it?

5. In the expression $5\,m^3$, what is the coefficient? what is the exponent? Write an expression in which 7 is the coefficient and 2 is an exponent.

6. What is the value of $2\,n^2$ when $n = 2$? when $n = \frac{1}{2}$?

7. If the edge of a cube is 2 in., how many cubic inches are there in the volume of the cube? What is the volume when the edge is e?

8. In Ex. 7, what is the result of adding all the edges in the first case? in the second case?

9. Write the expression $3 \times a^2 \times b$ without using the symbol $\times$. One way is $3 \cdot a^2 \cdot b$, but there is a better way.

10. What is the value of your result in Ex. 9 when $a = 2$ and $b = 7$? when $a = 1$ and $b = 1$?

11. What is the value of x^2y when $x = 2$ and $y = 3$? when $x = 3$ and $y = 2$? when $x = 1$ and $y = 1$?

12. What is the value of $3\,xy$ when $x = 1$ and $y = 2$? when $x = \frac{1}{3}$ and $y = 1$?

13. What is the value of $2\,x + x^2$ when $x = 3$? when $x = 1$?

DRILL CHART IN LITERAL EXPRESSIONS

Perform the following operations, in the order in which the terms appear, writing each result as a single term:

1. $2x + 7x$.
2. $15m - 9m$.
3. $6a + 8a$.
4. $4x + 7x$.
5. $17x - x$.
6. $x + 19x$.
7. $2x + 19x$.
8. $8y - 4y$.
9. $8m + 9m$.
10. $17b - 5b$.
11. $2b + 11b$.
12. $23c - 19c$.
13. $5x + 24x$.
14. $26a - 17a$.
15. $a + 24a$.
16. $\frac{1}{2}a + \frac{1}{2}a$.
17. $\frac{3}{4}a - \frac{1}{4}a$.
18. $\frac{7}{8}x - \frac{1}{2}x$.
19. $\frac{5}{8}x + \frac{7}{8}x$.
20. $\frac{3}{4}y - \frac{1}{8}y$.
21. $2.7x + 4.8x$.
22. $3.1y - 0.7y$.
23. $9x - 3x + 2x$.
24. $9x - 3x - 2x$.
25. $15a + 3a + a$.
26. $15a - 3a + a$.
27. $15a + 3a - a$.
28. $15a - 0.3a - a$.
29. $25n + 7n + 4n$.
30. $2.5n + 7n - 0.4n$.
31. $25n - 7n + 4n$.
32. $25n - 7n - 4n$.
33. $1.5w + 10w + 0.5w$.
34. $15w + 10w - 5w$.
35. $15w - 10w - 5w$.
36. $23b + 24b + 8b$.
37. $32b - 24b + 8b$.
38. $3.2b + 2.4b - 0.8b$.
39. $3.2b - 2.4b - 0.8b$.
40. $4\frac{3}{4}x + 2\frac{1}{2}x - \frac{1}{4}x$.
41. $3.9m + 7.4m - 6.9m$.
42. $5.75a^2 + 2.97a^2 - a^2$.
43. $6.23m^2 + 3.98m^2 - 0.67m^2$.
44. $3.66n^3 + 6.27n^3 - 5.77n^3$.

Order of Operations. If we have an expression containing only multiplications, we may perform the multiplications in any order that we wish.

For example, in finding the value of abc when $a = 2$, $b = 3$, and $c = 5$, the product of 2, 3, and 5 is always 30, whatever order we use in multiplying the factors.

In a case like $\frac{ab}{xy}$, we first multiply a and b, we then multiply x and y, and finally we divide the first result by the second. We no longer make use of literal expressions like $ab \div xy$, because the fractional form given above is much better for practical purposes.

In a case involving only the signs + and −, we perform the operations in the order in which they come. For example, in the case of $4\,b + 3\,b - 2\,b + b$, we have

$$4\,b + 3\,b = 7\,b,$$

then $$7\,b - 2\,b = 5\,b,$$

and, finally, $$5\,b + b = 6\,b.$$

In a case involving the operations indicated by +, −, ×, and the fraction bar (÷), we first perform the operations of multiplication and division in the order in which they come, and then perform the others in the order in which they come. In our present work, however, we shall not find difficult cases that involve so many operations.

In the case of $3\,ab^2$, the exponent 2 refers only to the b, and so, if $a = 5$ and $b = 4$, we have

$$3\,ab^2 = 3 \times 5 \times 4^2 = 3 \times 5 \times 4 \times 4 = 240.$$

If there are parentheses, we do the work within them before doing anything else. For example, we have

$$30\,a - (12\,a - a) = 30\,a - 11\,a = 19\,a,$$

and $$5\,(2\,a + 3\,a - a) = 5\,(5\,a - a) = 5 \times 4\,a = 20\,a.$$

DRILL CHART IN LITERAL EXPRESSIONS

Write each of the following in its simplest form:

1. aa.	6. aaa.	11. aab.	16. $3\,aab$.
2. bb.	7. bbb.	12. acc.	17. $3\,abb$.
3. xx.	8. $3\,aa$.	13. aac.	18. $4\,xxy$.
4. $2\,xx$.	9. $4\,bb$.	14. ccc.	19. $4\,xyy$.
5. xxx.	10. abb.	15. xxy.	20. $4\,xxy^2$.

21. Divide 2×3 by 3; 2×5 by 5; $2\,x$ by x; $2\,x$ by 2.

Write each of the following in its simplest form:

22. $\frac{4\,x}{4}$.	24. $\frac{5\,a^2}{5}$.	26. $\frac{7\,ab}{7}$.	28. $\frac{9\,xy}{y}$.
23. $\frac{4\,x}{x}$.	25. $\frac{5\,a^2}{a^2}$.	27. $\frac{7\,ab}{b}$.	29. $\frac{9\,xy}{x}$.

30. If $x = 5$, what is the value of $2\,x$? of x^2?

Given that $x = 2$ and $y = 3$, find the value of

31. xy.	34. y^2.	37. x^2y.	40. $2\,x^2y$.
32. $\frac{x^2}{x}$.	35. $\frac{3\,y}{y}$.	38. $\frac{x^3y}{y}$.	41. $\frac{x^2y^2}{x^2}$.
33. $\frac{2\,x^2}{2}$.	36. $\frac{3\,y^2}{3}$.	39. $\frac{x^3y}{x^3}$.	42. $\frac{xy^2}{x}$.

Write each of the following in its simplest form:

43. $2\,xy + xy$.	47. $7\,a^2bx - 3\,a^2bx$.
44. $6\,x^2y - 2\,x^2y$.	48. $5\,a^2b^2x^2 + 5\,a^2b^2x^2$.
45. $5\,ab^2c + 2\,ab^2c$.	49. $5\,a^2b^2x^2 - 5\,a^2b^2x^2$.
46. $6\,a^2b^2c^2 - 3\,a^2b^2c^2$.	50. $64\,a^3b^2x - 28\,a^3b^2x$.

REVIEW DRILL CHART IN LITERAL EXPRESSIONS

Given that $a = 3$, $b = 2$, $c = 5$, *find the value of*

1. $a + b + c$.
2. $a + b - c$.
3. $a - b + c$.
4. $a^2 + 2\,b$.
5. $a^3 - 3\,c$.
6. $a^3 - c^2$.
7. $a^2 + c^2$.
8. $\frac{125\,abc}{125}$.
9. $\frac{125\,abc}{abc}$.
10. $\frac{125\,abc}{bc}$.
11. $2\,a + 3\,b$.
12. $2\,a - 3\,b$.
13. $ab + c$.
14. $ab - c$.
15. $4\,ab - 3\,c$.
16. $4\,c - 3\,ab$.
17. $a^2 - c$.
18. $\frac{100\,a^2b}{10}$.
19. $\frac{100\,a^2b}{b}$.
20. $\frac{100\,a^2b}{a^2}$.
21. $a^2 + b^2 + c^2$.
22. $a^2 + b^2 - c$.
23. $a^2 + c^2 - b^2$.
24. $2\,abc - 10$.
25. $2\,a^2bc - 15$.
26. $3\,ab^2c - 5$.
27. $5\,abc^2 - c^2$.
28. $\frac{40(a+b)}{40}$.
29. $\frac{40(a+b)}{a+b}$.
30. $\frac{4\,a(a+b)}{a}$.

Given that $a = 7$, $b = 6$, $x = 1$, *find the value of*

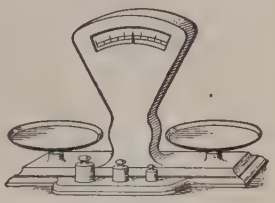
31. abx.
32. abx^2.
33. ab^2x.
34. a^2bx.
35. $15\,a^2b^2x$.
36. $\frac{20\,abx}{10}$.
37. $\frac{20\,abx}{bx}$.
38. $2\,a^2b$.
39. $20\,ab^2$.
40. $30\,ab^2x$.
41. $ab^2 - x$.
42. $a^2b^2 + x$.
43. $\frac{25\,a^2bx^2}{5}$.
44. $\frac{25\,a^2b^2x^2}{25\,a^2}$.
45. $a + b - x$.
46. $a^2 + b^2 - x^2$.
47. $a^2 - b^2 + x^2$.
48. $a^3 + b + 7\,x$.
49. $a + b^3 - 7\,x$.
50. $\frac{125\,a^3b^3x^3}{5\,b^3}$.
51. $\frac{125\,a^3b^3x^3}{125\,a^3b^3}$.

VIII. EQUATIONS

SILENT READING LESSON

Any statement of equality, like $3 + 7 = 10$, $x - 3 = 7$, or $A = lw$, is called an *equation*.

If we have a set of scales which balance like those here shown, and if we put equal weights on each side, the scales continue to balance. If we then add more equal weights, the scales still balance; if we take off equal weights, they still balance; if we put twice, or any other number of times, as much weight on each side, they still balance; and if we divide the weights by any number, so that we have only a part as much weight on each side, the scales still balance, as before. That is,



If a pair of scales balance, they continue to balance when we do the same thing (with respect to weight) to each side.

This is such a simple statement that each one of us, by using his common sense, can see that it is true without requiring any further proof.

For example, we can easily see that if we should put a 4-pound weight on each side, the scales would balance. If we should then add a 1-pound weight to each side, they would continue to balance, and if we should then take the 4-pound weight from each side, the scales would still balance. They would also continue to balance if we should multiply the weight on each side by 6 or if we should divide the weight on each side by 5.

What does the equation $12 = 12$ become if we add 6 to each side? if we take 3 from each side? if we multiply each side by 5? if we divide each side by 4?

Axioms. An equation is like a set of scales that balance, and the laws of equations, or the *axioms*, as they are called, are like the common-sense statement on page 363. For example,

1. If you hold 5¢ in each hand, and I add 2¢ more to each amount, how do the amounts then compare? Complete this sentence: *If equals are added to equals, the results* · · · .

2. If you hold 10 coins in each hand, and I take 3 coins from each, how do the coins remaining in each hand compare in number? Complete this sentence: *If equals are subtracted from equals, the* · · · .

3. If in your class the number of boys is equal to the number of girls, how will the numbers compare if each is doubled? Complete this sentence: *If equals are multiplied by the same number, the* · · · .

4. If two boards are of equal length, how do the lengths of one third of each board compare? Complete this sentence: *If equals are divided by the same number, the* · · · .

Stated concisely, the important axioms are as follows:

1. *If equals are added to equals, the results are equal.*

2. *If equals are subtracted from equals, the results are equal.*

3. *If equals are multiplied by the same number, the results are equal.*

4. *If equals are divided by the same number, the results are equal.*

Another axiom that is sometimes used is

5. *Like powers or like roots of equals are equal.*

In an equation we speak of the expressions on opposite sides of the sign of equality as the *sides* or *members* of the equation. We may then say, just as we did for the scales,

Whatever we do to one member of an equation, we must do to the other member.

First Axiom Applied. If we have $50¢$ and spend $10¢$, and someone then gives us $10¢$, we shall still have $50¢$; that is,

$$50¢ - 10¢ + 10¢ = 50¢.$$

Similarly, suppose that we are given an equation like

$$x - 10 = 30.$$

If we add 10 to both members, the results will be equal, and we have

$$x - 10 + 10 = 30 + 10,$$

or

$$x = 40.$$

$$x - 10 = 30.$$
$$x - 10 + 10 = 30 + 10.$$
$$x = 40.$$
Check. $40 - 10 = 30.$

To *solve* the equation $x - 10 = 30$ means that we are to find the value of the unknown quantity (that is, of x). We do this by finding, as above, that $x = 40$.

We may check our work by *substituting* 40 for x in the original equation $x - 10 = 30$. We then have

$$40 - 10 = 30;$$

that is, we have proved that our value for x is correct.

FIRST AXIOM APPLIED

Solve the following equations:

1. $x - 7 = 36$.	**6.** $x - \frac{1}{4} = \frac{1}{4}$.	**11.** $x - 0.5 = 2$.
2. $x - 8 = 58$.	**7.** $x - \frac{1}{2} = \frac{1}{2}$.	**12.** $x - 4.5 = 1$.
3. $x - 5 = 82$.	**8.** $x - \frac{1}{2} = \frac{1}{4}$.	**13.** $x - 0.2 = 2.8$.
4. $x - 6 = 41$.	**9.** $x - \frac{1}{8} = \frac{3}{8}$.	**14.** $x - 1.8 = 0.2$.
5. $x - 2 = 98$.	**10.** $x - \frac{7}{8} = \frac{3}{8}$.	**15.** $x - 3.7 = 5.8$.

16. If $x - 4 = 36$, what is the value of x?

17. Solve the equation $x - 8 = 32$ and check the result.

Solve the following equations and check the results:

18. $x - 8 = 39$. **19.** $x - 9 = 52$. **20.** $x - 7 = 64$.

Second Axiom Applied. Suppose that when 9 is added to a certain number, the result is 41. What is the number?

Letting x represent the number, the shorthand way of telling the story is the equation

$$x + 9 = 41.$$

Solving by subtracting 9 from both members, we have

$$x + 9 - 9 = 41 - 9,$$

or

$$x = 32.$$

$$x + 9 = 41.$$
$$x + 9 - 9 = 41 - 9.$$
$$x = 32.$$
Check. $32 + 9 = 41.$

After a little practice in solving equations, we omit the step

$$x + 9 - 9 = 41 - 9.$$

SECOND AXIOM APPLIED

Solve the following equations and check each result:

1. $x + 3 = 43.$
2. $x + 4 = 50.$
3. $x + 5 = 42.$
4. $x + 6 = 56.$
5. $x + 7 = 49.$
6. $y + 7 = 17.$
7. $y + 6 = 27.$
8. $y + 5 = 19.$
9. $n + 4 = 84.$
10. $n + 3 = 51.$
11. $z + 40 = 50.$
12. $z + 40 = 40.$
13. $z + 32 = 33.$
14. $m + 44 = 55.$
15. $m + 86 = 89.$

16. If $2x = x + 3$, what must we subtract from both members so as to have no x in the second member? If you subtract this from both members, what is left in the first member? in the second member? What is the value of x? Prove it.

Solve the following equations and check each result:

17. $2x = x + 9.$
18. $3x = 2x + 7.$
19. $5x = 4x + 11.$
20. $2x = x + 3.5.$
21. $2y = y + 2.8.$
22. $3z = 2z + \frac{1}{2}.$
23. $x + 9 = 19.$
24. $5n = 4n + 1.$
25. $2x - 9 = x.$
26. $2y - 7 = y.$
27. $2x + 9 = 3x.$
28. $3y + 1 = 4y.$

Third Axiom Applied. Suppose someone says that he is thinking of a certain number, and that half of it is 17. What is the number?

Letting x represent the number, the shorthand way of telling the story is the equation

$$\tfrac{1}{2}\,x = 17.$$

Multiplying these equals by 2, so as to change the $\frac{1}{2}$ to 1,

$$2 \times \tfrac{1}{2}\,x = 2 \times 17,$$

or

$$x = 34.$$

$$\tfrac{1}{2}\,x = 17.$$
$$2 \times \tfrac{1}{2}\,x = 2 \times 17.$$
$$x = 34.$$
Check. $\frac{1}{2}$ of $34 = 17$.

Instead of $\frac{1}{2}\,x$ we may write $\frac{x}{2}$ and proceed in the same way, omitting the second step after a little practice.

THIRD AXIOM APPLIED

Find the value of the letter in each of these equations:

1. $\frac{1}{2}\,x = 7$.	6. $\frac{1}{4}\,x = 7$.	11. $\frac{1}{5}\,x = 14$.	16. $\frac{1}{10}\,y = 2$.
2. $\frac{1}{2}\,x = 5$.	7. $\frac{1}{4}\,x = 9$.	12. $\frac{1}{6}\,x = 15$.	17. $\frac{1}{10}\,y = 2.1$.
3. $\frac{1}{3}\,x = 4$.	8. $\frac{1}{5}\,y = 8$.	13. $\frac{1}{8}\,x = 20$.	18. $\frac{1}{10}\,y = 0.3$.
4. $\frac{1}{3}\,x = 9$.	9. $\frac{1}{6}\,y = 11$.	14. $\frac{1}{9}\,x = 30$.	19. $\frac{1}{24}\,y = 10$.
5. $\frac{x}{4} = 60$.	10. $\frac{y}{8} = 100$.	15. $\frac{x}{10} = 140$.	20. $\frac{y}{20} = 200$.

21. If in the equation $\frac{1}{2}\,x + 3 = 16$ you subtract 3 from both members, what do you have? What is the value of x? Check the result.

Solve the following equations and check each result:

22. $\frac{1}{3}\,x + 2 = 12$.	25. $\frac{1}{8}\,x + 9 = 20$.	28. $\frac{1}{2}\,x + 20 = 21$.
23. $\frac{1}{4}\,x + 5 = 18$.	26. $\frac{1}{6}\,x + 7 = 30$.	29. $\frac{1}{3}\,x + 11 = 21$.
24. $\frac{1}{5}\,x + 7 = 17$.	27. $\frac{1}{10}\,x + 3 = 43$.	30. $\frac{1}{4}\,x + 11 = 30$.

Fourth Axiom Applied. Harry is thinking of a number which, when multiplied by 5, gives 75. What is the number?

Letting x represent the number, the shorthand way of telling the story is the equation

$$5x = 75.$$

Dividing both members of this equation by 5, we have

$$x = 15.$$

$5x = 75.$
$x = 15.$
Check. $5 \times 15 = 75.$

FOURTH AXIOM APPLIED

Find the value of the letter in each of these equations:

1. $3x = 6.$
2. $4x = 16.$
3. $5y = 40.$
4. $2x = 34.$
5. $3y = 90.$
6. $4y = 80.$
7. $5x = 20.$
8. $6x = 30.$
9. $7x = 63.$
10. $2.2x = 4.4.$
11. $3.3x = 6.6.$
12. $4.5x = 90.$

Solve the following equations and check each result:

13. $100x = 200.$
14. $117x = 585.$
15. $2.41x = 7.23.$
16. $3\frac{3}{4}x = 15.$
17. $15x = 3\frac{3}{4}.$
18. $2\frac{7}{8}x = 46.$
19. $1.213x = 8.491.$
20. $1.311x = 78.66.$
21. $0.742x = 667.8.$

22. Mary is thinking of a number which, when multiplied by $\frac{2}{3}$, becomes 18. What is the number?

23. In the equation $5x - 7 = 33$ what must be added to both members to leave $5x$ alone in the first member? What is the value of x? Check your result.

Solve the following equations and check each result:

24. $2x - 3 = 7.$
25. $3x - 2 = 13.$
26. $5x - 6 = 34.$
27. $6x - 2 = 16.$
28. $2x - \frac{1}{2} = \frac{3}{4}.$
29. $\frac{3}{4}x - \frac{1}{2} = 2.$
30. $5x - \frac{1}{8} = \frac{1}{2}.$
31. $8x - \frac{1}{5} = \frac{4}{5}.$
32. $\frac{1}{2}x - 1 = \frac{1}{2}.$

Further Work in Solving Equations. In some cases we combine two or more of the methods already learned, as in the following examples:

1. Solve the equation

$$\frac{x}{3}+7=11.$$

First subtracting 7, we have

$$\frac{x}{3}=4,$$

and then, multiplying by 3,

$$x=12.$$

$$\frac{x}{3}+7=11.$$

$$\frac{x}{3}=4.$$

$$x=12.$$

Check. $\frac{12}{3}+7=4+7$

$$=11.$$

Check. Substituting 12 for x in the given equation, we can show, as in the work above, that this result is correct.

2. Solve the equation

$$\frac{x}{2}+\frac{x}{3}-7=6.$$

Adding 7 to both members,

$$\frac{x}{2}+\frac{x}{3}=13.$$

If we now multiply both members by 2, $\frac{x}{2}$ will become x, and if by 3, $\frac{x}{3}$ will become x; that is, if we multiply both members by both 2 and 3, or 6, we have

$$\frac{x}{2}+\frac{x}{3}-7=6.$$

$$\frac{x}{2}+\frac{x}{3}=13.$$

$$3x+2x=78.$$

$$5x=78.$$

$$x=15.6.$$

$$6\times\frac{x}{2}+6\times\frac{x}{3}=6\times 13,$$

or $3x+2x=78;$

whence $5x=78,$

and, dividing by 5, $x=15.6.$

Check. $\frac{15.6}{2}+\frac{15.6}{3}-7=7.8+5.2-7=13-7=6.$

DRILL CHART IN EQUATIONS

Solve the following equations and check each result:

1. $x + 3 = 5$.
2. $x - 3 = 6$.
3. $3x = 9$.
4. $\frac{1}{3}x = 7$.
5. $2x = 6 - x$.
6. $2x = 6 + x$.
7. $7x = 2x + 5$.
8. $x - 9 = 12$.
9. $x + 9 = 12$.
10. $x - 12 = 9$.
11. $7x - 12 = 9$.
12. $3x - 12 = 9$.
13. $5x - 9 = 12$.
14. $\frac{1}{2}x + 7 = 16$.
15. $\frac{1}{2}x - 7 = 16$.
16. $\frac{1}{2}x - 16 = 7$.
17. $\frac{2x}{3} + 7 = 9$.
18. $\frac{8x}{3} - 5 = 35$.
19. $\frac{x}{2} + \frac{x}{3} = 12$.
20. $\frac{3y}{4} - \frac{2y}{3} = 17.2$.
21. $2x + 9 = 29$.
22. $2x - 9 = 29$.
23. $2x - 29 = 9$.
24. $3x - 29 = 1$.
25. $3x + 8 = 29$.
26. $5x + 6 = 18$.
27. $5x - 6 = 14$.
28. $5x - 14 = 6$.
29. $14 = 6 + 8x$.
30. $31 = 3x + 22$.
31. $17x - 31 = 20$.
32. $14x - 17 = 11$.
33. $2.6x - 2.2 = 3$.
34. $1.6x - 3.6 = 4.4$.
35. $4.1y - 1.75 = 10.55$.
36. $7.5y - 13 = 18.5$.
37. $\frac{3x}{4} - 7 = 14$.
38. $\frac{2y}{3} - 9 = 27$.
39. $\frac{2n}{5} + \frac{3n}{8} = 62$.
40. $\frac{5n}{8} + \frac{2n}{3} = 124$.

Directions for Solving Equations. 1. *If the equation contains fractions, multiply both members by an expression that will remove the fractions.*

For example, if we are given the equation

$$\frac{8x}{2x+3} = 3,$$

and multiply both members (Axiom 3) by $2x+3$, we have

$$8x = 6x + 9.$$

2. *Subtract from both members such terms as are necessary in order to leave the unknown quantity on one side and the known numbers on the other side.*

For example, in the last equation above we have

$$8x = 6x + 9.$$

Subtracting $6x$ from both members (Axiom 2), we have

$$8x - 6x = 9,$$

or

$$2x = 9.$$

This operation is called *transposition,* and we may say that we *transpose* the term $6x$.

3. *Divide both sides of the equation by the coefficient of the unknown quantity.*

For example, if in the equation $2x = 9$, given above, we divide both members (Axiom 4) by 2, we have

$$x = \tfrac{9}{2}, \text{ or } 4\tfrac{1}{2}.$$

4. *Check the solution by substituting in the original equation the value found for the unknown.*

Thus, substituting $\frac{9}{2}$ for x in the given equation, we have

$$\frac{8 \times \frac{9}{2}}{2 \times \frac{9}{2} + 3} = \frac{36}{9+3} = \frac{36}{12} = 3.$$

DRILL CHART IN EQUATIONS

Solve the following equations and check each result:

1. $2x + 7 = x + 19$.
2. $3x - 2 = x + 16$.
3. $5x + 6 = x + 34$.
4. $7x - 9 = 3x + 31$.
5. $3x + 9 = x + 35$.
6. $6x - 8 = 3x + 37$.
7. $9x + 13 = 6x + 85$.
8. $8x - 13 = 6x + 13$.
9. $5 + 8x = 17 + 4x$.
10. $9.1x + 1 = 6.1x + 34$.
11. $3x - 9 = 7 - x$.
12. $8x - 10 - 7x = 4$.
13. $8x - 37 = 2x - 7$.
14. $6x - 44 + 2x = 4x$.
15. $7x + 7 = 31 + x$.
16. $10 + 3x - 21 = 2x$.
17. $\dfrac{9x + 3}{2x + 2} = 3$.
18. $\dfrac{12x + 21}{2x + 5} = 5$.
19. $\dfrac{8x - 7}{3x + 2} = 2$.
20. $\dfrac{7x - 1}{x + 2} = 3$.
21. $5x + 24.2 = 48.7$.
22. $9y - 32.4 = y + 0.4$.
23. $3.8y + 4.3 = 80.3$.
24. $7.4 + 3y = y + 9.8$.
25. $6 - 2x + 8x = 25$.
26. $2 + 21x + x = 25$.
27. $7x + 2 = 3x + 6$.
28. $3x + 3 = x + 73$.
29. $5x + 29 = 2x + 32$.
30. $42x = 39x + 639$.
31. $9x + 3 = 4x + 28$.
32. $4x - 9 = 5 + 2x$.
33. $5 + 22x = 12x + 75$.
34. $5 + 12x = 3x + 50$.
35. $8 + 9x = 9 + x$.
36. $83 + 2.7x = 1.2x + 98$.
37. $\dfrac{35x - 49}{3x + 5} = 10$.
38. $\dfrac{8x - 1}{3x + \frac{1}{2}} = 2$.
39. $\dfrac{42x - 5}{x + 2\frac{1}{2}} + 4 = 30$.
40. $\dfrac{25x - 35}{9 - 2x} = 10$.

Solving Problems. The following examples show how we may use equations in solving problems:

1. Edward is thinking of a number such that half of it plus a third of it is 10. What is the number?

Letting x represent the number, we have the equation

$$\frac{x}{2}+\frac{x}{3}=10.$$

Multiplying both members by 2×3, or 6, we have

$$3x+2x=60,$$

or $$5x=60,$$

and, dividing by 5, $$x=12.$$

Check. $\frac{1}{2}$ of $12+\frac{1}{3}$ of $12=6+4=10$.

2. A carpenter wishes to cut a board 20 ft. long into two pieces such that one piece is four times as long as the other. Into what lengths should he cut the board?

At first reading, it looks as though we had two unknown lengths, but, if we know the length of one piece we can find the other by subtraction. Thus, letting n represent the *number of feet* in the shorter piece, we have

$$n+4n=20,$$

or $$5n=20,$$

and, dividing by 5, $$n=4;$$

that is, he should cut the board into lengths of 4 ft. and 16 ft.

Check. 16 ft. $\div$ 4 ft. $=4$, and 16 ft. $+$ 4 ft. $=20$ ft.

General Directions. In solving a problem we should (1) read it carefully; (2) determine what is to be found and represent this by x, n, or some other letter; (3) state the equation; (4) solve the equation; and (5) check the result by seeing if it meets the conditions given in the problem.

SOLVING PROBLEMS

Find the unknown number described in each of Exs. 1–14 and check each result:

1. If 8 is added to the number, the result is 47.

2. If 6 is subtracted from the number, the result is 8.

3. When the number is multiplied by 7, the result is 63.

4. When the number is divided by 9, the result is 8.

5. The result of adding the number to twice itself is 27.

6. When the number is added to 27, the result is 29.

7. When twice the number is added to 8, the result is 44.

8. If 9 is subtracted from seven times the number, the result is 82.

9. If from three fourths of the number half the number is taken, the result is 14.

10. When the number is increased by three fourths of itself, the result is 28.

11. The number is 42 more than half itself.

12. The number is $3\frac{1}{2}$ more than half itself.

13. The number is 9 more than three fourths of itself.

14. The number is $\frac{7}{8}$ more than three fourths of itself.

15. Ben is thinking of a number whose double is 48. What is the number of which he is thinking?

16. The number 69 is three times the number of which Kate is thinking. Of what number is Kate thinking?

17. If three fourths of the number of marbles a boy has is 18, how many marbles has he?

18. A man spent one fourth of a day's wages for food. If he spent \$1.35 for food, what were his wages?

19. A man sold his car for $450, which was five eighths of its cost. What was the cost?

20. When eight times a certain number is diminished by 9, the result is 23. What is the number?

21. In grinding wheat into flour there is a loss of 18%. How much wheat is needed to make 2132 lb. of flour?

22. To a building 60 ft. long and 40 ft. wide, a wing 30 ft. wide and having half the floor space of the original building is to be added. Find the length of the wing.

23. The top of a tree 30 ft. high is broken off by the wind in such a way that the broken part is twice as long as the part of the trunk that is left standing. Find the length of the broken part of the tree.

24. Seven times a certain number is equal to the number increased by 24. Find the number.

25. When seven times a certain number is decreased by 35, the result is twice the number. What is the number?

26. When a certain number is multiplied by 4, and the result is added to 13, the final result is 25. Find the number.

27. Find a number such that the result of adding nine times the number and eleven times the number is 480.

28. The greater of two numbers is seven times the less, and their sum is 96. Letting x represent the lesser number and $96 - x$ the greater, find the numbers.

29. Separate 90 into two parts such that the larger is twice the smaller.

30. Separate 100 into two parts such that the larger is three times the smaller.

31. The sum of three numbers is 21. If the second number is four times the first, and the third is half the second, what are the three numbers?

Further Work in Solving Problems. The following examples illustrate the uses of equations in solving other problems:

1. An agent, who made a profit of \$120 on the sale of some property, said that he made 5%. Find the selling price.

We know that $5\% = \frac{1}{20}$, and so, if we let x represent the *number of dollars* in the selling price, the shorthand way telling the story is

$$\frac{1}{20}x = 120.$$

Multiplying by 20, $x = 2400.$

That is, the selling price of the property was \$2400.

Check. $0.05 \times \$2400 = \$120.$

2. Find two consecutive numbers whose sum is 279.

Let n represent the smaller number. Then, since the numbers are consecutive, the next number must be 1 more than this (that is, $n + 1$ represents the larger number), and so

$$n + n + 1 = 279,$$

or $2n + 1 = 279.$

Subtracting 1, $2n = 278.$

Dividing by 2, $n = 139$, the smaller number,

and $n + 1 = 140$, the larger number.

Check. $139 + 140 = 279.$

3. Find a number such that the tens' digit is 4 more than the units' digit, and the sum of the two digits is 10.

If x is the units' digit, $x + 4$ is the tens' digit; that is,

$$x + x + 4 = 10.$$

Solving, $x = 3$, the units' digit,

and $x + 4 = 7$, the tens' digit.

That is, the required number is 73.

Check. $7 - 3 = 4$, and $7 + 3 = 10.$

SOLVING PROBLEMS

Find the number described in each of Exs. 1–7:

1. The tens' digit is 2 more than the units' digit, and the sum of the digits is 10.

2. The units' digit is 4 more than the tens' digit, and the sum of the digits is 12.

3. The tens' digit is one half the units' digit, and the sum of the digits is 9.

4. The tens' digit is 1 more than the units' digit, and the sum of the digits is 9.

5. The units' digit is 5 less than the tens' digit, and the number is 2 more than ten times the tens' digit. That is, if t is the tens' digit, the units' digit is $t - 5$, and the number is $10\,t + t - 5$, because it is made up of t tens, or $10\,t$, plus $(t - 5)$ units. Now solve the equation $10\,t + t - 5 = 10\,t + 2$, which expresses the conditions of the problem.

6. The tens' digit is four times the units' digit, and the number is 10 more than nine times the tens' digit.

7. The tens' digit is three times the units' digit, and the number is 2 more than ten times the tens' digit.

8. Find two consecutive numbers whose sum is 441.

9. Find two consecutive numbers whose sum is 481.

10. If 8 is subtracted from nine times a certain number, the result is 73. What is the number?

11. If eight times a certain number is decreased by one fourth of itself, the result is 124. Find the number.

12. What number, on being decreased by $33\frac{1}{3}\%$ of itself, becomes 88?

13. What number, on being increased by $12\frac{1}{2}\%$ of itself, becomes 54?

14. A man sold an automobile for \$1500, gaining 20% on what he paid for it. Find the price which he paid for it.

15. If a dealer sold a radio set for \$120, which was 25% above the cost price, what did the set cost the dealer?

16. At a bargain sale a dealer offered a discount of 15% on his entire stock. Find the original marked price of a coat for which the dealer received \$55.25.

17. If 3 is subtracted from three times a certain number, and if 9 is added to twice the number, the two results are equal. What is the number?

18. A rope 90 ft. long is cut into two parts in such a way that the longer part is three times the shorter. Find the length of each part.

19. If $37\frac{1}{2}\%$ of a certain sum of money is \$135, what is the sum of money?

20. A certain sum of money increased by \$14 is equal to eight times itself. Find the sum of money.

21. A boy 5 yr. ago was two thirds as old as he is at present. How old is he now?

22. In 26 yr. from now a boy will be three times as old as he is at present. How old is he now?

23. The length of a rectangle is 8 ft. more than the width, and the perimeter is 64 ft. Find the dimensions.

24. The length of a tennis court is 42 ft. more than the width, and the perimeter is 228 ft. Find the dimensions.

25. If 10 yr. ago a boy was one third as old as he is now, how old is he now?

26. A merchant's sales last year were 10% more than they were the year before. If the sales last year were \$38,500, how much were they the year before?

IX. FORMULAS

A SILENT READING LESSON

We have learned that representing numbers by letters is helpful in solving certain kinds of problems. We shall now consider another, and much more important, use of this device for shortening work.

For example, we know that the area of a circle is approximately $\frac{22}{7}$ times the square of the radius, and have seen that it is much easier to express this in mathematical shorthand as

$$A = \tfrac{22}{7} r^2.$$

Such an expression is called a *formula*. It is evident that a formula is much more easily written and remembered than a rule expressed in words.

In later work in mathematics we shall find the above formula written with the Greek letter π (pi) in place of $\frac{22}{7}$, because $\frac{22}{7}$ is only an approximation for a number which is really an endless decimal beginning 3.14159; that is, the formula for the area of a circle is usually written as

$$A = \pi r^2.$$

From this formula we can easily find the value of A if we know the value of r. For example, in finding the area of a circle with a radius of 7 in., we see that in this case

$$r = 7,$$

and so, taking $\frac{22}{7}$ as the value of π as usual, we have

$$A = \pi r^2 = \frac{22}{7} \times 7 \times 7 = 154.$$

That is, since the radius is given in inches, the area of the circle is 154 sq. in.

A little later we shall consider the case of finding the value of r from this formula when we know the value of A.

FINDING VALUES FROM FORMULAS

1. The formula for the area (A) of a rectangle with the base b and the height h is $A = bh$. Find the value of A when $b = 8$ and $h = 4$.

Using the formula $A = bh$, find the value of A when

2. $b = 4$, $h = 2$. **5.** $b = 8$, $h = 1\frac{1}{4}$. **8.** $b = 18$, $h = 5$.

3. $b = 7$, $h = 3$. **6.** $b = 6$, $h = 2\frac{1}{2}$. **9.** $b = 16$, $h = 8\frac{1}{4}$.

4. $b = 9$, $h = 5$. **7.** $b = 9$, $h = 3\frac{1}{3}$. **10.** $b = 20$, $h = 9\frac{3}{4}$.

11. The formula for the area (A) of a triangle with the base b and the height h is $A = \frac{1}{2}\,bh$. Find the value of A when $b = 4$ and $h = 2\frac{1}{2}$.

Using the formula $A = \frac{1}{2}\,bh$, find the value of A when

12. $b = 3$, $h = 2$. **15.** $b = 8$, $h = 3\frac{1}{2}$. **18.** $b = 12$, $h = 9$.

13. $b = 4$, $h = 7$. **16.** $b = 6$, $h = 2\frac{1}{4}$. **19.** $b = 9$, $h = 12$.

14. $b = 6$, $h = 5$. **17.** $b = 8$, $h = 5\frac{3}{4}$. **20.** $b = 16$, $h = 9\frac{7}{8}$.

21. The formulas for the circumference (C) of a circle with the diameter d and the radius r are $C = \pi d$ and $C = 2\,\pi r$. Find the value of C when $d = 4$, using $\frac{22}{7}$ for the value of π. Find also the value of C when $r = 2$. Explain why the two results are the same.

Using one of the formulas of Ex. 21, find the value of C when

22. $r = 7$. **24.** $r = 14$. **26.** $r = 3\frac{1}{2}$. **28.** $r = 2.1$.

23. $d = 14$. **25.** $d = 28$. **27.** $d = 7$. **29.** $d = 4.2$.

30. The formula for the area of the entire surface of a cube with the edge e is $A = 6\,e^2$. Find the total surface area of a cube whose edge is 5 in.

31. The formula for the volume of a cube with the edge e is $V = e^3$. Find the volume of a cube whose edge is 3 in.

A SILENT READING LESSON

The formula is also very convenient as a means of remembering the rules that are needed in business. For example, in finding the interest on \$250 for 6 mo. at 5%, we first find 5% of \$250, which is \$12.50, as the interest for 1 yr. We then take half this result, which is \$6.25, as the interest for 6 mo. If we were to write a rule for this, we should have

To find the interest, multiply the principal by the rate for 1 yr. and then multiply the result by the time expressed in years.

This rule is much too long for a modern business man to bother with. A formula is shorter and is more easily understood and remembered. Letting i stand for the number of dollars of interest, p for the number of dollars of principal, r for the rate of interest expressed as hundredths, and t for the time in years, the above rule may be written

$$i = prt.$$

Similarly, for finding the horse power of a steam engine, there is a long rule which any one of us would find very difficult to remember. But when we write this long rule as a formula, it is simply

$$h = \frac{plan}{33{,}000}.$$

This means that the horse power (h) is equal to the steam pressure (p), times the length (l) of the piston stroke in the cylinder, times the area (a) of the piston, times the number (n) of piston strokes per minute, divided by 33,000.

We are not expected to remember such a formula or even to understand the technical terms used. It is given simply to show how condensed and how clear a formula is. From such an illustration we can readily infer that the formula is an important and useful device in all kinds of work.

Deriving Formulas. Since formulas are equations, we can solve them for any letter we choose. That is, from one formula we can derive other formulas that we may need. The following examples show how we proceed in such cases:

1. From the interest formula, $i = prt$, find a formula for p, and from it find the principal that must be invested in order to yield \$12.50 in interest in 1 yr. at 5%.

Dividing $i = prt$

by rt, we have $\frac{i}{rt} = p,$

or, as usually written, $p = \frac{i}{rt}.$

Then, since $i = 12.50$, $r = 0.05$, and $t = 1$, we have

$$p = \frac{12.50}{0.05 \times 1} = 250;$$

that is, the principal that will yield the given interest is \$250.

2. From the formula for the area of a circle, $A = \pi r^2$, find a formula for r, and from it find the radius of a circle which has an area of 44 sq. in.

Dividing $A = \pi r^2$

by π, we have $\frac{A}{\pi} = r^2.$

Taking the square root of both members, we have

$$r = \sqrt{\frac{A}{\pi}}.$$

Then, since $\pi = \frac{22}{7}$ and $A = 44$, we have

$$r = \sqrt{44 \times \tfrac{7}{22}} = \sqrt{14}.$$

From the table on page 349, we have $\sqrt{14} = 3.742$; that is, the radius of the circle is 3.7 in., to the nearest 0.1 in.

DERIVING FORMULAS

1. From the formula $C = 2\pi r$ find a formula for r.

Find to the nearest 0.1 in. the radius of a circle which has a circumference of

2. 22 in. 3. 4.4 in. 4. 5 in. 5. 24.2 in. 6. 100 in.

7. The formula for the area of the curve surface of a cylinder is $S = 2\pi rh$. Find a formula for r.

Find to the nearest 0.1 in. the radius of a cylinder when the area of the curve surface (S) and the height (h) are as follows:

8. 44 sq. in., 4 in.
9. 88 sq. in., 10 in.
10. 100 sq. in., 8 in.
11. 400 sq. in., 20 in.

12. The formula for the volume (V) of a cylinder of radius r and height h is $V = \pi r^2 h$. Find a formula for h.

Find to the nearest 0.1 in. the height of a cylinder when the volume (V) and radius (r) are as follows:

13. 22 cu. in., 3 in.
14. 44 cu. in., 5 in.
15. 100 cu. in., 7 in.
16. 400 cu. in., 10 in.

17. From the formula of Ex. 12 find a formula for r.

Find to the nearest 0.1 in. the radius of a cylinder when the volume (V) and height (h) are as follows:

18. 66 cu. in., 4 in.
19. 88 cu. in., 11 in.
20. 100 cu. in., 7 in.
21. 150 cu. in., 14 in.

22. From the interest formula, $i = prt$, find a formula for r, and then find one for t.

23. Make up three examples applying the formulas of Ex. 22 and of Ex. 1, page 382, and solve each one.

DEVELOPING FORMULAS

1. If an agent sells \$500 worth of goods and receives a commission of 4%, what is the amount of his commission?

2. Letting s represent the amount of a sale and r the rate of commission, develop a formula for c, the amount of commission received.

3. From the formula in Ex. 2, find the amount of commission at 3% on a sale amounting to \$1200.

4. A man sells a house that cost him \$6500 and makes a profit of 10%. What is the amount of the profit that he makes on the transaction?

5. From Ex. 4 write a formula for the amount (p) of profit when the cost is c dollars and the rate of profit is r%.

6. From Ex. 5 find a formula for r when p and c are given.

7. An insurance policy for i dollars requires the payment of a premium p at the rate r per \$100. Write a formula for p.

8. From Ex. 7 find a formula for r.

9. A man's property is assessed at \$5000, and the tax rate is \$25 per \$1000. How large a tax does he pay?

10. From Ex. 9 write a formula for the amount (t) of the tax when the assessed valuation a and the tax rate r per \$1000 are given.

11. From Ex. 10 find a formula for r.

12. From Ex. 10 find a formula for a.

13. The formula for the area of a parallelogram with the base b and the height h is $A = bh$. From this derive two other formulas, one for b and one for h.

14. Use all three formulas of Ex. 13 in three problems that you make up about parallelograms.

15. One formula for the area of a trapezoid with the bases b and b' (read b-prime) and the height h, is $A = \frac{1}{2}h(b + b')$. By dividing both members by $b + b'$ and then multiplying both members by 2, find a formula for h.

16. Use the result of Ex. 15 to find the height of a trapezoid which has an area of 50 sq. in. and bases of 6 in. and 4 in.

17. If you drive your car 20 mi. per hour for 3 hr., what distance do you go? If you drive r miles per hour for t hours, what distance (D) do you go?

18. From the formula $D = rt$, find two other formulas, one for r and the other for t.

19. Use the three formulas of Ex. 18 in three problems that you make up about driving a car.

20. If one book costs \$2, how much will 3 books cost? If one book costs p dollars, what will be the cost (C) of n books at the same price?

21. From the formula $C = np$, derive two other formulas, and give an example illustrating the use of each one.

22. From the formula $S = 4\pi r^2$, which gives the area of the surface of a sphere, derive a formula for r.

23. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. Find the volume of a sphere with a radius of 21 in.

24. A ball with a diameter of 7 in. is placed inside a box having the form of a cube with an edge of 7 in., inside measurement. How many cubic inches are there in the box that are not occupied by the ball?

25. Solve Ex. 24 when the diameter and the edge are each 14 in. instead of 7 in.

26. The formula for the hypotenuse h of a right triangle in which the other two sides are a and b is $h = \sqrt{a^2 + b^2}$. Find the value of h when $a = 6$ and $b = 8$.

REVIEW OF FORMULAS

From the formula $T = 2\pi rh + 2\pi r^2$ *find the value of* T *when*

1. $r = 7, h = 5$.
2. $r = 7, h = 6$.
3. $r = 7, h = 7$.
4. $r = 7, h = 8$.
5. $r = 14, h = 28$.
6. $r = 63, h = 15$.

From the formula $V = \pi r^2 h$ *find the value of* V *when*

7. $r = 7, h = 6$.
8. $r = 3, h = 8$.
9. $r = 1, h = 7$.
10. $r = 7, h = 1$.
11. $r = 14, h = 1$.
12. $r = 32, h = 4$.

13. In the formula $i = prt$, find the value of i when $p = 8000$, $r = 0.04$, and $t = 2$. Then write the problem in words and state what the result means.

14. Using the formula of Ex. 13, find the interest on \$5000 for 4 mo. at 5%.

15. The amount (A) of a debt at a given time is the principal plus the interest. Write a formula for A in terms of p, r, and t; that is, using only p, r, and t to express the value of A.

16. Using the formula of Ex. 15, compute the amount, at the end of 2 yr., of a debt of \$5000 bearing interest at 5%.

Taking π *as* $\frac{22}{7}$, *write the following formulas in words:*

17. $V = \frac{4}{3}\pi r^3$, where V stands for the volume of a sphere and r for the radius.

18. $V = \frac{1}{3}\pi r^2 h$, where V stands for the volume of a cone, r for the radius of the base, and h for the height.

19. $S = 4\pi r^2$, where S stands for the area of the surface of a sphere and r for the radius.

20. From the formula $T = 2\pi rh + 2\pi r^2$, which gives the area of the entire surface of a cylinder, derive a formula for h.

21. If an automobile averages m miles an hour, what is the distance (d miles) traveled in h hours? Find the value of d when $m = 25$ and $h = 3\frac{1}{2}$.

X. REVIEW AND DRILL

MINIMUM ESSENTIALS

Add and also subtract as follows:

1.	2.	3.	4.	5.	6.
$3\frac{7}{8}$	$4\frac{7}{8}$	$7\frac{5}{8}$	$9\frac{1}{16}$	$8\frac{1}{32}$	$6\frac{7}{16}$
$2\frac{5}{16}$	$3\frac{1}{4}$	$2\frac{15}{16}$	$3\frac{5}{8}$	$2\frac{3}{4}$	$3\frac{5}{8}$

Multiply and also divide as follows:

7. $\frac{2}{3}$ by $\frac{4}{5}$. **8.** $\frac{7}{32}$ by $\frac{1}{8}$. **9.** $\frac{9}{10}$ by $\frac{2}{3}$. **10.** $37\frac{1}{2}$ ft. by $2\frac{1}{4}$.

Add and also subtract as follows:

11.	12.	13.	14.
\$7.25	\$8.32	16.4 mi.	26.32 in.
6.38	.76	7.9 mi.	9.76 in.

Multiply and also divide as follows:

15. 172.8 by 1.44. **16.** 473.48 by 6.23. **17.** 47.595 by 5.01.

Find the value in each of the following cases:

18. 45% of \$12. **20.** $37\frac{1}{2}$% of \$72.64. **22.** $6\frac{1}{4}$% of 80.

19. 15% of \$65. **21.** $66\frac{2}{3}$% of \$117.36. **23.** $66\frac{2}{3}$% of $37\frac{1}{2}$.

24. Using a short method, add 125% of 5280 ft. to $\frac{3}{4}$ mi.

Find, to the nearest 0.01, the value of

25. 28.76^2. **26.** 3.14×5.6^2. **27.** $\sqrt{138}$. **28.** $\sqrt{16.75}$.

Find the value of x in each of the following equations:

29. $x - 9 = 6$.

30. $x + 7 = 21$.

31. $2x = 38$.

32. $\frac{1}{4}x = 5$.

33. $\dfrac{2x-3}{5} = 3$.

34. $2x + 3 = x + 7$.

35. $3.4x = 0.1x + 6.6$.

36. $6x - 40 + 2x = 3x$.

37. $88 + 3.7x = 2.2x + 91$.

38. $\dfrac{4x-7}{5} = \dfrac{3x+1}{4}$.

PROBLEMS WITHOUT NUMBERS

All work oral

1. If you know the number of feet in a roll of carpeting, how do you find the number of yards?

2. If you know the number of pints of milk in a can, how do you find the number of quarts?

3. If you know the number of gallons of oil in a tank, how do you find the number of quarts?

4. If you know the number of pupils in each of two schools, how do you find the number in both schools?

5. If you know the number of oranges in one box and the number in another box, how do you find how many more oranges there are in one box than in the other?

6. When you buy something at a store and give the clerk more than the amount of the sale, how do you find how much change is due you?

7. If you are told the number of trees in each row in an orchard and the number of rows, how do you find the number of trees in the orchard?

8. If you know the number of passengers in a trolley car and the total amount collected in fares, how do you find the fare paid by each passenger?

9. If a man, who is going to buy a set of dining-room furniture, finds that by waiting a week he can buy it at a sale at a certain per cent off the regular price, how do you find the amount he can save by waiting?

10. If you know how much a man paid for rent last year and how much more he pays this year, how do you find the per cent of increase? How do you find the amount of the monthly increase in rent?

11. If you know the amount that a real-estate agent receives for selling a house at a certain price, how do you find the per cent of commission he receives?

12. If you know the face of a fire-insurance policy and the rate per hundred, how do you find the premium?

13. If you know a man's bank balance on the first of the month and the amount of his deposits and checks drawn during the month, how do you find his balance on the last day of the month?

14. If you know the market quotation on some railroad stock, how do you find the amount that you must pay your broker if you buy a certain number of shares?

15. If you know the quarterly rate of dividend on a stock, how do you find the amount received annually by the owner of a certain number of shares?

16. If you know the prices at which you can buy a first-class stock and a first-class bond, and the rate of dividend or of interest paid, how do you find which is the more desirable investment for you to make?

17. How do you find the square of a number?

18. If you know the side of a square, how do you find the area? Illustrate by an example.

19. If you know the area of a square, how do you find the side? Illustrate by an example.

20. How do you find the area of a parallelogram? the area of a circle? the volume of a cube?

21. How do you find the hypotenuse of a right triangle when you know the other two sides?

22. If you know the height of a cylinder and the diameter of the base, what other measurements can you find relating to the cylinder and how do you find them?

PROBLEMS FOR COMPLETION

1. Our school baseball team lost $16\frac{2}{3}\%$ of the games played last season, and they won 10 games. Complete the problem in any reasonable way and then solve it.

Complete and solve the following problems:

2. A boy has a newspaper route on which he delivers 75 papers each morning and night, 6 da. a week. He is paid $\frac{1}{2}¢$ for each paper delivered.

3. Between June 30 and Sept. 1 a boy earns \$6 a week by working in a grocery store. He saves $33\frac{1}{3}\%$ of his wages.

4. A steel girder 18 ft. 10 in. long weighs 48 lb. to the running foot.

5. In making a scale drawing of a rectangular garden that is 18 ft. long and 12 ft. wide, a girl wishes to make the drawing 6 in. long.

6. Harry, who is 5 ft. tall, finds that his shadow is 4 ft. long at the same time that the telephone pole in front of his house casts a shadow 30 ft. long.

7. A square has an area of 22 sq. in.

8. Another square has a diagonal of 10 in.

9. The edge of a cube is 16 in.

10. The area of one face of a cube is 289 sq. in.

11. The area of a circle is 44 sq. in.

12. A square courtyard with a perimeter of 144 ft. is to be cemented at a cost of \$2.90 per square yard.

13. A ladder 28 ft. long leans against a building. The foot of the ladder is 8 ft. from the building.

14. A cylindric water tank is 28 ft. in diameter and 32 ft. high. There are $7\frac{1}{2}$ gal. in 1 cu. ft.

15. A traveling salesman, who covers an eastern state, is paid a commission of $6\frac{1}{2}\%$ on his sales, and pays his own expenses. Last year his sales amounted to \$78,650 and his expenses to \$2350.75.

16. A man, who lives in a city where the property-tax rate is \$28.70 per \$1000, owns a house that is assessed for \$8800 and an automobile that is assessed for \$1250.

17. A department store borrows \$18,500 on a 60-day note which the bank discounts at $5\frac{1}{2}\%$.

18. A man wishes to insure his house, which is valued at \$25,000, for 80% of its value for a period of 5 yr. The 5-year rate is four times the annual rate, and on this type of property the annual rate is $47¢$ per \$100.

19. A man takes out a 20-payment life-insurance policy for \$15,000. The annual premium at his age for this type of policy is \$26.40 per \$1000.

20. Mr. Huntington invests \$12,500 in a mortgage that pays interest at $5\frac{3}{4}\%$ a year.

21. Mr. Driscoll buys a house for \$9800 and rents it at \$75 a month. In a certain year he pays \$156 for taxes, \$12.50 for insurance, and \$75 for repairs.

22. A man buys 50 shares of stock when it is quoted at $108\frac{3}{4}$. The brokerage is \$15 per 100 shares.

23. A man owns 25 shares of stock on which a dividend of \$2.75 a share is declared.

24. A man buys 30 shares of Western Union Telegraph stock at $127\frac{1}{2}$ and sells it at 130, paying the usual brokerage of \$20 per 100 shares.

25. Mr. Rowley has \$3000 to invest and finds that with it he can buy, including brokerage, bonds which have a par value of \$3200 and pay $4\frac{1}{4}\%$.

Testing Your Alertness. In this work, which will show how alert you are, you are to follow directions. For example, for the figures on page 393 the teacher may read directions something like these:

1. In Ex. 11 write 4 in a space that is in the triangle and one circle but not in both circles.
2. In Ex. 12 write 6 in a space that is in one and only one of the two large triangles and also in the circle.
3. In Ex. 13 write 7 in a space that is in the triangle and rectangle but not in the circle, and write 3 in a space that is in the triangle and the circle but not in the rectangle.
4. In Ex. 14 write the second letter of the word "number" in the first circle, the fourth letter in the third circle, and the fourth letter from the last in the second circle.
5. In Ex. 15 write 1 in the third square, 3 in the first square, and half their sum in the second.
6. In Ex. 16 write 7 in the triangle, 6 in the circle, and twice their product in the square.
7. In Ex. 17 write 8 in each circle that comes after a square and write 7 above each square that comes before a circle.

ALERTNESS TEST

Copy the following, and then, allowing 15 sec., insert in the parentheses the number announced by the teacher (it may be 15, 16, or 17) and place a circle (◯) before each correct statement and a cross (×) before each incorrect one:

1. $3 + (\ \ ) = 20.$
2. $(\ \ ) - 4 = 12.$
3. $2 \times (\ \ ) = 34.$
4. $(\ \ ) \div 5 = 3.$
5. $10 \times (\ \ ) \div 2 = 80.$
6. $5 + (\ \ ) = 4 \times 5.$
7. $9 \times (\ \ ) = 153.$
8. $(\ \ ) + (\ \ ) = 40.$
9. $10 \times (\ \ ) = 120.$
10. $(\ \ ) - 5 = 12.$

Copy the following figures, and, at a signal given by the teacher, do as you are directed, allowing 5 sec. for each example:

11.

12.

13.

14.

16.

15.

17.

Copy the following numbers as rapidly as possible, and then cancel in each case the numbers (if any) that should be omitted so that the numbers shall run along in a regular order according to some kind of rule or law (for example, Ex. 18 should appear as 9 8 7 6 5 4 2), allowing 5 min. for the work:

18. 9 8 7 6 5 4 2.

19. 1 7 2 3 4 5 6.

20. 3 4 6 8 10 12 14.

21. 4 7 10 12 13 16 19.

22. 1 2 4 8 10 16 32.

23. 64 32 24 16 8 4 2.

24. 7 11 15 19 23 27 30.

25. 8 4 3 2 1 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$.

26. 25 20 15 12 10 5 0.

27. 3 9 27 81 92 243 729.

28. 42 40 38 36 34 32 30.

29. 29 30 32 31 32 33 34.

30. 40 34 28 22 16 14 10.

31. 2 6 18 54 162 486 1024.

32. 93 83 73 68 63 53 43.

33. 40 20 10 8 5 $2\frac{1}{2}$ $1\frac{1}{4}$.

34. 81 27 9 3 2 1 $\frac{1}{3}$.

35. 7 7 6 7 7 7 7 7 7 7.

36. 2 8 32 128 512 729 2048.

37. 100 92 84 76 68 62 60.

38. $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$ $\frac{1}{24}$ $\frac{1}{32}$ $\frac{1}{64}$.

39. 0.3 0.9 0.27 0.54 0.81.

40. 1 0.1 0.01 0.001 0.0010.

41. 2.6 3.5 4.4 5.3 5.9 6.2.

SIMPLE PROBLEMS OF EVERYDAY LIFE

1. If 1 pk. of apples makes 3 qt. of sweet cider, 1 bu. of apples will make how many quarts? how many gallons?

2. If 2 bu. of apples make 12 qt. of sweet cider, how many bushels will it take to make 60 gal.?

3. If we put the 60 gal. of cider into quart bottles, how many bottles will it take to hold all the cider? How many pint bottles would it take?

4. A clerk in a grocery store put up 200 lb. of sugar in bags holding 5 lb. each. How many bags did he fill?

5. A bushel of potatoes weighs 60 lb. How many pounds shall we get if we buy 3 pk.? 6 pk.? 75 bu.?

6. On Saturday Helen and Ruth picked 32 qt. of berries in our strawberry patch. Mother canned the berries and found that it took 1 qt. of fresh berries to fill a pint can. How many pint cans did she fill? This was how many quarts?

7. Mother canned 32 qt. of cherries and used 8 lb. of sugar. How many quarts of cherries should she use with 24 lb. of sugar? How many pounds of sugar should she use with 24 qt. of cherries?

8. Mr. Bartley runs a hotel and finds that each guest uses, on an average, 1 oz. of butter at a meal. If he has 10 lb. of butter on hand, how many persons will this amount serve for three meals?

9. On rainy days Mr. Brown and his boys make potato crates in the barn. It takes 2 oz. of nails to make each crate. How many crates can they make with 1 lb. of nails? with 15 lb.? How many pounds of nails will they need for 72 crates?

10. When Ben had only 2 black rabbits, he fed them 2 lb. of cabbage a day. If he now has 16 rabbits, how many pounds of cabbage does he need for the day's feed?

11. Ben had a vegetable garden. On April 14 he planted some early potatoes, which he estimated would be ready for digging in 79 da. If he went on a camping trip in June, on what date did he have to be back to dig the potatoes?

12. Ben raised 42 bu. of potatoes which he sold at an average price of $1.40 a bushel. How much were his receipts?

13. In preparing a spray for his orchard Mr. Brown used 10 lb. of sulphur, 9 lb. of lime, and 1 lb. of arsenate of lead to 50 gal. of water. What part of a pound of each ingredient did he use for each gallon of water? How many pounds of each ingredient should he use for 400 gal. of water?

14. In making out a bill for 56 doz. eggs Ben made the mistake of multiplying the price per dozen by 65 instead of by 56. The result that Ben obtained was $24.70. What result should he have obtained?

15. Ben's uncle works in the chair factory in town. He is paid 70¢ an hour for an 8-hour day, with "time and a half" for overtime. How much does he receive if he works 8 hr. a day for 5 da.? if he works 9 hr. a day for 3 da.? if he works 8 hr. a day for 2 da. and 10 hr. a day for 3 da.?

16. The factory where Ben's uncle works has 12 men on its pay roll. As weekly wages, two of them receive $32.50 each, five receive $34.20 each, three receive $27 each, one receives $33.88, and one receives $34.10. How much money does the factory need for the week's pay roll? How many of each kind of bill or coin should it provide so as to give each man the right change in the most convenient form?

17. During one week that Ben's uncle worked he made $3.15 for overtime. The next week he made $4.20 for overtime, and during the next 3 wk. he made $3.68 a week in the same way. What was the average amount of overtime pay that he received per week during the 5 wk.?

EVERYDAY PROBLEMS IN PERCENTAGE

1. Mr. Brown's cherry crop has averaged 1950 lb. per year for several years. Last year it amounted to 2340 lb. What per cent was this above the average crop?

2. The average amount received annually from the cherry crop in Ex. 1 was \$117. Last year Mr. Brown received 4¢ a pound for his crop of 2340 lb. What per cent was this above or below the average yearly receipts?

3. For the annual meeting of the Mathematics Teachers' Association the railroads sell round-trip tickets for $1\frac{1}{2}$ times the regular one-way fare. What per cent of reduction is this on the regular round-trip cost?

4. If a gasoline-saving device installed on an automobile makes it possible for the car to go 15 mi. instead of 12 mi. on 1 gal. of gasoline, what is the per cent of increase in mileage?

5. With the increased mileage per gallon described in Ex. 4, how much gasoline will be saved on a trip that required 56 gal. of gasoline before the device was installed?

6. If an automobile travels 30 mi. in 50 min., what per cent of a mile will it go in 1 min.?

7. At 30 mi. per hour, what per cent of a mile will be traveled in $\frac{1}{4}$ min.?

8. If a produce dealer's expenses are 30% of sales and he wishes to make a profit of 10% on the selling price, how much must he charge for potatoes that cost him 60¢ a bushel in order to cover his expenses and make this profit?

9. If the enrollment in a school was 920 last year and has increased 35% this year, what is the present enrollment?

10. Ben weighed 110 lb. in June and 120 lb. in September. What was the per cent of increase?

11. If our rent is \$840 a year and the landlord raises it 15%, what will be the new monthly rental?

12. If the rent in Ex. 11 should be decreased 15% after being raised 15%, how much would it be then per year?

13. If we pay \$13.50 a ton for coal and the price is increased 10%, how much shall we have to pay then for 14 T. of coal?

14. If the present rate for natural gas in a Pennsylvania city is 60¢ per 1000 cu. ft., and the rate is increased 25%, what will be the cost of 15,000 cu. ft. of gas under the new rate?

15. An agent's sales in one day amounted to \$144. If he received 15% on his sales, how much did he make?

16. If Mr. Brown borrows \$100 at 6% in order to buy seed potatoes and repays the loan at the end of 6 mo., how much interest will he have to pay?

17. Mr. Brown wishes to make a profit of 25% on the total cost of his potato crop. His expense for labor is \$75, the interest on his loan at the bank is \$3, and his other expenses are \$162. How much must he receive for his crop in order to pay all expenses and make the desired profit? If he has 500 bu. to sell, how much must he charge per bushel?

18. If James bought 1 doz. melons for \$3, and the express was 60¢, what will he have to charge apiece for the melons in order to make a profit of 20% on the selling price?

19. On an invoice for a shipment of farm tools amounting to \$720, Mr. Brown takes advantage of a discount of 2% for paying cash. What is the amount that he actually pays?

20. One gas company requires each customer to deposit \$10 when the meter is installed in the house. If the company pays interest at 6% on these deposits, how much will a customer, who moves out at the end of 18 mo., receive from the company, including the returned deposit?

EVERYDAY HOUSEHOLD PROBLEMS

1. A recipe for cream candy calls for $\frac{3}{4}$ lb. of sugar, $\frac{1}{4}$ cup of vinegar, and $\frac{1}{2}$ cup of water. If Mollie wishes to make five times what the recipe allows, how much of each of these ingredients should she take?

2. A recipe for 2 lb. of a certain kind of candy calls for 4 cups of brown sugar, $\frac{1}{2}$ cup of milk, 3 teaspoonfuls of butter, and $\frac{3}{4}$ of a cup of English walnuts. What changes should you make in the recipe if you wished to make 1 lb.? to make 3 lb.? to make 5 lb.?

3. Rachel is making a cover for the library table. The cover is $51\frac{1}{2}$ in. long and $16\frac{3}{4}$ in. wide, and she wishes to put a fringe around it. Estimating the amount needed to the next greater $\frac{1}{4}$ yd., find how much Rachel will have to pay for the fringe at 35¢ a yard.

4. To make a fancy bag Mrs. Jewett needs a piece of homespun 20 in. long and 8 in. wide. She has a strip of homespun 2 yd. long and 32 in. wide. Draw a plan that shows how she should cut the cloth to make as many bags as possible.

5. Marie's mother has 7 yd. of cloth of the right width for towels. Each towel is to be 30 in. long when finished, and to have a hem $\frac{3}{4}$ in. wide at the top and bottom. How many towels can she make from the cloth?

6. If a living room is 24 ft. long and 15 ft. wide, how many strips of wall paper 18 in. wide will be needed to paper the four walls, no allowance being made for windows, doors, fireplaces, or other openings?

7. Make inquiries as to the probable cost of furnishing a room for yourself. Assume that you have $50 to spend and write a statement telling how much each item will cost, but do not let the total cost exceed $50.

8. A housekeeper buys a pound each of mustard seed, allspice, and cayenne pepper, paying 90¢ for the three. The mustard seed costs 18¢ a pound, and the allspice costs the same as the pepper. How much does the allspice cost?

9. A housekeeper bought a can each of cinnamon, nutmeg, and pepper, paying 39¢ for the three. The pepper cost 9¢, and the nutmeg cost twice as much as the cinnamon. How much did the cinnamon cost?

10. If a housekeeper can buy canned corn at 18¢ a can or at $1.90 per dozen cans, how much is saved on 24 cans in buying by the dozen?

11. If a housekeeper can buy canned soup at 24¢ a can or at $2.50 per dozen cans, how much is saved on 36 cans in buying by the dozen?

12. A dealer sells coffee in half-pound packages at 24¢ a package or in 5-pound cans at $2.15 a can. If a housekeeper wishes 5 lb., how much does she save in purchasing a can?

13. A housekeeper can buy Dutch cocoa in $\frac{1}{4}$-pound boxes at 28¢ a box or in 5-pound cans at $4 a can. How much does she save in purchasing by the can? What per cent of the $4 does she save by buying in this way?

14. A housekeeper can buy maple sirup at 85¢ a quart or in gallon cans at $2.85 a can. How much does she save in purchasing by the gallon? What per cent does she save?

15. When flour costs $12.25 a barrel (196 lb.) or 7¢ a pound, what per cent is saved in purchasing by the barrel?

16. A housekeeper's ice bill averages $3.60 a month. A better refrigerator, which will use only 75% as much ice, can be bought for $60. Not counting the saving due to the food being kept in better condition, in how many months will the saving in ice pay for the new refrigerator?

17. If each of 200,000 homes in a certain state wastes $\frac{1}{8}$ cup of milk a day, how many quarts (4 cups = 1 qt.) are wasted per day? How many quarts are wasted per year (365 da.)?

18. In Ex. 17, what is the cost of this waste in a year when the average price of milk is 11.6¢ a quart?

19. What would be the cost in Ex. 18 if, by taking more care, the waste should be reduced 35%?

20. A careless housekeeper wasted a cup of cereal each week. If the cereal costs 12¢ a pound and $2\frac{2}{3}$ cups weigh a pound, how much money is wasted in this way each year?

21. If each of 250,000 housekeepers in a certain state should waste as much as the one in Ex. 20, how much money would be wasted in this way in a year?

22. In making apple sauce and pies a woman pares the apples so carelessly that she loses 15% of the usable part of the fruit. If she buys apples costing 4¢ each and uses on an average $\frac{1}{2}$ doz. apples every fifth day in the year, how much money does she waste in this way every year?

23. A careless housekeeper threw away the water used in boiling meats and vegetables, and thereby lost material which could be used in making soup. If, as a result, she has to buy during the year 28 cans of soup at 15¢ a can, 2 doz. cans of soup stock at $1.25 a dozen, and soup meat costing $6.80, how much money is wasted in this way during the year?

24. In making a cake, a cook uses 3 eggs at 68¢ a dozen, $\frac{1}{2}$ cup of butter at 48¢ a pound, $\frac{1}{2}$ cup of milk at 14¢ a quart, 1 cup of sugar at 8¢ a pound, 2 cups of flour at 7¢ a pound, and 6¢ worth of other materials. Allowing 2 cups of butter or of sugar to the pound, 4 cups of flour to the pound, and 4 cups of milk to the quart, if she burns the cake so that it has to be thrown away, how much money is wasted?

DAIRY PROBLEMS

1. How many pounds of butter fat are credited to a farmer who sells to a creamery 28,000 lb. of milk which tests 4.2% butter fat?

2. If 6 lb. of butter fat are needed in making 7 lb. of butter, what is the value, at 44¢ a pound, of the butter produced from 2136 lb. of butter fat?

3. A farmer shipped to a creamery a quantity of milk which averaged 3.75% butter fat. If the butter fat weighs 890 lb., how many pounds of milk were there in the shipment?

4. How much is the butter fat referred to in Ex. 3 worth at 44¢ a pound?

5. Last month one of two cows belonging to a farmer gave 986 lb. of milk testing 3.1% butter fat, and the other 812 lb. testing 4.2%. Which cow gave the larger receipts for butter fat, and how much more did the farmer receive if the creamery paid 36¢ a pound for butter fat that month?

6. A creamery used 8500 lb. of milk in a certain week. The skim milk, containing 0.6% butter fat, amounted to 80% of the whole milk. During that week, how much butter fat was lost in the skim milk?

PROBLEMS ON SPRAYING

1. Bordeaux mixture, for spraying, contains $1\frac{1}{4}\%$ by weight of copper sulphate, $1\frac{1}{4}\%$ of unslaked lime, and the rest of the mixture is water. How many pounds of each substance are there in a barrel containing 250 lb. of Bordeaux mixture?

2. A cubic foot of Bordeaux mixture weighs 63 lb. From the facts given in Ex. 1 find the number of pounds of copper sulphate and unslaked lime in a tank 8 ft. long and 5 ft. wide, filled with Bordeaux mixture to a depth of 18 in.

3. Half of a peach tree was sprayed with Bordeaux mixture and the other half was not. From the sprayed half 340 lb. of fruit were taken, and from the other half 95% less. How many pounds of peaches were taken from the tree?

4. In preparing a spray for his trees, a farmer uses 3 lb. of arsenate of lead and 4 lb. each of lime and copper sulphate to 50 gal. of water. If the arsenate of lead costs 42¢ a pound, the lime 1¢ a pound, and the copper sulphate 16¢ a pound, and the man uses $2\frac{1}{2}$ gal. of the spray on each tree, what will be the cost of the material to spray 450 trees?

5. For spraying potato plants a man prepares a solution of Paris green, using $\frac{1}{2}$ lb. of Paris green costing 60¢ a pound to 100 gal. of water. He uses 300 gal. to an acre and sprays a field containing $4\frac{1}{2}$ A. If the labor costs \$18.50, what is the total cost? If the crop brings \$74 more than if he had not sprayed the field, how much is his gain?

6. In order to lessen the damage from potato scab a farmer soaks his seed potatoes in a solution made by adding $\frac{1}{2}$ pt. of formalin to 15 gal. of water. What is the per cent of formalin in the mixture?

7. In Ex. 6, what would be the per cent of formalin in the mixture if he had used $\frac{1}{8}$ pt. more to 15 gal. of water?

CAREFUL MANAGEMENT ON THE FARM

1. Frank's father owns two farms of about the same size and value. He runs one farm himself and lets the other on shares. In showing Frank the value of careful management on the farm, he made up a table showing the receipts last year from the home farm and from the tenant's farm. Here is the table from which he asked Frank to find the total receipts on each farm. How much did Frank find that they were?

Items	Home Farm	Tenant's Farm
Crops	$1075.30	$607.22
Dairy products	348.60	193.75
Domestic animals	695.75	388.17
Eggs and poultry	332.60	180.04
Wool	21.75	10.34

2. Frank then found for each item the per cent by which the home-farm receipts exceeded those of the tenant's farm. What results did Frank find?

3. The expenses of the two farms also showed poor management by the tenant. Here are the different items of expenses:

Items	Home Farm	Tenant's Farm
Feed	$52.60	$47.13
Fertilizers	86.75	10.00
Labor	575.45	502.43
Repairs	41.35	89.50
Taxes and miscellaneous	85.70	83.03

Find the difference between each pair of items, and between the total expenses of each farm.

4. From Exs. 1 and 3 find by how much the receipts exceeded the expenses on each farm, and then find the difference between these results.

PROBLEMS ON TRANSPORTATION

1. If a freight car is 38 ft. long, 8 ft. 4 in. wide, and 7 ft. 8 in. high, inside measurements, how many cubic feet does it contain?

2. If the width of the car in Ex. 1 were increased 1 in., the height $1\frac{1}{2}$ in., and the length 1 ft. 6 in., what would be the increase in capacity?

3. If a wooden coal car can carry 80,000 lb., and a steel car 140,000 lb., the steel car can carry what per cent more than the wooden car? The wooden car can carry what per cent less than the steel car?

4. A car 36 ft. long and 8 ft. 4 in. wide is loaded with wheat to a depth of 4 ft. 6 in. Taking a bushel as equivalent to $1\frac{1}{4}$ cu. ft. and the weight of a bushel of wheat as 60 lb., what is the weight of the wheat in the car?

5. If the car mentioned in Ex. 4 has a carrying capacity of 80,000 lb., to what depth must it be filled with wheat so as to load the car to capacity?

6. If a locomotive that weighs $115\frac{1}{2}$ T. can exert a pull of $22\frac{1}{4}\%$ of its own weight in starting a long train, how many pounds of pull does it exert?

7. A locomotive and coal tender together weigh 95 T., and there are seven cars to the train, averaging 21 T. each. If a horizontal pull equal to 0.5% of the total weight of the train is required to maintain a certain speed on a level track, how large a pull in pounds is required?

8. A 20,000-ton modern ship carries 12 times as much freight as the old-style ocean steamers, makes 25% better time, and the expenses per year are only 4 times as much. In a year a modern ship of this type will do how many times the work of the old kind? At the same cost, it will carry how many times as much freight?

PROBLEMS IN MANUFACTURING

1. Last year the sales of a manufacturing company with a capital of \$350,000 were $3\frac{1}{4}$ times its capital, and the profit was $7\frac{1}{2}\%$ of the sales. How much was the profit?

2. A dealer sells a lamp for \$21, which is 40% more than the cost to him. If the manufacturer who sold it to the dealer made a profit of 25% on the manufacturing cost, what was the cost of manufacture?

3. If the manufacturer allows a discount of 40%, how much does a dealer pay for 8 rockers listed at \$14.50, 9 dressers listed at \$29, and 16 bedsteads listed at \$37.50?

4. A manufacturer sells 8 doz. plates @ \$2.84; 9 doz. cups @ \$4.70; $\frac{3}{4}$ doz. covered dishes @ \$19.90; 16 doz. plates @ \$3.30; $\frac{5}{6}$ doz. pitchers @ \$13.50. If the manufacturer's selling price represents a profit of 20% on the manufacturing cost, find the cost of manufacturing the lot.

5. On which will a factory make the greater per cent of profit on the cost, an article which costs \$2.90 to make and sells for \$3.50 less 2% or one which costs \$8.70 to make and sells for \$16.50 less 40%? How much greater is the per cent?

6. A manufacturer makes a bid on a contract to turn out certain articles. He wishes to make a profit of 25% on the cost, and finds that the cost of labor and materials is \$3782.40. What bid, to the nearest \$1, should he make?

7. A company manufactures circular water tanks 20 ft. in diameter and 28 ft. high. Taking $7\frac{1}{2}$ gal. to the cubic foot, find the capacity in gallons that the company would use in describing the size of the tanks.

8. A factory pays its salesmen a commission of 12% on their sales. If a salesman sells \$66,725 worth of goods in a certain year, how much is his income that year?

PROBLEMS OF THE MERCHANT

1. How much is paid for an article marked \$82.50, when bought at a sale where a discount of 20% is given?

2. A dealer received \$31.50 for a suit of clothes which he sold at 10% off the marked price. What was the marked price?

3. A merchant, whose expenses of doing business average 20% of sales, sold for \$308 goods which cost him \$215.60. What is the per cent of profit on the cost? on the selling price?

4. A furniture dealer buys tables at \$16.50 each and sells them at \$25 each. His expenses of doing business are 24% of the cost. What per cent does he make on the cost?

5. If the dealer in Ex. 4 had calculated his cost of doing business as 24% of the selling price, what would have been his per cent of profit on the selling price?

6. A hardware dealer buys stoves at \$28.50 each. His cost of doing business is 28% of sales, and he wishes to make a profit of 15% on the selling price. Find the selling price which the dealer should place on the stoves.

7. A merchant buys a line of carpeting at \$1.84 a yard. His cost of doing business is 14% of sales, and his profit is to be 10% on the selling price. Find, to the next larger multiple of 10¢, the price per yard at which he should sell the carpeting.

8. A dealer buys some kitchen chairs at \$2.24 each, with a discount of 25%, 10%. If his cost of doing business is 16% of sales, and the profit is to be 12% on the selling price, find, to the next larger multiple of 25¢, the price at which he should sell the chairs.

9. A dealer pays \$972.40 for a lot of goods. His cost of doing business is 20% of sales, and the profit is 12% on the selling price. Find the profit on the sale of the entire lot.

PROBLEMS OF THE BUILDER

1. The cellar of a building is to be 9 ft. deep, 92 ft. long, and 60 ft. wide, outside measurements. The excavation is to be extended 18 in. beyond each of the four walls, so as to allow space for working. How many cubic feet of earth must be removed? How many cubic yards?

2. In figuring the material needed for a brick wall 68 ft. long, 6 ft. high, and 2 bricks thick, a builder allows 14 bricks per square foot of outside surface for a 2-brick wall. How many bricks does he need? At $17 per M, how much will the bricks cost if in buying them he gives his order to the next larger 500 to allow for breakage?

3. The walls of a brick building, which is to be 78 ft. long, 35 ft. wide, and 28 ft. high, are to be 18 in. thick. Deducting 750 cu. ft. for openings, and allowing 22 bricks to a cubic foot, find how much the bricks will cost at $18.50 per M if the builder orders to the next higher M to allow for waste.

4. If a builder makes a profit of $1080 on a house that he sells for $9400, what per cent does he make on the cost?

5. If a builder makes a profit of 20% on the cost of a house that he sells for $7200, how much profit does he make?

6. A builder plans to put up a house costing \$5800 on a lot costing \$1250. If he estimates that he will spend \$250 worth of his time in planning the house and supervising its erection, for what price should he sell it in order to make a profit of 10% on the total cost?

7. A semicircular window over a door has a diameter of 4 ft. 1 in. Find the area and the perimeter of the window.

8. A room is heated by 168 ft. of steam pipe with an outside diameter of 2 in. Find the area of the radiating surface.

9. The distance from the eaves to the ridge of a roof is 13 ft., and the span, the horizontal distance between the eaves, is 24 ft. Find the rise of the roof; that is, the height of the peak above the level of the eaves.

10. The span of an A-shaped roof is 26 ft., and the rise of the roof is 6 ft. What is the distance, to the nearest 0.1 ft., from the eaves to the ridge?

11. A paper hanger measures the rooms of a small house and finds them to be 14 ft. $\times$ 16 ft., 8 ft. $\times$ 12 ft., 15 ft. $\times$ 18 ft., 10 ft. $\times$ 12 ft., 16 ft. $\times$ 16 ft., and 12 ft. $\times$ 12 ft. For openings he deducts 6 ft. from the perimeter of each room and allows one roll of paper for every $3\frac{1}{2}$ ft. of perimeter. How many rolls does he need in all? If the paper for the first two rooms costs 35¢ a roll, for the third and fourth 40¢, and for the last two 55¢, what is the total cost of the paper for the house?

12. In building a concrete wall 66 ft. long, 8 ft. high, and 18 in. thick, the concrete which the builder plans to use is to be $\frac{1}{6}$ cement, which weighs 150 lb. per cubic foot and costs \$3.80 a barrel of 400 lb., and the rest is to be gravel costing \$1.75 a cubic yard delivered. In ordering the material the builder cannot order a fraction of a barrel or of a cubic yard. What will be the cost of the material for the wall?

13. A cubic yard of "1 : 2 : 3" concrete contains approximately $1\frac{1}{2}$ bbl. of cement, $\frac{1}{2}$ cu. yd. of sand, and $\frac{3}{4}$ cu. yd. of crushed rock, the expression "1 : 2 : 3" indicating the proportions of cement, sand, and crushed rock. The sum of the volumes of the different materials is more than 1 cu. yd., but since the smaller particles of the sand and cement fill in around the pieces of rock, thus adding very little to the volume of the mixture, they make only 1 cu. yd. of concrete. How much of each of these materials is needed for the foundation of a building requiring 650 cu. yd. of concrete?

14. A cubic yard of $1 : 2\frac{1}{2} : 3$ concrete contains approximately $1\frac{1}{2}$ bbl. of cement, 0.6 cu. yd. of sand, and 0.7 cu. yd. of crushed rock. How much of each of these materials is required for 240 sq. yd. of concrete pavement 9 in. thick?

15. How many cubic yards of 1 : 2 : 3 concrete will 1 bbl. of cement make? This is equal to how many cubic feet? How many square feet of 1 : 2 : 3 concrete sidewalk can be made for each barrel of cement used if the concrete is 6 in. thick?

16. How many cubic yards of $1 : 2\frac{1}{2} : 3$ concrete will 25 bbl. of cement make? How much sand and gravel (see Ex. 14) is needed to mix with this amount of cement?

17. A farmer plans to build three concrete watering troughs of 1 : 2 : 3 concrete. The inside of each trough is to be 10 ft. long, 3 ft. wide, and 20 in. deep, the walls are to be 6 in. thick, and the bottom is to be 10 in. thick. First computing to the next larger whole cubic yard the amount of concrete required for the three troughs, find how much of each of the materials named in Ex. 13 is needed.

18. In Ex. 17 find the total cost of the material needed if the cement costs \$3 per barrel, the sand \$1.50 per cubic yard, and the crushed rock \$1.75 per cubic yard.

PROBLEMS ABOUT INVESTMENTS

1. A man has $25,000 invested at various rates of income as here shown. Copy the table and fill in the blank column:

Amount	Rate	Annual Income
$3000	6%	
2000	$4\frac{1}{2}$%	
2000	5%	
5000	$5\frac{1}{2}$%	
6000	4%	
3000	8%	
2500	$4\frac{1}{4}$%	
1500	$7\frac{1}{2}$%	
$25000		

Given the amount invested and the rate of income paid on the investment as follows, find the amount of income received for the stated period:

2. $100, 6%, 1 yr.
3. $200, 5%, 2 yr.
4. $150, 4%, 1 yr.
5. $500, 6%, 1 yr.
6. $800, 5%, 6 mo.
7. $960, 4%, 3 mo.
8. $9000, 6%, 2 yr.
9. $7500, 5%, 6 mo.
10. $1500, $5\frac{1}{2}$%, 6 mo.
11. $2500, 6%, 2 yr.
12. $7500, 5%, $2\frac{1}{2}$ yr.
13. $8000, $4\frac{1}{2}$%, $1\frac{1}{2}$ yr.

14. Robert saved his money and bought two 100-dollar bonds, which pay interest at $4\frac{1}{4}$% annually, and one 50-dollar bond, which pays interest at $4\frac{1}{2}$% annually. How much interest does he receive every 6 mo.?

15. Which gives the better rate of income, a bond paying 5% that can be bought at 113, including brokerage, or a mortgage that pays $4\frac{1}{2}$%?

16. A man has a choice of three investments which his banker tells him are equally sound. The first is a stock that pays $1\frac{3}{4}\%$ quarterly and can be bought at $129\frac{3}{4}$, the second is a stock that pays 5% and can be bought at $92\frac{1}{2}$, and the third is a bond paying $5\frac{1}{2}\%$ that can be bought at 100.40, including brokerage. Remembering to add the brokerage in the case of the stock, find the rate of income on each investment. In which one should you advise the man to invest?

17. If the rate of dividend on a first-class stock is higher than the average rate of income on good investments, will the stock be selling above or below par? Give the reasons on which you base your answer.

18. If you receive a circular advertising at below par a stock that promises to pay a very high dividend rate, and another circular advertising a new bond issue which will yield the current market rate of interest, in which investment should you prefer to put your money? Give the reasons on which you base your answer.

19. A young man received a legacy of $1000 and had the opportunity of making the following investments: (1) Southern Pacific stock at 105, paying 6%; (2) Pacific Gas stock at 90, paying 6%; (3) a Florida orange grove of 10 A., with an average income per acre of $25; (4) a first mortgage, paying 8%, for two thirds of the value of a farm; (5) Gold B Oil at $10 per share, 50% to be used for drilling, stock to pay 500% when oil is struck. If you were the young man, in which should you invest? Give reasons for your answer.

20. A man bought a house for $6000 as an investment. He finds that the average yearly expenses of maintaining the property are as follows: insurance, $5; taxes, $125; water rent, $15; upkeep, $150. What monthly rent, to the next higher $1, must he charge in order to receive 10% on his investment?

21. A man who bought a house as an investment leases it at a rental of \$50 a month. He pays \$122.40 a year for taxes, \$5.60 for insurance, and an average of \$65 for repairs. How much does the owner have left from the rental each year?

22. The owner of the house in Ex. 21 sells it for \$6400 and pays \$150 to an agent for making the sale. If he then invests the proceeds in a mortgage which pays 6%, is his income increased or is it decreased, and how much?

23. The new owner of the house spends \$500 in improvements. What is now his total investment in the house? What is the annual interest on this investment at 6%?

24. Owing to the improvements made in Ex. 23 the taxes are increased to \$142.30 a year, and the insurance to \$6.40. Since he has just fixed over the house, he has no repair bill the first year. What is the monthly cost of owning the house the first year, including the interest on his investment which he loses by not being able to invest his money in other ways? How does this compare with the rent that the first owner charged for the house as mentioned in Ex. 21?

25. The second year that he owns the house he pays \$75 for repairs as well as the other expenses given in Ex. 24. At the end of the year his business takes him to a distant city, and so he sells the house for \$7500. In figuring out whether he has lost or gained, he adds the expenses, including the interest on his investment, and compares them with the amount received from the sale. What is the result?

26. In studying the result in Ex. 25 the man realizes that he has made no allowance for the fact that he has lived in the house for 2 yr. and has paid no rent. Owing to the improvements made he could have rented the house for \$60 a month. Taking this figure as a fair rental, at what amount should the man in Ex. 25 calculate his gain or loss?

Recreations. Most of the following problems have a long and interesting history. They are part of the inheritance of the school, and show some of the amusing features of arithmetic. They have no practical value, and should be omitted unless taken in the spirit intended; that is, as recreations which show the lighter side of arithmetic.

RECREATIONS AND CURIOSITIES

1. Multiply 142,857 by 7 and notice the result.

2. Multiply 37 by 3; by 6; by 9; by 12; by 15; by 18; by 21. Notice the results.

3. How can you write the number 100 with Arabic figures without using zeros?

4. Write an even number, using only the odd digits.

5. If a goose weighs 10 lb. and half of its own weight, what is the weight of the goose?

6. If a goose weighs 14 lb. when standing on both feet, how much does it weigh when standing on one foot?

7. How many days would it take to cut a piece of cloth 50 yd. long into 1-yard lengths, one length being cut off daily?

8. If a post in a pond stands $\frac{1}{3}$ in the ground, $\frac{1}{4}$ in the water, and 10 ft. above the water, how long is it?

9. A bottle and its stopper cost \$1.10, and the bottle cost \$1 more than the stopper. How much did the stopper cost?

10. A new steel coal car and its contents weigh 139,700 lb., and the contents weigh 60,300 lb. more than the car. How much does the car weigh?

11. Two men plan to cross the Rocky Mountains. X can walk 13 mi. a day and Y only 10 mi. a day, and so Y starts 9 da. ahead. How long will it take X to overtake Y?

XI. LITTLE EXAMINATIONS

I. 1. Find the interest on \$750 for 63 da. at 5%.
2. At $3\frac{1}{2}$%, find the commission on \$14,500.
3. If the tax on \$8600 is \$215, what is the rate?
4. Find the yearly income from \$2250 invested at $5\frac{3}{4}$%.
5. Find the rate of income when \$1500 yields \$67.50 a year.

II. 1. Find the area of a circle with a diameter of 6.3 in.
2. Find the number of gallons in a tank holding 160 cu. ft.
3. Show how to construct one triangle equal to another.
4. Find, to the nearest 0.01: $\sqrt{1625}$; $\sqrt{88.69}$; $\sqrt{92.75}$.
5. Find the length of the diagonal of a floor 21 ft. by 28 ft.

III. *Solve the following equations and check each result:*

1. $x + 10 = 18$.
2. $19x - 7 = 31$.
3. $5x + 3 = 3x + 9$.
4. $7x - 5 = 3x + 19$.
5. $4x + 3 = 2x + 7$.
6. $8x - 37 = 2x - 7$.
7. $8x - 10 = 5x - 7$.
8. $2(3 + x) = 9 + x$.

IV. *Using the formula* $A = \frac{1}{2}bh$, *find the value of* A *when*

1. $b = 6$ and $h = 8$.
2. $b = 7$ and $h = 9$.
3. $b = 12.2$ and $h = 18.6$.
4. $b = 22.4$ and $h = 18.7$.
5. $b = 16\frac{1}{2}$ and $h = 12\frac{3}{4}$.
6. $b = 165\frac{3}{4}$ and $h = 126\frac{1}{2}$.
7. $b = 26.57$ and $h = 18.24$.
8. $b = 117.58$ and $h = 109.67$.

V. *From the following formulas, find formulas as stated:*

1. $A = bh$; for h.
2. $V = lwh$; for l.
3. $C = 2\pi r$; for r.
4. $A = \frac{1}{2}bh$; for b.
5. $V = \pi r^2 h$; for h.
6. $S = 4\pi r^2$; for r.
7. $i = prt$; for p.
8. $V = \frac{1}{3}\pi r^2 h$; for r.

CHAPTER V

I. PROBLEM SURVEY OF CHAPTER I

USING ADDITION AND SUBTRACTION

After Page 4

1. My automobile speedometer showed that I traveled 128 mi. on Monday, 97 mi. on Tuesday, 136 mi. on Wednesday, and 145 mi. on Thursday. What was the total mileage for the 4 da.?

2. Last night my total mileage for the season was 9286 mi. and yesterday I drove 138 mi. Before I started yesterday, how much was my total mileage?

3. If, on a motor trip of 835 mi., you travel 214 mi. the first day, 196 mi. the second day, and 231 mi. the third day, how many miles do you have to travel on the fourth day in order to complete the trip?

4. On March 1 our gas-meter reading was 6739 cu. ft., and at the end of the month it was 7592 cu. ft. How many cubic feet of gas were used during that month?

5. If Mrs. Roberts spent \$1.28 for groceries on Monday, 96¢ on Tuesday, \$3.20 on Wednesday, 75¢ on Thursday, \$1.42 on Friday, and \$1.07 on Saturday, how much did she spend for groceries during the week?

6. If John buys a book for \$2.65, a fountain pen for \$1.75, and a silver pencil for \$1.39, how much change will he receive from a 10-dollar bill?

LINDBERGH GOES TO PANAMA

After Page 4

You all remember when Colonel Lindbergh flew to Paris and also when he flew to Panama. Some of you have seen him and *The Spirit of St. Louis.* The map on the opposite page shows the distances between the cities along his route from Washington to Panama.

From the map, find how far Lindbergh traveled in flying

1. From Washington to Mexico City.

2. From Washington to San Salvador.

3. From San Salvador to Panama.

4. From Washington to Panama.

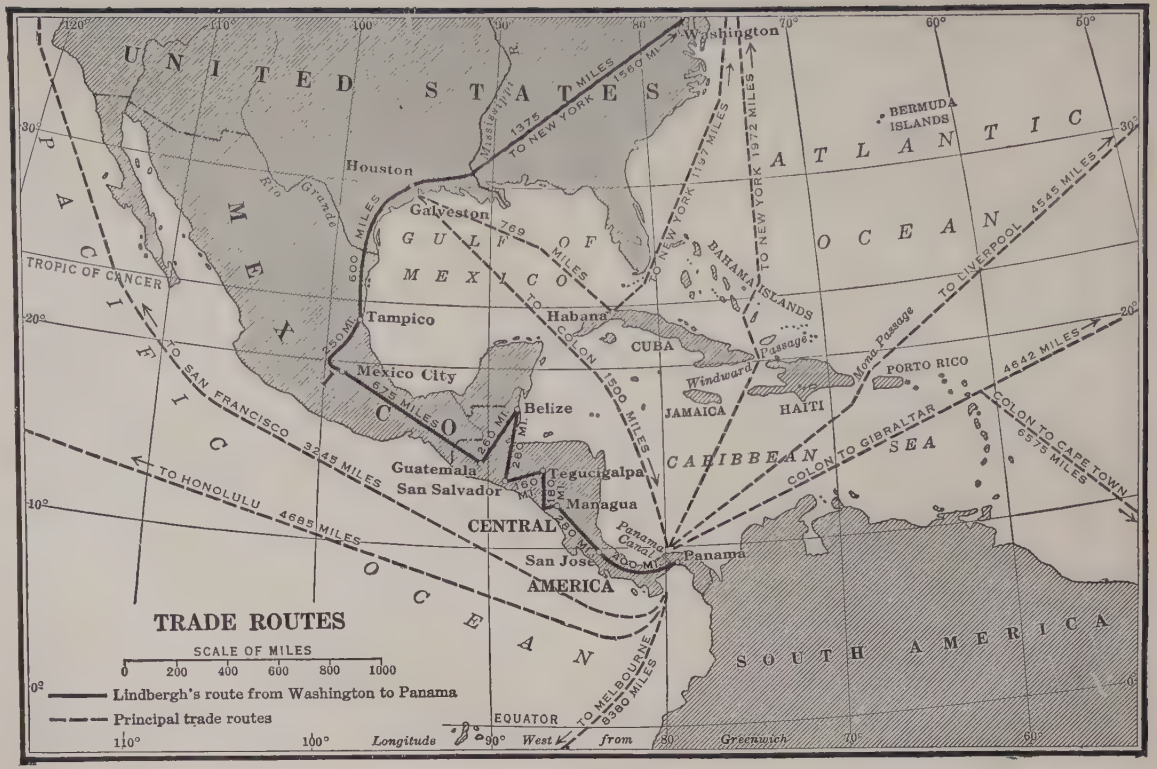
5. The air-route distance from Galveston to New York is the same as from Houston to New York. How much greater than the air-route distance is the steamship distance from Galveston to New York by way of Habana?

6. How does the steamship distance from Galveston to Colon compare with the length of Lindbergh's route from Houston to Panama? Which is the longer and how much longer is it?

7. How much longer is Lindbergh's route from New York to Panama than the steamship route through the Windward Passage (between Haiti and Cuba)?

8. If the straight-line distance from Guatemala to Panama is 825 mi., how much farther than this did Lindbergh fly in making his trip?

9. If the straight-line distance between Washington and Panama is 2075 mi., how much farther than this did Lindbergh fly in making his trip?



TRADE ROUTES
SCALE OF MILES
0 200 400 600 800 1000
Lindbergh's route from Washington to Panama
Principal trade routes
UNITED STATES
MEXICO
PACIFIC OCEAN
ATLANTIC OCEAN
GULF OF MEXICO
CARIBBEAN SEA
SOUTH AMERICA
CENTRAL AMERICA
Washington
Houston
Galveston
Tampico
Mexico City
Habana
CUBA
Belize
Guatemala
San Salvador
Tegucigalpa
Managua
San Jose
Panama
Panama Canal
BERMUDA ISLANDS
BAHAMA ISLANDS
JAMAICA
HAITI
PORTO RICO
Windward Passage
Mona Passage
Rio Grande
Mississippi R.
TROPIC OF CANCER
EQUATOR
TO NEW YORK 1560 MI.
1375 MILES
TO NEW YORK 1197 MILES
TO NEW YORK 1972 MILES
TO LIVERPOOL 4545 MILES
4642 MILES
COLON TO GIBRALTAR
COLON TO CAPE TOWN 6575 MILES
TO COLON 1500 MILES
769 MILES
600 MILES
250 MI.
675 MILES
260 MI.
280 MI.
160 MI.
180 MI.
280 MI.
200 MI.
TO SAN FRANCISCO 3245 MILES
TO HONOLULU 4685 MILES
TO MELBOURNE 8380 MILES
Longitude West from Greenwich

TRADE-ROUTE DISTANCES

After Page 7

In solving the following problems about trade routes, consult the map on page 417:

1. At the rate of 9 mi. per hour, how many days will it take a freight steamer to go from New York to Galveston by way of Habana?

2. How many days will it take a steamer averaging 15 mi. per hour, including all stops, to go from Liverpool to Melbourne by way of the Panama Canal?

3. How many days will it take a ship averaging 14 mi. per hour to make a trip from New York to San Francisco by way of the Windward Passage and the Panama Canal?

4. If the average speed is 14 mi. per hour, how many days will it take a steamer to go from New York to Honolulu by way of the Windward Passage and the Panama Canal?

5. If the average speed is 13 mi. per hour, how many days will it take a steamer to go to Honolulu from Liverpool by way of the Mona Passage (between Porto Rico and Haiti) and the Panama Canal?

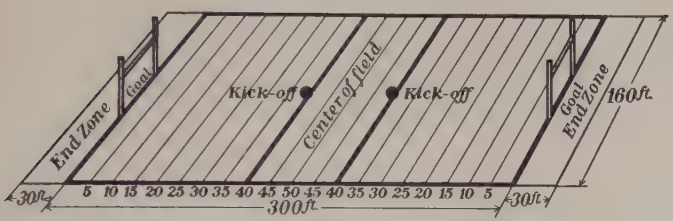
6. How long will it take an oil tanker averaging 9 mi. per hour to go from New York to Cape Town by way of Habana, Galveston, and Colon?

7. At the rate of 16 mi. per hour, how long will it take a ship loaded with oranges to go from San Francisco to Gibraltar by way of the Panama Canal?

8. Some types of airplanes can travel 50 mi. on 2 gal. of gasoline. How many gallons would such a plane need to cover the route of Lindbergh from Washington to Mexico City? from Washington to Panama?

OUTFITTING A FOOTBALL TEAM

After Page 7



Pants	$4.75	Elbow pads, each	$0.65
Shirt	3.50	Knee pads, each	.90
Stockings, pair	1.25	Shin guards, pair	1.95
Head gear	3.25	Belt	.75
Shoes	5.95	Football	6.50

1. The manager of the seventh-grade football team orders 7 new pairs of football pants, 7 new shirts, and 7 new belts. From the price list find how much they will cost.

2. What will 3 head gears, a football, 3 pairs of shoes, and 6 pairs of elbow pads cost?

3. What will be the cost of 15 pairs of stockings, 9 pairs of shin guards, and 9 pairs of knee pads?

4. Omitting the football, find the cost of a complete outfit for one player.

5. Find the cost of buying complete uniforms for two teams, each of which has 11 regular players and 2 substitutes.

6. The school sent 13 players, the coach, and the manager to the state championship game. If the fare was $2.95 each way, what was the total cost of the railroad fares?

TRAVELING BY RAILROAD

After Page 9

WESTBOUND		ALL TRAINS DAILY				EASTBOUND	
READ DOWN						READ UP	
No. 1	No. 2 Scenic Limited	Miles	STATIONS (*Mountain Time*)		Elevation in Feet	No. 3 Scenic Limited	No. 4 Denver and Eastern Express
4 00 PM	8 15 AM	0	Lv ..... DENVER	Ar	5280	**7 40** PM	**12 45** PM
5 59 PM	10 14 AM	52	Lv ..... Palmer Lake	Ar	7237	**6 05** PM	11 10 AM
6 45 PM	11 00 AM	75	Lv ... Colorado Springs ...	Ar	5989	**5 00** PM	10 05 AM
8 10 PM	**12 20** PM	119	Lv ..... Pueblo	Ar	4668	**3 30** PM	8 35 AM
9 15 PM	**1 17** PM	152	Lv ..... Florence	Ar	5199	**2 30** PM	7 28 AM
9 40 PM	**1 43** PM	160	Lv ..... Cañon City	Ar	5343	**2 11** PM	7 10 AM
12 09 AM	**4 15** PM	215	Lv ..... Salida	Ar	7050	11 45 AM	4 50 AM
.....	**5 10** PM	240	Lv ..... Buena Vista	Ar	7968	10 52 AM	3 52 AM
.....	**5 55** PM	276	Lv ..... Leadville	Ar	10200	10 15 AM	.....
3 00 AM	**7 00** PM	281	Lv .... Tennessee Pass	Ar	10240	9 30 AM	2 30 AM
3 37 AM	.....	294	Lv ..... Red Cliff	Ar	8608	...s....	...t...
4 10 AM	**8 05** PM	302	Lv ..... Minturn	Ar	7825	8 00 AM	12 50 AM
6 16 AM	**10 10** PM	360	Lv ..... Glenwood	Ar	5758	5 23 AM	**10 10** PM
6 50 AM	.....	373	Lv ..... New Castle	Ar	5562	...s....	**9 38** PM
7 18 AM	**11 02** PM	386	Lv ..... Rifle	Ar	5310	4 32 AM	**9 08** PM
7 54 AM	.....	404	Lv ..... Grand Valley	Ar	5104	...s....	.....
9 02 AM	.....	437	Lv ..... Palisade	Ar	4739	...s....	**7 10** PM
9 35 AM	1 01 AM	450	Ar .... Grand Junction	Lv	4583	2 45 AM	**6 51** PM
9 45 AM	1 11 AM	450	Lv .... Grand Junction	Ar	4583	2 35 AM	**6 41** PM
10 04 AM	.....	461	Lv ..... Fruita	Ar	4510	...s....	**6 14** PM
10 21 AM	1 45 AM	469	Lv ..... Mack	Ar	4540	...s....	**5 55** PM
12 18 PM	3 38 AM	528	Lv ..... Thompson	Ar	5160	12 08 AM	**4 07** PM
1 10 PM	4 30 AM	555	Lv ..... Green River	Ar	4080	**11 10** PM	**3 05** PM
3 30 PM	6 35 AM	619	Lv ..... Price	Ar	5546	**9 28** PM	**1 15** PM
4 00 PM	6 55 AM	627	Lv ..... Helper	Ar	5840	**9 10** PM	**12 51** PM
4 15 PM	.....	630	Lv ..... Castle Gate	Ar	6120	...s....	**12 27** PM
5 12 PM	.....	645	Lv ..... Colton	Ar	7170	...s....	11 53 AM
5 40 PM	8 15 AM	652	Lv .... Soldier Summit	Ar	7440	**8 05** PM	11 25 AM
6 53 PM	9 27 AM	681	Lv ..... Thistle	Ar	5033	**6 30** PM	9 55 AM
7 21 PM	.....	696	Lv ..... Springville	Ar	4555	...s....	9 23 AM
7 38 PM	10 08 AM	701	Lv ..... Provo	Ar	4517	**5 45** PM	9 12 AM
9 00 PM	11 30 AM	745	Ar .. SALT LAKE CITY ..	Lv	4224	**4 30** PM	7 55 AM

In solving the following problems about railroad routes, refer to the time-table given above:

1. When does Train No. 1 leave Denver? When does it leave Colorado Springs? Including stops, how long does it take this train to make the trip between these places?

2. How long does it take Train No. 3 to make the trip from Colorado Springs to Denver? How much longer does it take Train No. 2 to make the trip in the opposite direction?

3. Palmer Lake station is on a mountain between Denver and Colorado Springs. What is its elevation; that is, how high above sea level is it? What is the elevation of Denver? of Colorado Springs?

4. In Ex. 3, how many feet of elevation does a train climb in going from Denver to Palmer Lake? This is how many more feet than it climbs in going from Colorado Springs to Palmer Lake?

5. How many miles is it from Denver to Palmer Lake? Find, to the nearest foot, the average number of feet per mile that a train climbs in going from Denver to Palmer Lake.

6. Find the average number of feet per mile that a train climbs in going from Pueblo to Tennessee Pass.

7. Find the average number of feet per mile that a train climbs in going from Grand Junction to Tennessee Pass.

8. Limited trains usually make faster time and fewer stops than the regular trains. Train No. 2 is called the Scenic Limited. How much faster time does it make between Denver and Salt Lake City than Train No. 1?

9. Train No. 3 is also a limited train. How much less time does this train take between Salt Lake City and Denver than Train No. 4?

10. Find, to the nearest 0.1 mi., the average number of miles per hour made in going from Denver to Salt Lake City by Train No. 1; by Train No. 2.

11. Find, to the nearest 0.1 mi., the average number of miles per hour made in going from Salt Lake City to Denver by Train No. 3; by Train No. 4.

COST OF RAILROAD TRAVEL

After Page 9

In solving the following problems use 3.6¢ as the fare per mile and refer to the time-table on page 420:

1. What is the cost of a ticket from Denver to Pueblo? from Colorado Springs to Grand Junction?

2. How much more does it cost to buy a ticket from Denver to Salt Lake City than from Denver to Grand Junction?

3. What is the cost of a ticket from Salt Lake City to Soldier Summit? to Castle Gate? to Green River? to Tennessee Pass? to Cañon City?

4. What is the cost of a ticket from Grand Junction to Glenwood? to Tennessee Pass? to Castle Gate? to Pueblo?

5. How much will it cost Mr. Draper to travel by train from Colorado Springs to Salt Lake City and return?

6. If you had $20 to spend for car fare, and you got on Train No. 1 at Denver, how far could you ride? At what station should you have to get off?

7. If you start from Denver on Train No. 1 and pay $26.82 for your ticket, at what station should you get off the train? At what time should you arrive?

8. I got on Train No. 4 at Salt Lake City, and my ticket cost $24.12. At what station did I leave the train? If the train was 2 hr. late, when did I arrive?

9. What would be the cost of the tickets for a baseball team with 9 players and 4 substitutes from Denver to Palmer Lake and return?

10. How much money will a Scout troop of 25 boys have to earn in order to pay their fares from Pueblo to Colorado Springs and return?

MISCELLANEOUS PROBLEMS

After Page 10

1. If a man saved \$232.50 in 310 working days, what were his average savings per day?

2. The Rex Hardware Store is selling clothesline at \$1.50 for 50 ft. How much will 70 ft. cost?

3. In Ex. 2, how much clothesline can you buy with \$2.25?

4. Mrs. Reed bought a chicken weighing $4\frac{1}{2}$ lb. at $38¢$ per pound, 2 lb. of bacon at $34¢$ per pound, and $1\frac{3}{8}$ lb. of lamb chops at $48¢$ per pound. How much did she pay the butcher?

5. If a man in a shop is paid at the rate of $62\frac{1}{2}¢$ per hour, and $87\frac{1}{2}¢$ per hour for overtime, how much does he receive for working 18 hr. and 2 hr. overtime?

6. Jones Brothers are selling a new brand of coffee at $40¢$ a pound, or 5 lb. for \$1.80. If a housekeeper needs 20 lb., how much will she save by buying at the 5-pound rate?

7. Mrs. Case sent Arthur to buy a roast of beef, and he bought $4\frac{3}{4}$ lb. at $36¢$ a pound. If Arthur gave the butcher a 5-dollar bill, how much did he receive in change?

8. Mary saves $55¢$ a week from her allowance toward her college expenses. Taking 52 wk. to the year, at the end of 2 yr. how much will Mary have saved?

9. If the oven burners on a gas stove consume 8 cu. ft. per hour, and the gas company charges \$1.25 per 1000 cu. ft., how much will it cost to roast a leg of lamb for $2\frac{1}{2}$ hr.?

10. A farmer shipped 400 bbl. of apples to a city dealer. The dealer sold the apples for \$4.80 a barrel, and charged the farmer $24¢$ a barrel for selling, $8¢$ a barrel for storage, \$41.25 for freight, and \$32.25 for cartage. How much did the farmer receive from the dealer?

PROBLEMS INVOLVING STATISTICS

After Page 23

1. The census report for 1920 showed that the population of a certain small city had increased $46\frac{2}{3}\%$ since 1910, when the population was 12,015. What was the population of the city in 1920?

2. The chamber of commerce published the following report of the post-office receipts for the city for a period of several years. Copy the table and fill in the blanks:

Year	Total Receipts	Per Cent of Increase
1909	$54,928.40	17 % over 1904
1914		35 % over 1909
1919		22% over 1914
1924		31% over 1919

3. In 1924 the city engineer issued 360 permits for new buildings in the city. It is estimated that in 1930 he will probably issue $13\frac{1}{3}\%$ more. What is the estimated number for 1930?

4. The telephone company had 3729 telephones in use in the city in 1925. The estimated increase for 1930 is 26%. How many more telephones is it estimated that there will be in 1930?

5. Of the 3729 telephones in use in 1925, 63% were in houses and the rest were business telephones. How many telephones were used for business purposes?

6. In 1925 there were $60\frac{1}{2}$ mi. of sidewalks in the city, and it is estimated that by 1930 the mileage will have increased 34%. What is the estimated mileage for 1930?

7. In 1925 there were $36\frac{1}{4}$ mi. of paved streets in the city. If by 1930 there is an increase of 54% over 1925, how many miles of paved streets will there be then?

8. The report of the police department showed that in 1927 there was a decrease of $16\frac{2}{3}\%$ in the number of arrests as compared with 1926, in which there were 744 arrests. What was the decrease in the number of arrests?

9. The distance from the city to the state capital by automobile is 136 mi., of which $37\frac{1}{2}\%$ is along a new concrete road. How many miles of concrete must be built in extending this road the entire distance?

10. The enrollment in the public schools of the city was 2373 in 1925. In planning a new school it was estimated that the enrollment in 1930 would be $133\frac{1}{3}\%$ of that in 1925. What was the estimated enrollment for 1930?

11. Bank deposits in the city increased 74.7% from 1920 to 1928. In 1920 they amounted to $4,686,589. What was the amount of the deposits in 1928?

12. In the county in which the city is located there were 488 fruit farms in 1925. If the increase is $37\frac{1}{2}\%$ by 1930, how many more fruit farms will there be then?

13. The county shipped 223,080 bu. of grain in 1930. Of this amount $8\frac{1}{3}\%$ was barley, $16\frac{2}{3}\%$ was oats, and the remainder was wheat. What per cent of the grain shipped was wheat? How many bushels of each were shipped?

14. The value of the county's farm crops was $967,465 in 1920, and it is estimated that by 1930 this will have increased 39%. If the estimate is correct, what will be the value in 1930?

15. In 1920 there were 473 A. of timberland in the county. If the estimate for 1930 is 27% less, approximately how many acres of timberland will there be then?

OUR SCHOOL PLAY

After Page 26

1. Our school auditorium has 614 seats, and at the school play last year we sold tickets at four different prices. The table below shows the number of tickets for sale and the number sold at each price. Find the total number sold.

Price of Tickets	Number for Sale	Number Sold
\$1	120	80
75¢	130	126
50¢	164	152
25¢	200	200

2. To the nearest 0.1%, what per cent of the total number of tickets available were sold?

3. To the nearest 0.1%, what per cent of the total number available were offered for sale at \$1? at 75¢? at 50¢? at 25¢?

4. Of the total number of tickets in each price group, what per cent, to the nearest 0.1%, were sold?

5. Find the amount of money received from the sale of each group of tickets and also the total receipts.

6. To the nearest 0.1%, find what per cent of the total receipts came from the sale of each group of tickets.

7. After the play we voted to give the athletic association 35% of the total receipts to buy new football equipment. How much money did we give them?

8. We also voted to take 25% of the total receipts to buy new scenery for the stage. How much did we spend for this?

9. We decided to put the rest of the money in the bank for future use. How much did we put in the bank?

TIME SAVED BY AIR MAIL

After Page 28

Copy the following table, which relates to the air-mail and railway-mail services, and fill the blank column, giving each result to the nearest 1%:

	From San Francisco to	Time in Hours		Per Cent of Rail Time Saved by Air
		By Air	By Rail	
1.	Albany, N. Y. . . .	43	89	
2.	Atlanta, Ga. . . .	54	97	
3.	Baltimore, Md. . .	44	93	
4.	Boston, Mass. . . .	47	97	
5.	Buffalo, N. Y. . . .	40	83	
6.	Chicago, Ill. . . .	24	69	
7.	Cleveland, Ohio . .	31	79	
8.	Detroit, Mich. . .	34	76	
9.	Memphis, Tenn. . .	50	93	
10.	Milwaukee, Wis. .	29	71	
11.	Minneapolis, Minn.	39	69	
12.	New York, N. Y. .	36	93	
13.	Omaha, Neb. . . .	30	56	
14.	Pittsburgh, Pa. . .	38	83	
15.	Portland, Me. . . .	54	101	
16.	Rochester, N. Y. .	42	85	
17.	St. Louis, Mo. . . .	36	75	
18.	Washington, D.C. .	45	94	

FREIGHT ON AUTOMOBILES

After Page 34

Copy the following table, which gives the factory prices at Flint, Michigan, and the weights of different models of a well-known make of automobile. The freight rate from Flint to San Francisco is $4.65 per 100 lb. Fill the blank columns by finding the freight charges to San Francisco on each automobile and the per cent, to the nearest 0.1%, that this freight charge is of the factory price:

	Factory Price	Weight in Pounds	Freight Charge	Per Cent of Factory Price
1.	$1195	3310		
2.	1195	3090		
3.	1225	3140		
4.	1195	3215		
5.	1275	3300		
6.	1295	3370		
7.	1375	3400		
8.	1495	3920		
9.	1575	3930		
10.	1465	3835		
11.	1995	4085		
12.	1925	3980		
13.	1495	3655		
14.	1765	3890		
15.	1525	3735		
16.	1850	3925		

SELLING AT A LOSS

After Page 39

1. If a dealer was forced to sell for \$1050 an automobile costing \$1200, what was his per cent of loss on the cost?

2. Mr. Brown's motor boat originally cost him \$8500. This year he wanted a larger boat, and sold the old one for \$6000. What was his per cent of loss on the cost?

3. At a forced sale, fur coats costing \$275 were sold for \$190. What was the per cent of loss on the cost?

4. Mr. Morris sold his house which cost him \$9000 at a loss of 12% on the cost. What was the selling price?

5. If I sold a piece of land and lost \$150, which was 12% of the cost, how much did the land cost me?

6. The expenses for the basket-ball season in a large high school amounted to \$750. The receipts from the games were only \$675. The loss was what per cent of the expenses?

7. A carload of watermelons valued at \$729 was in a wreck. The railroad company sold the uninjured melons for \$81. The loss was what per cent of the value of the melons?

8. Mr. Williams sold an automobile at a loss of 25% on what it cost him. If he lost \$300 on the sale, what did he pay for the car? For how much did he sell it?

9. A merchant bought 85 yd. of silk at \$1.25 a yard and sold it at a loss of 8% on the cost. How much was his loss?

10. A dealer bought some glassware for \$578.50. Through his fault some of the glassware was broken, and he sold the rest for \$600. If his expenses are estimated at \$126.75, what per cent, to the nearest 0.1%, of the total cost (cost plus expenses) does he gain or lose on the transaction?

SELLING AT A PROFIT

After Page 42

A dealer in used cars finds that his expenses, such as rent, labor, insurance, and so on, average 17% of the selling price of every car he handles. He plans to sell his cars so as to cover the expenses and to make a profit of 15% on the selling price of each car. Find, to the nearest $5, the price he must charge for each of the following cars, bought at the prices given:

1. Sedan, $225.
2. Touring, $175.
3. Touring, $135.
4. Touring, $200.
5. Sedan, $230.
6. Coupé, $210.
7. Touring, $430.
8. Coupé, $350.
9. Coupé, $865.
10. Brougham, $950.
11. Coupé, $250.
12. Sedan, $1550.
13. Sedan, $1250.
14. Coach, $850.
15. Coach, $450.
16. Touring, $500.
17. Sedan, $300.
18. Roadster, $375.
19. 1-ton Truck, $340.
20. Coupé, $330.
21. Sedan, $1080.
22. Coupé, $975.
23. Touring, $550.
24. 1-ton Truck, $280.
25. $1\frac{1}{2}$-ton Truck, $1650.
26. Coupé, $1360.
27. Limousine, $1950.
28. $2\frac{1}{2}$-ton Truck, $1500.
29. Sedan, $1850.
30. Beach Wagon, $750.
31. Sedan, $1230.
32. Roadster, $650.
33. Coupé, $1980.
34. 5-ton Truck, $2250.
35. Coupé, $915.
36. Special Sedan, $1575.
37. School Bus, $5500.
38. Sport Roadster, $785.

REVIEW OF COMMISSION

After Page 50

1. Mr. Ross sold his house for $9000. He paid a commission of 2% to the real-estate agent who found a buyer. How much money did the real-estate agent receive?

2. Mr. Fox collects bills for the merchants in a small city, at a commission of 10%. Last month he collected $2318.70. How much commission did he receive?

3. Ben's father pays him a commission of 5% on all the farm produce he sells to tourists. At the end of the summer Ben's bank account showed that he had earned $130.05. What was the amount of his sales?

4. A college student received a commission of 40% for selling books during the summer. His commission amounted to $550. How many dollars' worth of books did he sell?

5. An automobile company receives a commission of 8% for selling cars. If the company received a commission of $1200 last month, what was the amount of the total sales for the month?

6. Tim's father drives through the country and buys poultry for a dealer in the city. He is paid a commission on the value of the poultry he buys from the farmers. Last week he bought $1575 worth of poultry, which with his commission amounted to $1653.75. What was the rate of his commission?

7. If a poultry-buyer's purchases, plus his commission of 5%, amounted to $2257.50, what was the value of the poultry which he bought? How much was his commission?

8. If an apple buyer pays the owner of an orchard $4500 for his apple crop, and it takes a check for $4590 to pay for the apples and his commission, what is his rate of commission?

NET PRICE BY ONE MULTIPLICATION

After Page 55

Using only one multiplication, find the net prices of the automobiles listed in Exs. 1–10:

1. Sedan listed at \$1195; discount, 5%.
2. Sport touring car listed at \$1225; discount, 5%.
3. Country Club coupé listed at \$1275; discount, 8%.
4. Town brougham listed at \$1350; discount, 15%.
5. Sedan listed at \$1500; discount, 18%.
6. Coupé listed at \$1995; discount, 20%.
7. Sedan listed at \$1850; discount, 20%.
8. Sport touring car listed at \$1525; discount, 25%.
9. Roadster listed at \$1495; discount, 30%.
10. Master-six sedan listed at \$2250; discount, 30%.

11. A 36-foot motor boat listed at \$7250 is offered for sale at a discount of 20%. What is the net price?

12. A 50-foot house boat was originally offered for sale at \$14,750, but in order to make a quick sale it is now offered at a discount of 40%. What is the net price?

13. What is the net price of a farm sold for \$15,000, less 30%?

14. Find the net price of a dozen baseball bats listed at \$1.25 each, with a discount of 15%.

15. Find the net price of a dozen fielder's mitts listed at \$2.50 each, with a discount of 10%.

16. What is the net price of a balloon tire listed at \$38.50, with a discount of 15%?

17. What is the net price of a balloon tire listed at \$22.50, with a discount of 8%?

REVIEW OF DISCOUNTS

After Page 60

Find the net price of each of the following articles recently advertised in a newspaper:

1. Hats, formerly \$15, now at a reduction of $66\frac{2}{3}\%$.

2. Dresses, formerly \$45, $\frac{1}{3}$ off; \$65, $\frac{1}{3}$ off; \$85, $\frac{1}{4}$ off.

3. Bags, formerly \$5, \$10, \$12.50, \$15, \$20, all 20% off.

Find the per cent of reduction, to the nearest 0.1%, on the articles advertised in the same paper, as follows:

4. Evening wraps, formerly \$85, now \$42.50; formerly \$150, now \$100; formerly \$110, now \$66.

5. Coats, formerly \$45, now \$20; formerly \$65, now \$40; formerly \$110, now \$70; formerly \$195, now \$95.

6. Dresses, formerly \$45, now \$30; formerly \$40, now \$25; formerly \$85, now \$40; formerly \$120, now \$70.

Find the net amount of each of the following orders:

7. 60 arithmetics @ 92¢, 84 advanced geographies @ \$1.60, 72 spelling books @ 60¢. Discount, 20%.

8. 1500 readers @ 88¢, 1460 histories @ \$1.50, 540 Latin books @ \$1.25. Discount, 25%.

9. 2 cases of tomato soup @ \$5.75, 3 doz. cartons of sugar @ \$1.20, 2 doz. bottles of pickles @ \$2.35. Discount, 3%.

10. 1 doz. hammers @ 75¢ each, 3 kegs of nails @ \$2.15, 6 small stoves @ \$6.65. Discount, 10%, 10%, 2%.

11. 2 doz. quart cans of floor paint @ \$5.75, 1 doz. pint cans of enamel @ 28¢ each. Discount, 15%, 10%, 2%.

12. 1 doz. dining-room chairs @ \$1.75 each, 4 dining tables @ \$10, 2 small sideboards @ \$8.75. Discount, 15%.

FIRE-INSURANCE PREMIUMS

After Page 62

1. Mrs. Edwards insured her house for $4500 at a rate of $1.60 per $100. Find the amount of the premium.

2. Mr. Hubbard's house is insured for $8000. The rate is only 80¢, because the roof of his house is noncombustible and he lives in a neighborhood where the fire hazards are slight. What is the amount of his premium?

3. The Kane Drug Company took out a policy for $1500 on drugs, medicines, and supplies. Find the premium on this policy at a rate of $1.67.

4. The State Asylum takes out a policy for $15,000 on its hospital building at a rate of $2.34. Find the premium.

5. Mr. McCoy pays a rate of $2.60 for 3 yr. on his house and furniture. The building is insured for $7000 and the furniture for $1500. What does it cost him?

6. Mr. Sprague insures his farmhouse for $4000, its contents for $1000, and his garage for $500. Find the total premium at a rate of $2.30.

7. The Stoddard Molding Company placed a policy for $5000 on its lumber. The rate was $4.45 for 1 yr. Find the cost of this insurance. Give any reasons for this high rate that you can.

8. A trucking company insures a 3-ton truck for $4000 at a rate of $1.40. Find the cost of the insurance.

9. The Colorado Coal Company carries a policy for $21,000 on its stock of coal and wood. At a rate of $2, what is the amount of the premium?

10. Mr. Sweet carries a policy for $3000 on his house. If the rate is 90¢, what is the premium?

WORKMEN'S COMPENSATION LAWS

After Page 70

In the following problems refer to the facts given on page 70:

1. A workman's wages are \$32 a week. In case of his death, how much does his widow receive per week?

2. A truck-driver receiving \$45 a week was killed at a railroad crossing. How much will be paid to his widow each week during her life if she does not remarry?

3. The truck-driver mentioned in Ex. 2 had 3 children under the age of 18 yr. How much will be paid for their support per week? for a year of 52 wk.?

4. In an accident in a steel mill, the father of 2 small children was killed. If the man's wages were \$42 per week, how much would be paid for the support of each child per week? How much would be paid for the support of both children and their mother for a year of 52 wk.?

5. A painter earning \$36 a week fell and broke his right arm, which disabled him for 14 wk. He received a disability allowance of 60% for 14 wk. How much did he receive per week? during the 14 wk.?

6. The father of 3 children, whose mother is not living, was earning \$45 per week at the time of his death from an accident. Compensation was paid for one of the children for $6\frac{1}{2}$ yr., for another for $4\frac{3}{4}$ yr., and for the third for 2 yr. Find the total amount of compensation paid, taking 52 wk. to the year.

7. A bridge-builder, who was earning \$1.25 an hour and working 44 hr. a week, was disabled for 20 wk. He was allowed $\frac{2}{3}$ of his wages during that time. If his doctor's bill amounted to \$35, how much was left for the support of the family for the 20 wk. during which he was disabled?

MISCELLANEOUS PROBLEMS

After Page 88

1. If Mr. Sinclair has an income of $3000 a year and he spends 22% of it for rent, how much does he spend for rent per year? per month?

2. In Ex. 1, suppose that Mr. Sinclair's rent is increased $10 per month. He will then have to spend what per cent of his income for rent?

3. If a man who has an income of $6400 a year spends $5280, what per cent of his income does he save?

4. John earned $1.60 one day by mowing lawns for the neighbors and spent 40¢ of it for a knife. What per cent of what he earned did he spend?

5. Mr. Williams, who is a traveling salesman for a manufacturing company, receives a salary of $2000 a year and a commission of 6% on the amount of the sales which he makes. If during the year he sells goods amounting to $48,450, what is his total income for the year?

6. A man buys a house for $8000 as an investment and rents it for $720 a year. The rent which he receives is what per cent of the cost?

7. If $37\frac{1}{2}$ T. of iron are obtained from 705 T. of ore, what per cent, to the nearest 0.1%, of the ore is iron?

8. An article listed in a catalogue at $92.50 is sold at a discount of 10%, with an extra discount of 2% for cash. Find the total amount that a dealer pays if he buys for cash.

9. A grocer insured his store, which was worth $10,000, for 80% of its value, at a rate of $2.35 per $100. If fire caused a total loss, how much did he lose, including the premium paid?

II. PROBLEM SURVEY OF CHAPTER II

INTEREST FOR SEVERAL YEARS

After Page 101

1. Mr. Kingsley bought a house for $10,000. He paid $6500 in cash and agreed to pay interest at 6% a year on the balance. How much interest did he have to pay at the end of the first year? How much interest does he pay during 6 yr.?

2. In Ex. 1, if Mr. Kingsley had not bought the house but had invested his $6500 at 6% for the 6 yr., how much interest would he have received in all?

3. If a man has $2000 which is drawing interest at 6% a year and uses the money to buy an automobile, how much interest does he lose in 2 yr.?

4. A man who was paying $50 a month for rent bought for $9000 the house in which he was living. He owned the house for 5 yr., during which time he had to pay a total of $450 for insurance and repairs and $140 each year for taxes on the property. At the end of 5 yr. he sold the house for $8500. If he could have invested the $9000 so as to receive interest at 6% a year, how much would he have saved in the 5 yr. by continuing to rent the house instead of buying it?

5. If a man uses $5000, upon which he has been receiving interest at the rate of 5%, to buy a motor boat, how much interest will he lose in 3 yr.?

6. If a widow who received $10,000 from her husband's life insurance invests $8000 at 5% and the remainder at 6%, how much interest will she receive in 5 yr.?

7. A business man borrows $4270, agreeing to pay interest at $5\frac{1}{2}\%$. If he pays the principal and interest at the end of 3 yr., how much does he pay, including principal and interest?

REVIEW OF INTEREST

After Page 112

Copy the following table and fill the blank spaces:

	PRINCIPAL	RATE	TIME	INTEREST
1.	$250	4%	1 yr.	$——
2.	$750	5%	——	$18.75
3.	$850	5%	6 mo.	$——
4.	$500	——%	1 yr.	$30
5.	$1200	4%	3 mo.	$——
6.	$3600	5%	——	$270
7.	$4200	——%	2 yr.	$189
8.	$5000	5%	2 mo.	$——
9.	$900	6%	——	$36
10.	$1250	$5\frac{1}{2}$%	30 da.	$——
11.	$2500	7%	45 da.	$——
12.	$5000	$4\frac{1}{2}$%	——	$112.50

13. By using the six-per-cent method, find the interest at 6% on $1750 for 60 da., for 90 da., for 15 da., for 10 da., for 45 da.; on $3500 for 60 da., for 90 da., for 15 da.

14. On May 15 a dealer borrowed $75,000 at 6% with which to purchase his stock of summer merchandise. When he repaid the loan on October 15, how much did he pay to settle the account?

15. A man puts $2500 into an investment that bears interest at $5\frac{1}{2}$% a year and has 4 yr. to run. How much income will he receive from his investment during this period?

BUDGETS AND SAVINGS

After Page 122

1. In making up a budget, a family having an income of $3600 a year allows 15% of the income for expenditures for charity, education, books, travel, and entertainment, and includes all these items under the heading "Advancement." The household account shows that the expenditures for advancement were $35.25 in January, $38.25 in February, and $46.50 in March. The total of these expenditures is what per cent of the income for these months? By how much do the expenditures differ from the amount allowed in the budget?

2. A family having a yearly income of $2600 made the following budget for the year's estimated expenditures:

Rent, heat, light . .	$700	Advancement	$224
Food.	860	Savings.	184
Clothing	400	Miscellaneous	232

Each item is what per cent of the total income?

3. A man has a salary of $1750 a year. He pays 15% of his salary for rent, 25% for groceries, 12% for meats, 13% for clothing, $112.50 for other expenses, and saves the rest. How much does he spend for each of these items, and what per cent of his salary does he save?

4. If a man saves $5 a day each day of the year (365 da.), how much richer, disregarding interest, will he be at the end of the year? at the end of 15 yr.?

5. A family budget allows for food 26% of an income of $180 a month. If the food last month cost $36, this amount was what per cent of the total monthly income? The amount spent was how much less than the budget allowed?

REVIEW OF BANKING

After Page 144

1. A man deposited \$1500 on Jan. 1 in a savings bank which pays 4% a year, compounded semiannually. If he makes no withdrawals, what will his balance be at the end of 3 yr.? How much more or less will he have received in interest than if he had invested the money at simple interest at 4%?

2. What is the interest at 6% on a 60-day note for \$750?

3. What is the interest at 6% on a 30-day note for \$1250?

4. A business man borrows \$2250 on a 90-day note. If he has to pay 6% for the use of the money, to what will the interest on the note amount?

5. Suppose that you borrow \$1500 from a bank, giving your note payable on demand with interest at 6%. If you pay the note 24 da. later, how much do you pay?

6. A man borrowed \$2500 on June 1, 1929, at 5%, and \$3500 on Aug. 5, 1929, at $5\frac{1}{2}$%. He paid the principal and interest on both loans on July 1, 1930. How much did he pay?

7. If I borrow \$3600 on a 90-day note and have to pay the bank $5\frac{1}{2}$% for lending me the money, to what will the interest on the loan amount?

8. A manufacturer sells a dealer an order of goods amounting to \$850 and takes the dealer's note, due in 90 da. at 6%. If the manufacturer discounts this note at his bank 30 da. later at 5%, what are the proceeds?

9. On an invoice of goods amounting to \$2975 a merchant is allowed a trade discount of 20%, 10%, and an extra discount of 2% for payment within 10 da. If, in order to take advantage of the cash discount of 2%, he borrows \$2150 for 90 da. at 6%, does he gain or lose by so doing, and how much?

PROBLEMS IN SUBTRACTING MEASURES

After Page 152

1. When we stopped at the gasoline station, the gasoline gauge on our car read $5\frac{1}{2}$ gal. If the tank holds 15 gal. 3 qt., how much gasoline would it take to fill it?

2. The gasoline tank on our motor boat holds 59 gal. 1 qt. Last Saturday afternoon we went for a trip on Long Island Sound. The tank was filled when we started, and when we reached home it contained 32 gal. 3 qt. How much gasoline did we use on the trip?

3. One Saturday morning there were 763 gal. 2 qt. of gasoline in one of the tanks at the filling station. By night it was all sold except 75 gal. 2 qt. Find the quantity sold.

4. There were 72 lb. 6 oz. of sugar in a large sack in a grocery store, and one day later the sugar had all been sold except 9 lb. 8 oz. How much had been sold?

5. Ben used a piece of his mother's clothesline for his sled. The clothesline was originally 84 ft. 7 in. long, and Ben cut off a piece 11 ft. 9 in. long. How much was left?

6. There were 53 T. 760 lb. of coal in a freight car. After a coal dealer had unloaded 17 T. 1320 lb., how much coal was left in the car?

7. A Chicago and Northwestern train takes 12 hr. 15 min. to make the trip from Chicago to Omaha. After you have been riding 5 hr. 55 min. on this train, how much longer will it take to reach Omaha if the train is on time?

8. From a piece of linoleum containing 42 sq. yd. 1 sq. ft. there was cut a piece containing 13 sq. yd. 2 sq. ft. How much was left in the original piece? This was how much more than half of the original piece?

PROBLEMS IN MEASURES

After Page 154

1. How many 12-ounce tins can be filled from a quantity of cocoa weighing 516 lb. 12 oz.? How many 8-ounce tins?

2. How many glass jars holding $6\frac{1}{2}$ oz. can be filled from a vat containing 1218 lb. 12 oz. of peanut butter?

3. How many rubber mats 2 ft. 3 in. long can be cut from a piece 80 ft. 9 in. long of the right width?

4. A western railroad is building 120 mi. of new track. The railroad ties are 1 ft. 8 in. apart, measured from the center of one tie to the center of the next. How many ties will be needed for 1 mi. of track? for 120 mi.?

5. How many rails, each 34 ft. long, will be needed to lay 170 mi. of double-tracked railroad?

6. Our car uses 1 qt. of oil every 150 mi. On a trip last summer we used $8\frac{1}{2}$ gal. How many miles did we go?

7. It takes $1\frac{3}{4}$ yd. of ribbon to tie up a certain size of candy box. How many of these lengths can be cut from a piece of ribbon 12 ft. 6 in. long? 9 ft. 4 in. long? 11 ft. 2 in. long?

8. How many $\frac{1}{4}$-pound packages of creamery butter can be obtained from a quantity weighing 499 lb. 12 oz.?

9. If it takes $8\frac{1}{3}$ oz. of jelly to fill one jar, how many jars can be filled with 24 lb. $7\frac{2}{3}$ oz. of jelly?

10. How much is saved per ton in buying coal at \$14.75 a ton instead of at 55¢ per basket of 50 lb.?

11. When seed potatoes were worth \$1.15 a bushel, a farmer paid \$57.50 for enough to plant 5 A. How many bushels did he allow for planting each acre?

12. How many telegraph poles, placed $2\frac{1}{2}$ rd. apart, are needed for 75 mi. of line?

13. For a school sale the girls filled some boxes with salted peanuts, each box containing 4 oz. If the girls filled 72 boxes, how many pounds of peanuts did they use?

14. If you wish to cut a strip of molding 12 ft. 6 in. long into pieces $14\frac{3}{8}$ in. long, how many such pieces can you cut, and how much of the strip will be wasted?

15. How many square feet of wire screening are there in a strip $9\frac{1}{2}$ yd. long and 30 in. wide? How much is it worth at 8¢ per square foot?

16. A coal dealer sends a truck load of coal to a school. The total weight of the truck and the coal is 16,225 lb., and the empty truck weighs 5225 lb. How much should the school pay for the coal at $8.20 a ton?

17. Find the cost of $12\frac{1}{4}$ yd. of wire screening 22 in. wide at 5¢ per square foot.

18. How many jelly glasses holding $\frac{1}{2}$ pt. each will be needed for 12 qt. of jelly?

19. A class of 26 pupils made some badges, each requiring 8 in. of ribbon. How many yards did they need for 26 badges?

20. Which is cheaper, to buy a gross of articles for $21.60 or to buy 12 doz. at $1.90 a dozen? How much cheaper is it?

21. If gold leaf is 0.001 in. thick, how many sheets of gold leaf, laid one on top of another, would it take to make a pile that is $2\frac{1}{2}$ in. high?

22. From New York to Albany the distance by one automobile route is 146 mi. How long will it take to cover this distance at an average rate of 18 mi. per hour?

23. If a farmer feeds his horses 4 qt. of oats per day per horse, how many bushels of oats per week will be needed to feed 6 horses?

RECTANGLES, TRIANGLES, AND PARALLELOGRAMS

After Page 159

1. Making no allowances for openings, find the total number of square yards in the walls and ceiling of a room which is 14 ft. long, 13 ft. wide, and 9 ft. high.

2. A section of land is 1 mi. square. How many rods of woven wire will it take to build a fence around the section?

3. An electric sign is to be built in the shape of a triangle with a base 16 ft. long and a height of 15 ft. If there is to be a light for every 2.5 sq. ft. of surface, how many electric lights will be needed?

4. A kitchen floor $16\frac{1}{2}$ ft. by 12 ft. is to be covered with linoleum costing $1.80 a square yard. Making no allowances for waste in cutting and laying, how many square yards of linoleum are needed, and how much will it cost?

5. How much would the linoleum cost for a floor twice as long and twice as wide as the one in Ex. 4?

6. A city lot is in the form of a parallelogram which has a base 66 ft. long and a height of 30 ft. Find the area in square feet; in square yards.

7. A school pennant is in the form of a triangle with a base 18 in. long and a height of 45 in. Find what part of a square yard there is in the pennant.

8. A traveling salesman carries some samples of dress goods, each of which is a triangle with a base of 6 in. and a height of 5 in. How many square yards are there in 600 samples?

9. On each side of a private street 900 ft. long and 36 ft. wide are 10 lots, each with the same frontage. It will cost $5.50 per square yard to pave the street. If the owners of the lots share the cost equally, how much will each pay?

USING THE DIAMETER AND RADIUS

After Page 162

1. A draftsman has to draw a circle with a circumference of 6.6 in. What radius should he use?

2. The circumference of the iron flagpole in our school yard is 22 in. measured close to the ground. The janitor said that he thought the diameter was 8 in. Was he right? If not, what is the diameter?

3. We have a circular swimming pool in our new park. In planning for some races we wanted to know the distance across the pool. Measuring the circumference with a tape, we found that it was 264 ft. How far is it across the pool?

4. A wheelwright finds that it takes a strip of iron 201 in. long to go around a wagon wheel and to allow 3 in. overlap for welding. What is the diameter of the wheel?

5. The circumference of the geography globe in our schoolroom is 44 in. What is the diameter of the globe?

6. We have an athletic field, 500 ft. square, in which the boys want to lay out a circular race track 15 ft. wide. Is there room for a track $\frac{1}{2}$ mi. long, measured along the inside edge, or will they have to plan for a $\frac{1}{4}$-mile track?

7. Uncle Tom is sending Louise a circular fish bowl for Christmas. It is 44 in. around the top and 9 in. deep. He plans to make a wooden box to pack it in, the box to be 4 in. larger, inside dimensions, than the dimensions of the bowl, in order to allow plenty of room for packing material. Find what the inside dimensions of the box should be.

8. How many rivets, spaced approximately $3\frac{1}{8}$ in. apart between centers, are there in a seam that runs directly round a boiler 6 ft. 3 in. in diameter?

MEASURING LUMBER

After Page 167

1. A piece of board that is 12″ by 12″ and 1″ thick contains 1 bd. ft. of lumber. How many cubic inches are there in such a piece?

2. How many cubic inches are there in a piece of board 24″ long, 6″ wide, and 1″ thick? How many board feet are there?

3. How many cubic inches are there in a piece of board 48″ long, 3″ wide, and 1″ thick? How many board feet are there?

4. How many cubic inches are there in a piece of plank 12″ long, 6″ wide, and 2″ thick? How many board feet are there?

5. How many cubic inches are there in a piece 12″ long, 3″ wide, and 4″ thick? How many board feet are there?

6. How many cubic inches are there in a piece of plank 24″ long, 6″ wide, and 4″ thick? How many board feet are there?

7. Suppose that you have 12 pieces of board, each 12″ by 12″ and 1″ thick. If you pile them one on top of the other, what is the thickness of the pile? How many cubic feet of lumber are there in the pile? How many board feet does each piece contain? How many board feet are there in the pile?

8. Find the number of cubic feet of lumber contained in a board that is 12′ long, 12″ wide, and 1″ thick. How many board feet are there?

9. How many cubic feet are there in a plank 12′ long, 12″ wide, and 2″ thick? How many board feet are there?

10. How many cubic feet of lumber are there in a piece of timber 24″ long, 12″ wide, and 12″ thick? How many board feet are there?

11. For lumber 1″ or more in thickness, the number of board feet is how many times the number of cubic feet?

Find the number of board feet in each of the following pieces, the length, width, and thickness being given in this order:

12. $4' \times 6'' \times 1''$; $8' \times 6'' \times 1''$; $8' \times 6'' \times 2''$.

13. $12' \times 2'' \times 4''$; $12' \times 4'' \times 2''$; $16' \times 4'' \times 3''$.

14. $12' \times 6'' \times 3''$; $12' \times 8'' \times 2''$; $18' \times 10'' \times 2''$.

15. A carpenter has 6 boards, each 12′ long, 6″ wide, and 1″ thick. How many board feet has he?

16. A plan for a partition calls for 24 pieces of $2'' \times 4''$ joist, each piece to be 12′ long. How many board feet are required?

17. At \$48 per M, what will be the cost of 10 planks, each 12′ long, 10″ wide, and 2″ thick?

18. At \$54 per M, what will be the cost of 72 18-foot pieces of $2'' \times 10''$ floor timber?

Find the cost of each of the following lots of lumber:

19. 16 pieces, 12′ long, 8″ wide, and 2″ thick, at \$38 per M.

20. 24 pieces, 16′ long, 2″ wide, and 4″ thick, at \$46 per M.

21. 160 pieces, 16′ long, 6″ wide, and 3″ thick, at \$52 per M.

22. 180 pieces, 14′ long, 8″ wide, and 8″ thick, at \$68 per M.

23. At \$49 per M, what will be the total cost of 12 planks, each 14′ long, 8″ wide, and 2″ thick, and 18 planks, each 16′ long, 6″ wide, and 2″ thick?

24. A carpenter needs 8 boards, each 18′ long, 8″ wide, and 4″ thick; 36 boards, each 14′ long, 6″ wide, and 2″ thick; and 18 boards, each 8′ long, 8″ wide, and 2″ thick. At an average price of \$62 per M, what is the total cost?

25. At an average price of \$56 per M, find the total cost of the following pieces of lumber: 12 pieces, each 16′ long, 10″ wide, and 6″ thick; 32 pieces, each 14′ long, 8″ wide, and 3″ thick; and 84 pieces, each 12′ long, 8″ wide, and 2″ thick.

MISCELLANEOUS PROBLEMS

After Page 184

1. A shoe factory shipped 137 cases of shoes on Monday, 259 on Tuesday, 376 on Wednesday, 592 on Thursday, 403 on Friday, and 117 on Saturday, each case containing 36 pairs. What was the average number of pairs shipped per day?

2. During the season of 3 wk., a small canning factory uses the peas grown on $37\frac{1}{2}$ A. and puts up 24,000 cans of peas. Allowing 6 da. to the week, find how many acres of peas are used per day, and how many cans are put up per day.

3. One farmer owns $5\frac{1}{2}$ A. of the $37\frac{1}{2}$ A. mentioned in Ex. 2. How many cans are put up from his peas?

4. One of the workmen in the factory mentioned in Ex. 2 nails an average of 180 cases, which hold 24 cans each, per day. How many days will it take him to nail the cases for the 24,000 cans?

5. In sealing 30 cans, a canning factory uses 8 oz. of solder, which is 20% lead. If the output is 1500 cans per day, how many pounds of solder are used in a week of 6 da.? How many pounds of this is lead?

6. If a factory worker who is receiving \$2.25 a day for an 8-hour day has his pay increased to $38\frac{1}{2}$¢ an hour, what is the per cent of increase?

7. A builder sends in an order for 2800 bd. ft. (board feet) of sheathing at \$62 per M, 2450 bd. ft. of flooring at \$87.50 per M, and 14,000 clapboards at \$15.30 per M. Find the total amount of the order.

8. If steel rails weighing 100 lb. to the yard are used on a double-tracked road for a distance of 275 mi., how many tons of rails, not counting switches, are used?

III. PROBLEM SURVEY OF CHAPTER III

KEEPING ACCOUNTS

After Page 202

1. A farmer's account book showed the following entries: *May 1*, cash on hand, \$372.48; *3*, sold 7 doz. eggs @ 39¢; *4*, bought cultivator, \$54.50; paid help, \$12.25; *7*, sold 11 lb. butter @ 37¢; *11*, repairs to barn, \$102.30; *12*, sold 5 T. hay @ \$19.50; *14*, sold 4 doz. eggs @ 38¢; received from H. T. Marston for milk, \$7.52; *19*, sold chickens, \$21.42; *24*, bought feed, \$6.72; *27*, sold 14 lb. butter @ 37¢; *29*, paid help, \$28.50. What was his balance on June 1?

2. A farmer's cashbook showed \$245.75 in cash on hand on Dec. 1, and the following transactions during the month on the dates specified: *3*, sold 9 lb. butter @ 43¢; *5*, bought groceries, \$5.95; *6*, sold 7 T. hay @ \$17.75; *7*, paid help, \$16.50; *10*, sold 14 doz. eggs @ 51¢; *11*, sold cow, \$78; *15*, sold 8 lb. butter @ 43¢; *19*, bought groceries, \$4.87; sold 5 bu. potatoes @ \$1.95; *23*, sold 7 doz. eggs @ 51¢; *27*, paid help, \$19.30. What was his balance at the end of the month?

3. Suppose that your bank balance on June 1 is \$238.34, and that during the month you deposit \$38.50, \$40.65, and \$75.20. If during the month you draw checks for \$28.75, \$9.75, \$8.25, \$3.50, \$6.38, \$11.40, and \$39.25, what is your bank balance on June 30?

4. A business man's bank balance was \$1276.33 on Nov. 1. During the month he made deposits totaling \$1425.72, and also borrowed \$1500 from the bank, which was credited to his account. During the month he drew checks totaling \$3476.28, including one for the interest on the loan. How much does he need to deposit on Dec. 1 in order to pay back the loan and still have a balance of \$1000?

MISCELLANEOUS PROBLEMS

After Page 202

1. A tailor made 5 coats of the same size and style, using $28\frac{3}{4}$ yd. of cloth. How many yards did he use for each coat?

2. Frank has a board which is $18\frac{7}{8}$ in. long. In squaring up the ends he saws a piece $\frac{5}{8}$ in. long from each end. If he then cuts off a piece $10\frac{1}{4}$ in. long, how long is the piece of the board that is left?

3. If Marie has $8\frac{1}{2}$ lb. of candy to put into boxes holding $\frac{1}{4}$ lb. each, how many boxes does she need?

4. How many bottles holding $\frac{7}{8}$ qt. each does Mrs. Harris need for 28 qt. of sirup?

5. Mrs. Riley works in a factory. If she is paid \$1.44 for every dozen caps that she makes, how much does she receive for making $6\frac{3}{4}$ doz.?

6. At 60¢ per pound, how much will $\frac{3}{4}$ lb. of tea cost?

7. At 52¢ a dozen, how much will $3\frac{1}{2}$ doz. eggs cost?

8. When sugar is selling at $8\frac{1}{3}$¢ per pound, how many pounds will 75¢ buy?

9. In a store a tag attached to a roll of cloth shows that there were $68\frac{1}{4}$ yd. in the roll on Feb. 1, and that since that time there have been sold $7\frac{5}{8}$ yd., $14\frac{3}{4}$ yd., $6\frac{1}{2}$ yd., and $9\frac{7}{8}$ yd. How much cloth is left in the roll?

10. Mrs. Royce's ice-cream freezer holds 8 qt., but it should not be over three quarters full in order to allow for expansion in freezing. How many quarts of the mixture that is to be frozen may be put in the freezer?

11. A linoleum mat containing $\frac{3}{4}$ sq. yd. is priced at \$1.20, and another mat containing $\frac{7}{8}$ sq. yd. is priced at \$1.68. Which mat costs less per square yard, and how much less?

REVIEW PROBLEMS

After Page 205

1. After working 6 mo. for \$15 per week, a clerk in a city grocery store received an increase of 20%. What were his new wages per week?

2. If in a class of 35 pupils 14 are boys, what per cent of the class are boys?

3. If a bank clerk who receives a weekly salary of \$50 is able to save \$6.25 each week, what per cent of his salary is he saving?

4. A farmer who raised 1750 bu. of wheat last year increased his crop 350 bu. this year by using fertilizer. What is the per cent of increase?

5. Mr. Caldwell pays \$58 a month for the rent of the apartment in which he lives. If this is 20% of his total income, what is his income for the year?

6. A family with an income of \$3500 a year allowed 30% for food, 20% for rent, 18% for clothing, 12% for advancement, 10% for other expenses, and the rest for investment. How much was allowed for each of these purposes during the year?

7. In making bronze composed of 18%, by weight, of tin and the rest copper, how many pounds of tin must be melted with 2132 lb. of copper?

8. Two men purchased some property together, one putting in \$1500 and the other \$1750. If later they sell the property for \$4800 and divide the profit in accordance with the amounts invested, how much profit does each man receive, and what per cent is it on his investment?

9. Of three partners, the first is entitled to $37\frac{1}{2}\%$ and the second to $43\frac{3}{4}\%$ of the profits. If they make \$4800, what is the share of each of the three men?

10. Experiments with a wagon having high wheels showed that it required a pull of 84.48 lb. to move 1 T. on a dry, level, gravel road, and a pull of 30.4% more for a wagon with low wheels. How much power is required for the latter?

11. If a shipment of butter contains 86.6% butter fat, how many pounds of butter represent 433 lb. of butter fat?

12. If milk produces $10\frac{1}{2}\%$ of its weight as cheese, how many pounds of milk are required to produce 75 lb. of cheese?

13. If milk produces $6\frac{1}{4}\%$ of its volume as cream, what volume of milk is required to produce 130 gal. of cream?

14. A laborer earns $\$4.66\frac{2}{3}$ a day for 6 da. in the week. If he works for 51 wk. and spends 25% of his wages for rent, how much money is left for other expenses?

15. A boy is offered a salary of \$12 a week for the first year in a store, 25% more the second year, and an increase of 50% the third year. If he works 50 wk. in a year, what is his total salary for the first 3 yr.?

16. A man who has been earning \$120 a month is offered an increase of 20% or a salary of \$32 a week. Allowing 52 wk. to the year, which offers the better income, and how much better, for the year?

17. Flint glass contains by weight 72% sand, 14% soda, 12% lime, and 2% alumina. Find the number of pounds of each of these substances in 150 lb. of flint glass.

18. A bell weighing 1250 lb. is composed of 77% copper, 21% tin, and the rest zinc. How many pounds of each metal does the bell contain?

19. If the leading team in one league won 90 games out of 105, and that in another league won 78 out of 94, which team had the higher per cent of games won?

REVIEW OF DECIMALS

After Page 208

Copy the following table and fill the blank spaces:

	This Quantity	is this Fraction	or this Decimal	or this Per Cent	of this Quantity
1.	$\frac{1}{8}$ in.	$\frac{1}{8}$	0.125	——%	1 in.
2.	$\frac{1}{8}$ in.	——	——	——%	$\frac{1}{2}$ in.
3.	——¢	$\frac{1}{2}$	——	——%	50¢
4.	—— oz.	——	——	$12\frac{1}{2}$%	$\frac{1}{2}$ lb.
5.	1 hr.	——	0.10	——%	—— hr.
6.	1 pt.	——	——	——%	2 gal.
7.	1 gal.	——	——	——%	1 pt.
8.	40 lb.	$\frac{1}{50}$	——	——%	—— T.

9. The average ages of the boys in the four classes of a school are 18.4 yr., 17.9 yr., 16.8 yr., and 15.7 yr. What is the average age of the boys in the school?

10. From a piece of iron rod that was originally 35.8 in. long there were cut four pieces measuring 12.8 in., 4.3 in., 8.9 in., and 5.6 in., respectively, in length. How many inches remain?

11. If a certain kind of gun metal is composed of 90.5 parts, by weight, of copper to 9.5 parts of tin, how many ounces of tin are there in a pound of the gun metal? How many ounces of copper are there in a pound of the gun metal?

12. If 15 burners, each consuming 5.8 cu. ft. of gas per hour, are used on an average of 4 hr. per day for 365 da., what will be the gas bill at 80¢ per M?

13. If rails cost $28.50 per ton and weigh 98.5 lb. per yard, what will the rails for 5 mi. of double track cost?

REVIEW PROBLEMS

After Page 210

Copy the following table and fill the blank spaces:

	Amount of Purchase	Rate of Discount	Amount of Discount	Net Price
1.	$300	——%	$24	$——
2.	$37.50	——%	$7.50	$——
3.	$48	8%	$——	$——
4.	$75	12%	$——	$——
5.	$24	——%	$2.40	$——
6.	$450	$33\frac{1}{3}$%	$——	$——

7. If I can buy a book marked $2.25 at a discount of 20%, how much will the book cost me?

8. What is the net price of a suit which is marked $45 but can be bought at 15% off?

9. If a table is listed at $25 with a discount of 20%, 15%, what is the net price?

10. In bidding on a contract for supplying lumber, one company makes a bid of $1528 less 35%, 15%, and a second company bids $1420 less 30%, 20%. Which is the lower bid, and how much lower is it?

11. In buying a piano Mr. Wallace was able to save $60 by buying at a special sale at which a discount of 20% was given. What was the price of the piano before the sale?

12. A furniture dealer sold a rug that was marked $115 at $95.50 and another that was marked $79 at $62.50. On which sale was the per cent of discount the greater, and how much greater was it?

REVIEW PROBLEMS

After Page 211

Find the missing items in Exs. 1–4 and then solve Exs. 5–11:

1. Sale, \$450; rate of commission, 5%; commission, ——.

2. Sale, \$960; rate of commission, ——; commission, \$48.

3. Sale, ——; rate of commission, 4%; commission, \$196.

4. Sale, \$950; rate of commission, 5%; commission, ——.

5. A washing-machine salesman is paid a commission of $7\frac{1}{2}\%$ of the amount of his sales. How much does he receive in a month in which he sells 39 machines at \$89.50 each?

6. At 3% of the cost, how much will an architect receive for furnishing a set of plans for a house which is to cost \$11,850?

7. A commission merchant receives 7% for selling fruit for a farmer. If the amount of the sale is \$265, how much does the farmer receive?

8. A new clerk in a grocery store receives \$12 a week and a commission of 5% of his sales. In order to receive an income of \$25 a week, to what must his sales amount during the week?

9. An agent who sells dress goods to retail dealers is paid a commission of $2\frac{1}{2}\%$. Find the amount of his commission on a sale amounting to \$16,500.

10. The pupils in the Lincoln School wished to earn some money for decorating the school. They secured 400 subscriptions to a magazine at \$2 each. If they received 5% on all subscriptions, how much did they earn?

11. A real-estate dealer sold one piece of property for \$7500 and another for \$8250. If his commission was $2\frac{1}{2}\%$ on the first sale and 3% on the second, how much commission did he receive? This is how much more or less than he would have received if the commissions had been $2\frac{3}{4}\%$ on both sales?

REVIEW PROBLEMS

After Page 213

1. A butcher sells bacon that cost 32¢ a pound at a price which gives him a profit of 25% on the cost. What part of a pound does he sell for 30¢? for 20¢? How much does he sell for 60¢? for \$1.20? for 80¢? for \$1.60?

2. A grocer bought coffee at 32¢ a can. Since it did not sell fast enough, he sold 4 doz. cans at a loss of 12% on the cost. How much did he lose on each can sold? How much did he lose on the 4 doz. cans?

3. On some goods which cost \$9840 a merchant expects to make a profit of $12\frac{1}{2}\%$ on the cost. Find the amount of the profit which he expects to make.

4. A merchant sells some goods for \$725 and makes a profit of 20% on the selling price. How much is his profit?

5. After a fire a merchant sells some goods for \$1250 and suffers a loss of 8% on the selling price. How much is his loss?

6. A store's profit was \$7.92 on an article that sold for \$52.80. Find the per cent of profit on the selling price.

7. A man bought a piece of land for \$1250 and sold it at a profit of 15% on the cost. What was the selling price?

8. A builder sold a house for \$9200, thereby making a profit of 15% on the total cost. Find the cost of building the house.

9. A manufacturer sells a piece of machinery for \$225. If it costs him \$180 to manufacture it, what is the per cent of profit on the manufacturing cost?

10. Last year a merchant's sales totaled \$52,000, the expenses of running his store averaged 25% of sales, and the cost of the goods sold was \$32,760. How much profit did he make during the year? His profit averaged what per cent of sales?

EVERYDAY USES OF PER CENTS

After Page 221

1. An iron tire expands 1.6% on being heated for shrinking on a wheel. If a wheel takes a tire 16 ft. 8 in. in circumference, how much longer will the tire be when heated?

2. After a rough casting that weighs 282 lb. is turned in a lathe, it is found to weigh 271 lb. The loss in weight of the casting is what per cent of the weight in the rough?

3. In Ex. 2, the loss in weight is what per cent of the weight when finished?

4. A man saved $1350.40 last year, which was 32% of his income. How much was his income?

5. If a locomotive weighing 124 T. can exert a pull equal to $22\frac{1}{2}\%$ of its weight, how great a pull can it exert?

6. On a boiler that supplies steam for an engine there is a gauge to show how many pounds of pressure the steam exerts against every square inch of the boiler surface. If the steam pressure increases from 110 lb. to 160 lb. per square inch, it is then what per cent greater than before?

7. A man sells some lumber for $360, thereby gaining 20% on the cost. What per cent of the cost is the selling price? How much is the cost?

8. A line shaft is driven by a 15 H.P. motor. There is, however, a power loss of $2\frac{1}{2}\%$ due to the slipping of the belt connecting the motor and the line shaft. Find the actual horse power delivered to the line shaft.

9. It is estimated that water in freezing expands approximately 9% of its volume. How many cubic feet of water are needed to make 1199 cu. ft. of ice? How many gallons (1 cu. ft. $= 7\frac{1}{2}$ gal.) of water are needed?

GOVERNMENTAL FINANCE

After Page 224

1. In a suburban town which has a total assessed valuation of $18,500,000 the tax rate last year was $27.80 per $1000. What was the total amount raised by taxes?

2. Mr. Russell, who lives in the town mentioned in Ex. 1, owns real estate that is worth $10,500. If the property is assessed at two thirds of its value, what is the amount of Mr. Russell's tax bill?

3. In Ex. 1, what would be the amount of the tax bill of a man who owned a house assessed at $5800, land assessed at $1800, and a garage assessed at $450?

4. In a city in which the total assessed valuation last year was $85,575,000, it was necessary to raise $2,225,000 to meet the year's expenses. Find the tax rate, expressing it (1) as so much per $1000, (2) as so much per $100, and (3) as so many mills per $1.

5. If a man's property is assessed at $26,000 and the amount of his tax bill is $554.84, what is the tax rate per $1000?

6. In a year when the local tax rate was $1.75 per $100, a man paid a tax bill of $612.50. What was the assessed valuation of his property?

7. In Ex. 6, if a man's tax bill was $464.10, what was the assessed valuation of his property?

8. When the duty on pineapples is $7.50 per 1000, what is the amount of the duty levied on a shipment containing 16,200 pineapples?

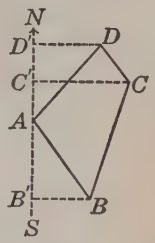
9. The duty on perfumery is 40¢ per pound and 75% ad valorem (on the value). Find the total duty on a shipment of 150 lb. of perfumery valued at $9750.

IRREGULAR FIGURES

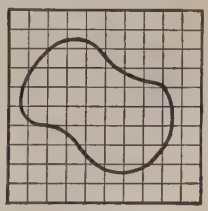
After Page 228

1. A garage is 22 ft. long and 20 ft. wide, and the walls are 8 ft. high at the eaves and 14 ft. high at the peak of the A-shaped roof. How many square yards of stucco will be needed to cover the outside walls of the garage, allowing for one door 6 ft. 6 in. by 3 ft., two doors each 8 ft. by 8 ft., and five windows, each 5 ft. by 3 ft.?

2. In surveying the field $ABCD$ here shown, a surveyor first ran the north-and-south line NS through A. He then laid off BB', CC', and DD' at right angles to NS, and found by measuring that $BB' = 24$ rd., $CC' = 40$ rd., $DD' = 28$ rd., $B'A = 32$ rd., $B'C' = 48$ rd., and $C'D' = 16$ rd. Find the area of the field in acres.



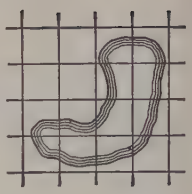
3. In finding the approximate area of an irregular figure, we may draw the outline on squared paper, as shown below, and then, by counting the squares, find the area inclosed. In doing this, we exclude any square which does not lie at least half within the outline of the figure, we count as a half square one that is practically evenly divided, and we count as a full square one that lies more than half within the outline. If each square in this picture represents 1 sq. in., what is the approximate area of the irregular figure?



4. In Ex. 3, if each square represents 0.1 sq. ft., what is the approximate area of the figure?

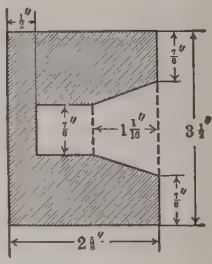
5. In Ex. 3, if each square represents 1 sq. ft., what is the approximate area of the figure? This is how many times the area found in Ex. 4?

6. This figure shows a lake on a portion of a map of a township on which the section lines, which are 1 mi. apart, are drawn. By counting the squares as suggested in Ex. 3, find the approximate area of the lake in square miles.

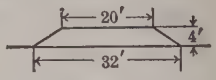


7. Copy the following figures on squared paper and, by counting the squares, find the approximate area of each one.

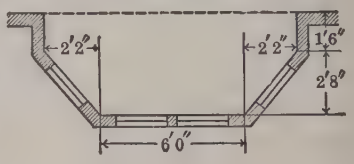
8. This figure shows a working drawing, such as might be used in a machine shop, of a metal support for a bearing. From the measurements given in the figure, find the area of the shaded portion of the support. Check your result by drawing the figure on squared paper, letting each square represent $\frac{1}{64}$ sq. in.



9. In constructing a state road across a swamp, the roadbed was filled in until its cross section had the shape and dimensions here shown. Find the area of this section.



10. In planning to add a bay window to his living room, a man draws the sketch here shown. If he builds the window according to this plan, how much additional floor space will the window give to the room?



RECTANGULAR SOLIDS

After Page 245

1. A railroad car is 38 ft. long and 8 ft. 6 in. wide, and its sides are 4 ft. high. If filled level full of sand, how many cubic yards of sand are there in the car?

2. A rectangular block has square ends 6 in. on a side and contains 99 cu. in. Find the length of the block.

3. A rectangular water tank is 9 ft. long and 5 ft. 6 in. wide, inside measurements. How much water must be drawn off in order to lower the surface of the water 1 ft.?

4. A coal bin 32 ft. long and 19 ft. wide is filled with coal to the depth of 7 ft. Allowing 35 cu. ft. of coal to the ton, find the number of tons of coal in the bin.

5. An ice dealer delivers a cake of ice 12 in. wide, 8 in. thick, and 20 in. long. Taking the weight of 1 cu. ft. of ice as $57\frac{1}{2}$ lb., find the weight of the cake.

6. Find the weight of a bar 6 ft. long, 2 in. wide, and $1\frac{3}{4}$ in. thick, made of steel weighing 490 lb. per cubic foot.

7. The sill of a barn is 20 ft. long, 8 in. wide, and 10 in. thick. How many board feet of lumber does the sill contain?

8. In a rectangular irrigation ditch, which is 3 ft. wide, the water is flowing at the rate of 8 ft. per minute, and the average depth of the water is 1 ft. 6 in. Taking $7\frac{1}{2}$ gal. as the practical equivalent of 1 cu. ft., find the amount of water that flows past a fixed point in 1 hr.

9. At 54 lb. per cubic foot, find the weight of a wooden beam 8 in. by 10 in. and 18 ft. long.

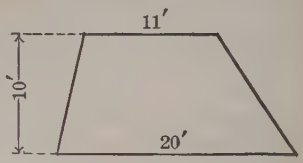
10. To the nearest 100 cu. yd., how much gravel is needed to grade $3\frac{1}{2}$ mi. of road if the gravel is to be laid to an average depth of 6 in. over a width of 27 ft.?

PRISMS AND CYLINDERS

After Page 255

1. The base of a prism has an area of 17.35 sq. in., and the height of the prism is 6.18 in. Find its volume.

2. A railway embankment extends 700 ft. along a river bank. A cross section of the prism is a trapezoid 10 ft. high, with a lower base of 20 ft. and an upper base of 11 ft. Find the number of cubic yards of earth in the embankment.



3. A tinsmith wishes to make 4 doz. cylindric cups, open at the top. The diameter of the base of each cup is to be 3 in. and the height of each cup is to be 4 in. Not allowing for overlapping and waste, find the number of square feet of tin that will actually be put into the cups.

4. A tin water pipe has a circumference of 10 in. and a length of 16 ft. Making no allowances for joints, find the number of square feet of tin used in making the pipe.

5. Allowing $7\frac{1}{2}$ gal. to the cubic foot, find the capacity in gallons of a cylindric gasoline tank $3\frac{1}{2}$ ft. long and 18 in. in diameter, inside measurements.

6. Taking the weight of cast iron as 450 lb. per cubic foot, find the weight of a hollow cast-iron column which is 12 ft. long and has an inside diameter of 5 in. and an outside diameter of 7 in.

7. A workman has a cylindric can 14 in. in diameter and about 15 in. high. He knows that it is either a 5-gallon or a 10-gallon can, but he has forgotten which. Allowing 231 cu. in. to a gallon, find which it is.

8. A pipe 28 ft. long has an inside diameter of 6 in. Find the number of cubic feet of water needed to fill the pipe.

PYRAMIDS, CONES, AND SPHERES

After Page 261

1. Find the volume of a cone with a height of 5.8 in. and a radius of 2.1 in.

2. A push ball 6 ft. in diameter is to be covered with leather. Allowing one third extra for waste and for the seams, find the cost, at $1.75 per square foot, of the leather cover.

3. Taking the earth as a sphere with a radius of 4000 mi., find to the nearest 1000 sq. mi. the area of the earth.

4. Find the area of the curve surface of a cone which has a slant height of 10.2 in. and a circular base that is 22.8 in. in circumference.

5. How many cubic feet of space are there in a conic tent which is 14 ft. in diameter and 9 ft. high?

6. Find the volume of a pyramid with a height of 9 in. and a base that has an area of 22 sq. in.

7. How large is the surface of a ball 44 in. in diameter?

8. A funnel in the shape of a cone is closed at the bottom. If the circular top is 3 in. across, and the funnel is 4 in. deep (to the vertex), how many cubic inches of water will it hold?

9. Find the number of square feet of slating on a church steeple in the form of a pyramid with a slant height of 40.5 ft. and a square base 18.0 ft. on a side.

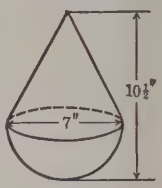
10. Find the area of the largest sphere that can be cut from a cube of wood 12 in. on an edge.

11. If an iron ball 4 in. in diameter is dropped into a cylindric jar 8 in. in diameter in which water is standing to a depth of 6 in., what is then the depth of the water?

12. How many square feet of sheet copper are needed to cover a hemispheric roof which measures 35 ft. across?

13. Making no allowances for seams or waste in cutting, how many square inches of flannel will it take to cover a tennis ball 3 in. in diameter?

14. An iron casting is made in the form of a cone on a hemisphere, as shown in this figure. The total length of the casting is $10\frac{1}{2}$ in., and the base of the cone upon which the hemispheric base is constructed is 7 in. in diameter. Taking the weight of 1 cu. in. of cast iron as 0.26 lb., find to the nearest ounce the weight of the casting.



15. Find the area of the curve surface of a cone with a slant height of 8.4 in. and a radius of 4.2 in.

16. Find the number of cubic yards of earth in a pile in the shape of a cone 35 ft. in diameter and 15 ft. high.

17. Taking the radius of the earth as 4000 mi. and the atmosphere as extending 50 mi. above the surface of the earth, find the volume of the atmosphere to the nearest 1000 cu. mi.

18. Find the area of the curve surface of a cone with a slant height of 8.4 in. and a diameter of 6.3 in.; of a cone with a slant height of 6.3 in. and a diameter of 8.4 in.

19. The volume of a pyramid is 26 cu. ft. 936 cu. in. and each side of its square base is 3 ft. 6 in. Find the height.

20. A pile of sand is in the form of a cone. The circumference of the base is 33 ft. and the height of the pile is 5 ft. How many cubic yards of sand are there?

21. At 26¢ per square inch, what is the cost of gilding a ball 14 in. in diameter on top of a flagpole, no allowance being made for the support?

22. Find the diameter of a sphere in which the number of square inches in the area of its surface is equal to the number of cubic inches in its volume.

MISCELLANEOUS PROBLEMS

After Page 265

This table shows the dimensions of a number of figures. In Exs. 1–6 find the areas, and in Exs. 7–12 find the volumes:

	FIGURE	DIMENSIONS
1.	Square	3.5 in. on a side
2.	Rectangle	Length, 3.25 in.; width, 2.75 in.
3.	Triangle	Base, 4.5 in.; height, 3.2 in.
4.	Parallelogram	Base, 8.8 in.; height, 3.9 in.
5.	Trapezoid	Bases, 7.5 in., 5.4 in.; height, 2.2 in.
6.	Circle	Radius, 4.9 in.
7.	Cube	6.5 in. on a side
8.	Prism	Base, 4.75 sq. in.; height, 1.25 in.
9.	Cylinder	Radius, 2.1 in.; height, 3.5 in.
10.	Sphere	Radius, 4.5 in.
11.	Cone	Radius, 0.75 in.; height, 3.75 in.
12.	Pyramid	Base, 7.5 sq. in.; height, 3.0 in.

13. A cubic inch of iron weighs $4\frac{4}{9}$ oz. Find the weight in pounds of an iron bar 1 in. square at the end and 1 yd. long; of a bar 1 in. square at the end and 15 ft. long.

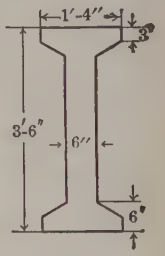
14. The drive wheels of a locomotive are 6 ft. 8 in. in diameter. How many revolutions a minute does each wheel make when the locomotive is traveling 48 mi. an hour?

15. Allowing 1000 shingles to 120 sq. ft., find how many thousand shingles it will take to cover a pitched roof each side of which is 58 ft. long and 24 ft. wide.

16. Allowing $7\frac{1}{2}$ gal. to the cubic foot, find the number of gallons of water in a well 5 ft. in diameter, when the depth of the water is $10\frac{1}{2}$ ft.

17. Find the area of a field in the form of a right triangle in which the two sides forming the right angle are 60 rd. and 40 rd.

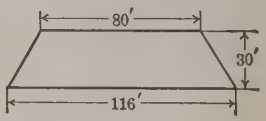
18. Find the area of the cross section of the steel girder shown in this figure.



19. How many cubic yards of earth were removed in excavating a tunnel which is $\frac{3}{4}$ mi. long and whose cross section is a semicircle 18 ft. in diameter?

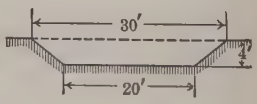
20. The front wheels of a wagon are 3 ft. 6 in. in diameter and the rear wheels are 4 ft. 1 in. In going 1 mi., how many more revolutions will a front wheel make than a rear wheel?

21. Find the area of the cross section of this railway embankment, which is 30 ft. high, 80 ft. wide at the top, and 116 ft. wide at the bottom.



22. During a rainfall of $\frac{1}{2}$ in., to what depth will a circular well 5 ft. 6 in. in diameter be filled by the drainage from a rectangular asphalted courtyard which is 30 ft. long and 26 ft. 6 in. wide?

23. Where a road runs through a cut for a distance of 200 ft., the cross section of the cut is a trapezoid with the dimensions here shown. How many cubic yards of material were removed in making the cut?



24. A square plot of land 127 yd. on a side has a path 1 yd. wide running round the inside of it. Find the cost of graveling this path at 35¢ per square yard.

PROBLEMS IN RATIO AND PROPORTION

After Page 279

Copy and complete the statements in Exs. 1–3:

1. If a rectangle is 3 in. wide and 4 in. long, I may say that the width is —— of the length, or that the ratio of the width to the —— is ——.

2. In a rectangle 2 ft. long and 18 in. wide, the ratio of the width to the length is ——, or ——.

3. An expression of equality between two ratios is called a ——. The —— which expresses the fact that the ratio of 3 to 4 equals the ratio of 6 to 8 is —— = ——.

4. How is the ratio of 10 to 30 affected if we multiply each term by 2? divide each term by 2? add 2 to each term? subtract 2 from each term?

5. A solution contains ammonia and water in the ratio of 3 parts of ammonia to 7 parts of water. What is the ratio of ammonia to solution? What is the per cent of ammonia?

6. Suppose that you wish to cut a 20-foot pole into two parts whose ratio is $\frac{1}{3}$. Find the length of each part.

7. If 200 ft. of copper wire weighs 60 lb., what is the weight of 125 ft. of the same kind of wire?

8. The ratio of the areas of two fields is $\frac{3}{4}$, and the smaller field contains 60 A. Find the number of acres contained in the larger field.

9. A plaster model of a building is 20 in. long, 12 in. wide, and 18 in. high. If the finished building is to be 36 ft. wide, what are its other dimensions?

10. Two boys make a profit of \$8.60 by selling lemonade at a baseball game. If they divide the profit in the ratio $\frac{2}{3}$, what is the share of each boy?

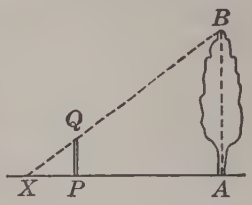
MEASURING HEIGHTS AND DISTANCES

After Page 283

1. How high is a tree which casts a shadow 18 ft. long at the same time that a tree 20 ft. high casts a shadow 12 ft. long?

2. If, when a boy 5 ft. tall casts a shadow 4 ft. long, the school building casts a shadow 36 ft. long, what is the ratio of the height of the building to its shadow? What is the height of the building?

3. This figure shows how a boy found the height of a tree. He set a 16-foot pole upright at a point P, and located a point X from which he could just see the top of the tree over the pole. By measuring he found that AP and PX were 24 ft. and 8 ft. long respectively. How tall was the tree?



4. At the time a man 6 ft. tall casts a shadow 4 ft. long, how long is the shadow cast by a building 60 ft. high?

5. How long is the shadow cast by a yardstick at the same time that a foot rule casts a shadow 9 in. long?

6. How high is a pole which casts a shadow 39 ft. long at the same time that a post 6 ft. 6 in. high casts a shadow 5 ft. long?

7. A telephone pole casts a shadow 30 ft. long at the same time that a post 5 ft. high casts a shadow 4 ft. long. How tall is the pole?

8. If a tree casts a shadow 56 ft. long at the same time that a yardstick held vertically casts a shadow $2\frac{1}{2}$ times its own length, what is the height of the tree?

9. The ratio of the height of a flagpole to the length of its horizontal shadow is $\frac{1}{2}\frac{4}{5}$. If the length of the shadow is 120 ft., what is the height of the flagpole?

ECONOMY IN THE HOME

After Page 291

1. When a cereal is quoted at 18¢ per package or at \$2 per dozen, what per cent does a purchaser save on the package rate if he buys by the dozen?

2. A housewife can buy coffee at 35¢ a pound or at \$1.65 for 5 lb. What per cent does she save on the pound rate by purchasing 5 lb. at a time?

3. Mrs. Blair finds that, by paying cash and carrying her purchases home, she can save $\frac{1}{10}$ of her grocery bill. How much does she save on 2 lb. of coffee at 38¢, 8 lb. of sugar at $7\frac{1}{2}$¢, 3 lb. of rice at 6¢, $1\frac{1}{4}$ lb. of starch at 9¢, and 4 cakes of soap at $4\frac{1}{2}$¢?

4. In Ex. 3, if Mrs. Blair spends, on an average, \$6.50 a week for groceries, how much will she save in a year (52 wk.) by the "cash-and-carry" plan?

5. Which is the lower net price per dozen, one of \$3.50 less 2%, or one of \$4 less 7%? How much would be saved in buying 6 doz. at the cheaper price?

6. When olives are selling at 40¢ per bottle or at \$4.50 per dozen bottles, what per cent is saved in buying by the dozen?

7. When fancy bacon sells at 50¢ a jar or at \$5.50 per dozen jars, what per cent is saved in buying by the dozen?

8. I can buy a set of furniture for \$250 cash or for \$25 down and \$25 a month for 10 mo. Disregarding interest, what per cent of the installment price can I save by paying cash?

9. A man who spent 75¢ a day (including Sundays) for tobacco gave up smoking and deposited in a savings bank the money that he thus saved. Not counting the interest, how much would he save in 10 yr., including three leap years?

IV. PROBLEM SURVEY OF CHAPTER IV

CORPORATION PROBLEMS

After Page 306

1. At \$100 per share, what is the number of shares issued by a corporation that is capitalized at \$2,250,000?

2. If the corporation mentioned in Ex. 1 declares regular quarterly dividends of $1\frac{3}{4}\%$, what amount does it distribute annually among its stockholders?

3. A manufacturing company declares a semiannual dividend of $3\frac{1}{4}\%$. How much does an investor receive who owns 15 shares of stock?

4. If a corporation capitalized at \$1,500,000 makes a net profit of \$52,500 in 6 mo., and reserves none of it for surplus, what semiannual dividend rate can it declare?

5. If a company capitalized at \$750,000 earned \$21,846.30 above expenses in the last 3 mo., and wishes to carry about \$2500 to the next quarter, what quarterly dividend rate can it declare?

6. In Ex. 5, how much would be received by the holder of 125 shares of stock?

7. A corporation with a capital of \$250,000 sets aside \$17,500 of its earnings this year for dividends. What is the annual rate of dividend?

8. If a corporation with a capital of \$375,000 appropriates \$13,125 for dividends, how much does the owner of 25 shares of stock receive?

9. If the quarterly dividend rate of a company is cut from $1\frac{3}{4}\%$ to $1\frac{1}{4}\%$, how much less income per year does the holder of 250 shares receive?

BUYING AND SELLING STOCKS

After Page 308

In solving the following problems, include the brokerage charges as given on page 307:

1. If a man buys 40 shares of stock @ $102\frac{1}{2}$, how much does the stock cost him?

2. If a man sells 75 shares of stock @ $158\frac{5}{8}$, how much does he receive from the sale?

3. If a man buys 50 shares of stock @ $98\frac{7}{8}$, how much does the stock cost him?

4. If a man sells 30 shares of stock @ $107\frac{5}{8}$, how much does he receive from the sale?

5. How much must a man pay for 125 shares @ $110\frac{3}{8}$?

6. Find the amount received by a man who orders his broker to sell 85 shares of stock @ $127\frac{1}{4}$.

7. How much does a buyer pay for 30 shares of stock @ $158\frac{1}{8}$, and how much will the seller receive?

8. A man bought 75 shares of Midwest Utilities @ $117\frac{1}{8}$, 20 shares of United Drug @ $138\frac{3}{4}$, and 180 shares of Lehigh Valley @ $78\frac{1}{4}$. Computing the brokerage separately on each stock, find how much he paid in all.

9. A man ordered his broker to buy 200 shares of one stock quoted @ $98\frac{1}{4}$ and to sell 250 shares of another @ $126\frac{1}{2}$. How much did the man receive from the transaction?

10. A man bought 30 shares of American Sugar @ $118\frac{1}{2}$, 20 shares of U.S. Rubber @ $46\frac{1}{8}$, and 40 shares of United Drug @ $67\frac{3}{4}$. How much did he pay?

11. A man sold 20 shares of General Electric @ $254\frac{1}{8}$, 10 shares of Union Pacific @ $125\frac{3}{4}$, and 15 shares of N.Y. Central @ $113\frac{1}{4}$. How much did he receive from his broker?

STOCKS AND BONDS

After Page 314

In solving the following problems, include when necessary the brokerage charges as given on page 307:

1. A man has \$7000 worth of 5% bonds, \$5000 worth of $4\frac{1}{2}\%$ bonds, and \$8000 worth of stock that regularly pays 2% quarterly. Find his total annual income from these securities.

2. Find the cost of a share of stock of par value \$100 when it is quoted at $98\frac{1}{4}$. If the stock pays a regular quarterly dividend of $1\frac{1}{2}\%$, what is the rate of return on the investment?

3. What is the rate of return on the investment in purchasing stock that pays quarterly dividends of $2\frac{1}{2}\%$ and is quoted at $137\frac{1}{2}$?

4. A man bought 125 shares of stock when it was quoted at $103\frac{1}{4}$ and sold it later when the price was $113\frac{3}{4}$. How much did he make on the transaction?

5. A man bought 75 shares of Union Pacific at $118\frac{5}{8}$, kept it a week, and sold it at $117\frac{7}{8}$. Find his net loss on the investment.

6. When the current rate of return on sound investments averages 5.87%, the rate of return on the investment in a stock that pays $6\frac{1}{2}\%$ and is quoted at 112 is how much more or less than the average rate?

7. A man bought 40 shares of stock quoted at $102\frac{1}{2}$, 75 shares at $98\frac{3}{8}$, and 125 shares at $86\frac{5}{8}$. He sold the first stock when it was quoted at $101\frac{3}{4}$, the second at $102\frac{1}{8}$, and the third at $91\frac{3}{8}$. Find his total gain or loss.

8. In order to finance an extension of its plant, a corporation issues \$175,000 worth of bonds bearing interest at 7%. How much must the company set aside each 6 mo. to provide for the payment of the interest?

INVESTING MONEY WISELY

After Page 318

1. How much is received each 6 mo. by the holder of bonds having a face value of \$15,000 and bearing interest at $5\frac{3}{4}\%$, payable semiannually?

2. A bond house advertises six issues of bonds, the first to yield 4.75% on the investment, the second, 5.00%, the third, 5.10%, the fourth, 5.37%, the fifth, 6.00%, and the sixth, 7.00%. What is the average rate of return?

3. Suppose that a man invests \$2500 in the first securities mentioned in Ex. 2, \$3500 in the second, \$2500 in the third, \$5000 in the fourth, \$6500 in the fifth, and \$1500 in the sixth. What annual income will he receive?

4. In Ex. 3, the annual income is how much more or less than the income that the man would receive if he invested the whole amount at the average rate found in Ex. 2?

5. A man invests \$5000 in bonds which bear interest at 7%, payable semiannually. If he can reinvest the interest as it comes due at the same rate, at the end of 2 yr. how much interest will he have received from his original investment?

6. A woman has \$8500 invested in bonds that bear interest at $5\frac{1}{2}\%$ and \$6250 in stocks which pay regular dividends of 7%. On which investment, if either, is the annual income the greater, and how much greater is it?

7. Mr. Jacobs bought a house for \$9000 and rented it for \$65 a month. He paid annually \$100 for taxes, \$15 for insurance, and \$90 for repairs. At the end of 3 yr. from the time he purchased the property he sold it for \$10,400. How much more did he gain than he would have gained by investing the money for 3 yr. at 6%, simple interest?

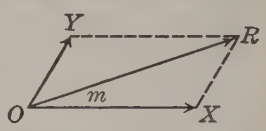
SCALE DRAWINGS

After Page 326

1. On a map of the United States in which each inch represents 32 mi., what distance on the map represents 48 mi.?

2. The infield of a baseball diamond is 90 ft. square. Draw a plan to the scale of 1 in. to 40 ft.

3. It is proved in physics that two forces acting in the directions OX and OY, as here shown, have the same effect as a single force acting in the direction OR, which is the diagonal of the parallelogram of which OX and OY are adjacent sides. Draw a figure like this to scale,



letting OY represent a force of 150 lb. acting at an angle of 60° with a force OX of 300 lb. Using a ruler, find the strength of the resultant force OR, and with a protractor find the size of the angle (m) which this force makes with OX.

4. A double tennis court is 78 ft. long and 36 ft. wide, and the net is halfway between the ends. The lines for the "alleys" are parallel to the long sides and $4\frac{1}{2}$ ft. inside them. The "service" lines between the alleys are parallel to the ends of the court and 18 ft. inside them, and are bisected by a line running lengthwise of the court. Draw a plan to the scale $\frac{1}{200}$.

5. A plant stand is 22 in. long and 1 ft. wide. Draw a plan of the top, using the scale $\frac{1}{6}$.

6. The floor of a garage is 24 ft. long and 10 ft. wide. Draw a plan of the floor, using the scale of $\frac{1}{8}$ in. to 1 ft.

7. If $\frac{7}{8}$ in. on a map represents an actual distance of $3\frac{1}{2}$ mi., what distance on the map represents 28.4 mi.?

8. If a map $4\frac{1}{2}$ ft. long and 3 ft. wide is reduced until it is 3 ft. long, what will the width be then?

PROBLEMS USING SQUARE ROOT

After Page 346

Find the length of the side of a square which has an area of

1. 1521 sq. ft.
2. 7744 sq. ft.
3. 13.69 sq. in.
4. 79.21 sq. in.
5. 105,625 sq. ft.
6. 1656.49 sq. yd.

Find to the nearest tenth the length of the hypotenuse of a right triangle in which the other sides are

7. 6 in., 8 in.
8. 11 in., 14 in.
9. 12 ft., 18 ft.
10. 3.6 in., 4.2 in.
11. 2.4 in., 1.8 in.
12. 3.5 ft., 2.6 ft.
13. 24 cm., 17 cm.
14. 45 cm., 28 cm.
15. 2.9 m., 3.7 m.

16. A square building lot contains about 24,400 sq. ft. of land. Find to the nearest 0.1 ft. the length of a side of the lot.

17. Find to the nearest millimeter the diameter of a circle that has an area of 27.2 sq. cm.

18. What distance is saved by cutting diagonally across a lot 58 ft. by 46 ft. instead of walking around two sides?

19. A baseball diamond is a square which is 90 ft. on a side. Find the distance across the diamond in a straight line from first base to third base.

20. How long must a rope be to reach from the top of a 16-foot pole to a point on the ground 20 ft. from its base?

21. What diameter should a pipe have in order to have a cross-section area of 6.4 sq. in.? of 5.8 sq. ft.?

22. A man has a fruit orchard in the form of a square. If it contains 576 sq. rd., how long is one side? How many rods of fence would be required?

23. A florist had a square flower bed containing 144 sq. ft. He enlarged it to a square 9 times as large. What were the dimensions after it was enlarged?

REVIEW PROBLEMS

After Page 412

1. If a horse is tethered by a 40-foot rope, the length of the largest circle that he can walk around is how many times what it would be if he were tethered by a 20-foot rope?

2. A steel rod 4 ft. 8 in. long has a triangular cross section 2 sq. in. in area. Taking the weight of a cubic foot of water as 62.5 lb., and steel as being 7.83 times as heavy as water, find how much the rod weighs.

3. A manufacturer has a water tank that is 8 ft. 3 in. wide, 12 ft. 6 in. long, and 9 ft. 8 in. deep. Taking the weight of a cubic foot of water as $62\frac{1}{2}$ lb., find the weight of the water required to fill the tank.

4. If a pressure of 120 lb. to the square inch is applied to a plunger 2 ft. in diameter, what is the total pressure?

5. If the drive wheel of an engine makes 140 revolutions a minute, and the cylinder is 9 in. in diameter and 14 in. long, and is filled with steam twice in each revolution, how many cubic feet of steam are used per minute?

6. In pumping oil from an oil-field, how many cubic feet of oil are there per mile of length in a pipe line which has an inside diameter of 14 in.? At 4.2 cu. ft. to the barrel, how many barrels of oil are there?

7. Water is flowing into a rectangular tank, whose base is 6 ft. 8 in. long and 4 ft. 6 in. wide, at the rate of 2 cu. ft. in 3 min. How long will it take the water to fill the tank to a depth of 3 ft.? of $4\frac{1}{2}$ ft.?

8. A circular lamp shade is 21 in. in diameter. Allowing 1 in. for overlapping, how much fringe must be purchased to go round the shade?

9. How many cubic feet of lumber are there in 15 planks, each 14 ft. 6 in. long, 6 in. wide, and 2 in. thick? How many board feet are there?

10. How far is it round a piece of land represented by a rectangle 14 in. by 18 in. on a map to the scale of 1 in. = 0.8 mi.?

11. The foot of a 37-foot ladder is 12 ft. from the wall of a building against which the top rests. How high does the ladder reach on the wall?

12. If a farmer fed 48 hogs 192 bu. of corn, how many bushels would he need in order to feed 72 hogs at the same rate?

13. On seven successive days a farmer delivered to a creamery the following number of pounds of cream: 32, 33, 31, 34, 33, 35, 32. This cream averaged 30.2% butter fat, worth 28¢ a pound. What was the value of the butter fat?

14. An agent bought for a dealer 240 baskets of peaches at 75¢ a basket, and charged a commission of $3\frac{1}{2}\%$. Find the total cost to the dealer.

15. The total assessed valuation of the property in a city is \$48,250,000; the amount to be raised from taxes is \$337,750. Find the tax on a piece of property assessed at \$75,000.

16. A manufacturing company has a capital of \$500,000, and has \$12,500 available for a semiannual dividend. Find the rate of dividend.

17. The Harrison Hardware Company orders the following items from a jobber: 5 doz. #18 bolts @ \$1.75; 8 doz. #25 E chain bolts @ \$10.50; 6 doz. #10 K locks @ \$22.50; $4\frac{1}{2}$ doz. #36 A hinges @ \$4.80; 16 doz. #48 brass plates @ \$3.60. At discounts of 25%, 10%, 8%, find the net amount of the order.

18. An iron casting weighing 425 lb. costs \$18.75. At this rate what is the cost of a similar casting weighing 379 lb.?

19. A bell weighing 875 lb. is composed by weight of 77% copper, 2% zinc, and the rest tin. How many pounds of each metal does the bell contain?

20. A dealer sold some fruit for $12\frac{1}{2}\%$ less than it cost, thereby losing \$70. What was the selling price?

21. A real-estate agent sold a lot for \$1775 and received \$71 commission. What per cent did he receive?

22. The duty on certain goods is 60¢ a pound, plus 45% ad valorem. If a shipment weighs 168 lb. and is worth \$380, what is the total duty?

23. A man takes out a life-insurance policy for \$4500 in one company at \$24.30 per \$1000 and a policy for \$3500 in another company at \$24.75. What do the two premiums together amount to in a year?

24. A farmer who has 75 trees on 1 A. of woodland decides to cut 60% of the trees. If the wood is worth \$5.75 a cord, and he can cut 3 cd. from 5 trees, how much will he receive for the wood?

25. If beans bought at \$1.60 a bushel are sold at 7¢ a quart, what is the per cent of gain or of loss on the cost?

26. A real-estate agent charged \$295.50 for selling a piece of property for \$9850. What was the rate of his commission?

27. Find the net amount of a bill for \$470, with trade discounts of 20% and 5%. What single rate of discount is equivalent to these two discounts?

28. A commission merchant sold 45 bbl. of apples at \$2.80 a barrel, and charged 6¢ a barrel for storage and a commission of $5\frac{1}{2}\%$. Find the net proceeds of the sale.

29. An ice-cream cone is 4 in. deep and 2 in. across the top. How many cubic inches of ice cream does the cone hold when packed level full?

MATERIAL FOR FREQUENT DRILL

How to Use the Exercises. This material is arranged to give practice in the fundamental operations. By going through each exercise, first using the number designated by (a), then the number designated by (b), and so on, nearly seven hundred different examples will be found.

EXERCISE 1

Taking (a) *7.1*, (b) *8.3*, (c) *15.4*, (d) *21.7*, (e) *24.9*, (f) *26.8*, (g) *35.7*, *or* (h) *28.2*, *as directed*,

1. Add it to $48.3 + 27.9 + 3.6 + 0.9 + 27.0$.
2. Subtract it from $23.4 + 76.5 + 62.8$.
3. Multiply it by itself.
4. Divide it by 0.65, finding the result to the nearest 0.01.
5. Find $33\frac{1}{3}\%$ of it, using the shortest method.

EXERCISE 2

Taking (a) *26.4*, (b) *31.4*, (c) *28.9*, (d) *34.8*, (e) *37.8*, (f) *39.1*, (g) *36.8*, *or* (h) *37.4*, *as directed*,

1. Add it to $27.9 + 36.7 + 4.2 + 29.9 + 0.8$.
2. Subtract it from $48.3 + 51.6 + 4.8$.
3. Multiply it by twice itself.
4. Divide it by 0.071.
5. Find 25% of it; 125% of it; $133\frac{1}{3}\%$ of it.

EXERCISE 3

Taking (a) *34.1*, (b) *35.7*, (c) *32.8*, (d) *38.7*, (e) *36.7*, (f) *39.8*, (g) *38.4*, *or* (h) *37.6*, *as directed*,

1. Add it to itself plus 9 times itself.

2. Subtract it from $429.3 + 570.6 + 0.1 + 4.7$.

3. Multiply it by 3.5.

4. Divide it by half itself; by twice itself; by 0.71.

5. Add it to the result of $2\frac{1}{4} \times 96$.

EXERCISE 4

Taking (a) *41.2*, (b) *42.3*, (c) *43.4*, (d) *44.5*, (e) *45.6*, (f) *46.7*, (g) *45.4*, *or* (h) *47.3*, *as directed*,

1. Add it to 99 times itself.

2. Subtract it from 1001 times itself.

3. Multiply it by the result of $625 \div 5$.

4. Divide it by the result of dividing it by 47.6.

5. Find 200% of it; 50% of it; 0.1% of it.

EXERCISE 5

Taking (a) *51.8*, (b) *52.7*, (c) *53.6*, (d) *54.5*, (e) *55.4*, (f) *56.3*, (g) *53.2*, *or* (h) *54.9*, *as directed*,

1. Add it to 94.3 less itself.

2. Subtract it from 26 times itself.

3. Multiply it by the result of $\frac{2}{3} \times \frac{1}{8}$.

4. Divide it by the result of $\frac{3}{4} \div \frac{7}{8}$.

5. Divide it by 125%; find 125% of it.

EXERCISE 6

Taking (a) *62.7*, (b) *63.6*, (c) *64.5*, (d) *65.4*, (e) *66.3*, (f) *67.2*, (g) *64.7*, *or* (h) *65.9*, *as directed*,

1. Add it to $4.6 + 39.8 + 0.9 + 53.7$.
2. Subtract it from $8.2 + 19.6 + 82.6 + 0.8$.
3. Multiply it by $2\frac{1}{2} + 3\frac{2}{5}$.
4. Divide it by $3\frac{7}{8} + 4\frac{3}{8}$.
5. Find $12\frac{1}{2}\%$ of it; $1\frac{1}{4}\%$ of it; 125% of it.

EXERCISE 7

Taking (a) *71.9*, (b) *72.8*, (c) *73.7*, (d) *74.6*, (e) *75.5*, (f) *76.4*, (g) *73.9*, *or* (h) *74.8*, *as directed*,

1. Add it to $48.7 + 6.9 + 39.4 + 95.0 + 0.2$.
2. Subtract it from $87.4 + 0.9 + 3.4 + 9.8 + 2.7$.
3. Multiply it by 0.625; by 62.5.
4. Divide it by $62\frac{1}{2}\%$; by 6.25; by $6\frac{1}{4}\%$.
5. Find 20% of it; 40% of it; 60% of it.

EXERCISE 8

Taking (a) *82.7*, (b) *83.6*, (c) *84.5*, (d) *85.4*, (e) *86.3*, (f) *87.2*, (g) *86.9*, *or* (h) *87.8*, *as directed*,

1. Add it to 999 times itself.
2. Subtract it from $6\frac{1}{2}$ times itself.
3. Multiply it by $2\frac{1}{2}\%$; by 25%; by 250%.
4. Divide it by $2\frac{1}{2}\%$; by 25%; by 250%.
5. Add it to the result of $42\frac{3}{4} \div \frac{3}{8}$.

EXERCISE 9

Taking (a) *2 ft. 8 in.*, (b) *3 ft. 6 in.*, (c) *5 ft. 4 in.*, (d) *6 ft. 2 in.*, (e) *7 ft. 10 in.*, (f) *8 ft. 4 in.*, (g) *8 ft. 7 in.*, *or* (h) *9 ft. 6 in.*, *as directed,*

1. Add it to 19 times itself.
2. Subtract it from 10 ft. 1 in.
3. Multiply it by 2; by $2\frac{1}{8}$; by $21\frac{1}{4}$.
4. Divide it by 2; by 4; by 6.
5. Divide it by 3 in; by 5 in.; by 2 ft.

EXERCISE 10

Taking (a) *92.7*, (b) *97.2*, (c) *93.4*, (d) *94.3*, (e) *98.6*, (f) *96.8*, (g) *97.7*, *or* (h) *98.4*, *as directed,*

1. Add it to $137\frac{7}{8}$ less itself.
2. Subtract it from 126 times itself.
3. Multiply it by the result of $196.4 \div 7.856$.
4. Divide it by 7.264; by $\frac{1}{2}$ of 7.264.
5. Multiply 34 ft. 2 in. by it.

EXERCISE 11

Taking (a) *1 yd. 15 in.*, (b) *58 in.*, (c) *2 yd. 8 in.*, (d) $2\frac{1}{2}$ *yd.*, (e) $3\frac{1}{2}$ *yd.*, (f) *43 in.*, (g) *62 in.*, *or* (h) $1\frac{1}{2}$ *ft.*, *as directed,*

1. Add it to 5 ft. 8 in. + 3 ft. 8 in.
2. Subtract it from 6 yd. 1 ft. + 2 yd. 11 in.
3. Multiply it by $2\frac{1}{2}$; by 25; by 25%.
4. Divide it by 3; by 5; by 7; by 12.
5. Divide it by 1 in.; by 0.1 in.; by $\frac{1}{4}$ in.

TABLES FOR REFERENCE

Length

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

Weight

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

Liquid Measure

2 pints (pt.) = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
12 months (mo.) = 1 year (yr.)

A common year has 365 da.; a leap year, 366 da.; for figuring interest, the commercial year is usually taken as 360 da., and the month as 30 da.

Money

10 mills = 1 cent (c., ¢, or ct.)
100 cents = 1 dollar ($)

Angles and Arcs

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)

Counting

12 units = 1 dozen (doz.)
12 dozen, or 144 units = 1 gross (gr.)

Paper

24 or 25 sheets = 1 quire
20 quires = 1 ream

For folded paper a quire is usually 24 sheets; for unfolded paper it is 25 sheets. Paper is sold also by weight.

Weight per Bushel

Barley . . .	48 lb.	Clover seed . .	60 lb.	Peas . . .	60 lb.
Beans . . .	60 lb.	Corn, shelled .	56 lb.	Potatoes .	60 lb.
Beets . . .	60 lb.	Flaxseed . . .	56 lb.	Rye . . .	56 lb.
Buckwheat .	48 lb.	Oats	32 lb.	Wheat . .	60 lb.

These weights vary in different parts of the country.

Practical Equivalents

1 bu. contains $1\frac{1}{4}$ cu. ft., or 2150 cu. in.
1 bbl. contains 4.2 cu. ft., or $31\frac{1}{2}$ gal.
1 gal. contains 231 cu. in.
1 cu. ft. contains $7\frac{1}{2}$ gal.
1 cu. ft. of water weighs $62\frac{1}{2}$ lb.
1 gal. of water weighs 8.3 lb.
1 T. of hay occupies 500 cu. ft.
1 cd. (cord) of 4-foot wood occupies 128 cu. ft.
1 T. of hard coal occupies 35 cu. ft.
1 T. of soft coal occupies 42 cu. ft.
1 cu. ft. of brickwork contains 22 bricks (and mortar)

These are generally only approximations, but they give results sufficiently close for ordinary computations.

Convenient Rules for Farmers

Bushels in a Bin. *Take $\frac{4}{5}$ of the product of the three dimensions.*

This gives the number of bushels for a rectangular bin when the dimensions are expressed in feet.

Bushels in a Circular Bin. *Multiply $\frac{5}{8}$ of the square of the diameter by the depth.*

This gives the number of bushels in a circular bin when the dimensions are expressed in feet.

Bushels of Shelled Corn. *Multiply the product of the three dimensions by 0.45 for old, dry corn; by 0.4 for new, fresh corn.*

This gives the number of bushels of shelled corn for a rectangular bin when the dimensions are expressed in feet.

Feet of Lumber in a Log. *Subtract 4 from the number of inches in the smallest diameter of the log, square the result, and multiply by $\frac{1}{16}$ of the number of feet in the length.*

Tons of Hay in a Stack. 1. *Find the approximate height by subtracting the width from the overthrow and dividing by 2. Then divide the product of the three dimensions by 500.*

The overthrow is measured, over the top, from the ground at one side to the ground at the other side. The dimensions are to be expressed in feet. For wild hay divide by 412; for timothy, by 450; for clover and alfalfa, by 512. The rule gives only an approximate result.

2. *Divide the product of the three dimensions in yards by 30.*

The divisor varies from 30 to 36, according to the kind of hay. This does not give such close results as the first rule and assumes that there is considerable slope to the stack.

Tons of Hay in a Mow. *Divide the product of the three dimensions in yards by 15 for well-packed hay.*

If the hay is not well packed, the divisor may vary from 15 to 18, according to conditions.

Gallons in a Tank or Cistern. *Multiply the product of the three dimensions by 7.5 for rectangular tanks; multiply the product of the depth and the square of the diameter by 5.9 for round (cylindric) cisterns.*

The dimensions in both cases are to be expressed in feet.

Approximate Tonnage of a Silo. The following table is useful:

Height in Feet	Inside Diameter in Feet					
	10	12	14	16	18	20
30	47 T.	67 T.	91 T.			
35	59 T.	84 T.	114 T.	137 T.		
40	70 T.	101 T.	138 T.	180 T.	229 T.	
45				215 T.	269 T.	330 T.
50					310 T.	382 T.

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Page 5. 1. 3304. 2. 12,376. 3. 31,140. 4. 76,608. 5. 278,614. 6. 176,320. 7. 188,592. 8. 236,929. 9. 1,699,920. 10. 241,980. 11. 279,772. 12. 858,088. 13. 272,916. 14. 430,360. 15. 772,255. 16. 2,307,690. 17. 435,780. 18. 2,210,560. 19. 3,626,000. 20. 4,560,000. 21. 318,132. 22. 453,882. 23. 318,955. 24. 387,408. 25. 206,360. 26. 5,836,380. 27. 3,763,260. 28. 7,209,380. 29. 1,082,610. 30. 3,323,180. 31. 288,782. 32. 306,306. 33. 163,649. 34. 652,832. 35. 241,063. 36. 4,415,560. 37. 4,731,000. 38. 5,605,220. 39. 3,916,170. 40. 995,860. 41. 269,397. 42. 95,282. 43. 834,344. 44. 318,160. 45. 332,681. 46. 175,560. 47. 435,600. 48. 553,076. 49. 821,601. 50. 208,872.

Page 6. 1. 37. 2. 66. 3. $215\frac{1}{2}$. 4. $131\frac{1}{2}$. 5. $35\frac{2}{3}$. 6. $258\frac{2}{3}$. 7. 454. 8. 92. 9. $86\frac{1}{4}$. 10. $71\frac{2}{5}$. 11. 259. 12. 92. 13. 35. 14. $124\frac{7}{8}$. 15. 180. 16. 41. 17. 72. 18. $115\frac{2}{3}$. 19. $17\frac{4}{7}$. 20. $54\frac{1}{6}$. 21. $225\frac{3}{4}$. 22. $404\frac{1}{2}$. 23. $63\frac{5}{7}$. 24. $236\frac{1}{3}$. 25. 147. 26. $235\frac{2}{3}$. 27. $95\frac{3}{8}$. 28. $77\frac{3}{7}$. 29. 124. 30. 181. 31. 59. 32. 37. 33. $31\frac{1}{4}$. 34. $43\frac{1}{8}$. 35. $85\frac{3}{5}$. 36. 236. 37. $89\frac{8}{9}$. 38. $53\frac{1}{4}$. 39. 148. 40. 212. 41. $137\frac{1}{2}$. 42. 47. 43. $74\frac{6}{7}$. 44. $37\frac{3}{8}$. 45. $226\frac{1}{4}$. 46. 45. 47. 74. 48. $25\frac{1}{5}$. 49. $75\frac{1}{3}$. 50. $244\frac{3}{4}$. 51. $269\frac{1}{3}$. 52. 102. 53. 113. 54. 75. 55. $234\frac{1}{2}$. 56. 140. 57. $45\frac{7}{8}$. 58. $64\frac{4}{7}$. 59. 99. 60. $301\frac{2}{3}$.

Page 7. 1. 250. 2. 326. 3. 241. 4. $301\frac{1}{4}$. 5. $208\frac{1}{3}$. 6. $297\frac{13}{21}$. 7. $334\frac{2}{3}$. 8. $215\frac{6}{11}$. 9. $170\frac{1}{11}$. 10. $222\frac{28}{31}$. 11. $210\frac{1}{4}$. 12. $210\frac{1}{2}$. 13. 157. 14. $78\frac{1}{2}$. 15. 122. 16. $122\frac{1}{2}$. 17. 102. 18. 202. 19. $200\frac{1}{2}$. 20. 221. 21. $170\frac{7}{16}$. 22. 235. 23. $335\frac{1}{2}$. 24. 410. 25. 406. 26. $406\frac{1}{2}$. 27. 201. 28. $77\frac{5}{16}$.

29. $151\frac{19}{24}$. 30. 152. 31. 21. 32. $21\frac{1}{3}$. 33. $123\frac{4}{9}$. 34. 110. 35. $110\frac{1}{8}$. 36. $120\frac{1}{2}\frac{1}{8}$. 37. 20. 38. 21. 39. $21\frac{1}{2}$. 40. $88\frac{20}{95}$. 41. 110. 42. $110\frac{1}{2}$. 43. 110. 44. 111. 45. $115\frac{2}{7}$. 46. $101\frac{1}{7}$. 47. $60\frac{1}{8}$. 48. 43. 49. 403. 50. 210. 51. 211. 52. $34\frac{1}{3}$. 53. $100\frac{1}{3}$. 54. $20\frac{1}{8}$. 55. 50. 56. $50\frac{1}{2}$. 57. 202. 58. $210\frac{1}{4}$. 59. 91. 60. 51.

Page 8. 1. $1\frac{1}{4}$. 2. $\frac{1}{4}$. 3. $\frac{3}{8}$. 4. $1\frac{1}{2}$. 5. $\frac{2}{3}$. 6. $2\frac{1}{8}$. 7. $1\frac{1}{8}$. 8. $\frac{13}{16}$. 9. $3\frac{1}{4}$. 10. $\frac{4}{13}$. 11. $4\frac{5}{8}$. 12. $1\frac{1}{8}$. 13. $5\frac{1}{32}$. 14. $1\frac{9}{14}$. 15. $\frac{12}{23}$. 16. $7\frac{7}{8}$. 17. $2\frac{3}{8}$. 18. $14\frac{3}{32}$. 19. $1\frac{19}{22}$. 20. $\frac{22}{41}$. 21. $37\frac{1}{2}$. 22. 6. 23. $\frac{5}{8}$. 24. $\frac{3}{8}$. 25. $\frac{1}{16}$. 26. 4. 27. $\frac{1}{4}$. 28. $2\frac{7}{8}$. 29. $1\frac{7}{8}$. 30. $1\frac{3}{16}$. 31. $4\frac{3}{4}$. 32. $\frac{4}{19}$. 33. $5\frac{3}{8}$. 34. $1\frac{7}{8}$. 35. $6\frac{11}{32}$. 36. $2\frac{1}{14}$. 37. $\frac{14}{25}$. 38. $10\frac{1}{8}$. 39. $2\frac{5}{8}$. 40. $23\frac{29}{32}$. 41. $1\frac{7}{10}$. 42. $\frac{30}{61}$. 43. $28\frac{1}{8}$. 44. 2. 45. $\frac{5}{6}$. 46. $\frac{1}{6}$. 47. $\frac{1}{8}$. 48. $1\frac{1}{2}$. 49. $\frac{2}{3}$. 50. $4\frac{1}{6}$. 51. $3\frac{1}{8}$. 52. $1\frac{5}{8}$. 53. $7\frac{1}{3}$. 54. $\frac{3}{22}$. 55. $7\frac{1}{3}$. 56. $4\frac{1}{3}$. 57. $8\frac{3}{4}$. 58. $3\frac{8}{9}$. 59. $\frac{9}{35}$. 60. $9\frac{1}{2}$. 61. $3\frac{5}{8}$. 62. $18\frac{8}{9}$. 63. $2\frac{6}{17}$. 64. $\frac{17}{40}$. 65. $103\frac{35}{64}$. 66. 3.

Page 9. 2. 0.25; 25%. 3. $\frac{3}{4}$; 75%. 4. $\frac{1}{10}$; 0.10. 5. 0.125; $12\frac{1}{2}$%. 6. $\frac{3}{8}$; $37\frac{1}{2}$%. 7. $\frac{5}{8}$; 0.625. 8. 0.875; $87\frac{1}{2}$%. 9. $\frac{1}{3}$; $33\frac{1}{3}$%. 10. $\frac{2}{3}$; $0.66\frac{2}{3}$. 11. $0.16\frac{2}{3}$; $16\frac{2}{3}$%. 12. $\frac{5}{6}$; $0.83\frac{1}{3}$. 13. 0.40; 40%. 14. $\frac{3}{5}$; 60%. 15. $\frac{4}{5}$; 0.80. 16. 0.05; 5%. 17. $\frac{3}{50}$; 0.06. 18. $\frac{2}{25}$; 8%. 19. $\frac{3}{25}$; 0.12. 20. $\frac{3}{20}$; 15%. 21. $0.08\frac{1}{3}$; $8\frac{1}{3}$%. 22. $\frac{1}{16}$; 0.0625. 24. 1; 100%. 25. 3; 3.00. 26. 6.25; 625%. 27. 6.75; 675%. 28. $2\frac{1}{8}$; $212\frac{1}{2}$%. 29. $2\frac{1}{4}$; 2.25. 30. $2\frac{1}{3}$; $233\frac{1}{3}$%. 31. $2\frac{1}{6}$; $2.16\frac{2}{3}$. 32. $3.66\frac{2}{3}$; $366\frac{2}{3}$%. 33. $4\frac{2}{3}$; $466\frac{2}{3}$%. 34. 2; 2.00. 35. $3\frac{3}{8}$; $337\frac{1}{2}$%. 36. 2.625; $262\frac{1}{2}$%. 37. $6\frac{7}{8}$; $687\frac{1}{2}$%. 38. $2\frac{5}{6}$; $2.83\frac{1}{3}$. 39. $1.16\frac{2}{3}$; $116\frac{2}{3}$%. 40. $2\frac{1}{12}$; $208\frac{1}{3}$%. 41. 5.00; 500%. 42. 7; 700%. 43. $6.83\frac{1}{3}$; $683\frac{1}{3}$%. 44. $7\frac{1}{2}$; 7.50.

Page 10. 1. \$47.25. 2. 28¢. 3. \$4; \$2; \$5. 4. \$24.18. 5. \$83.20; 2.2. 6. \$4.11. 7. 8; \$12.

Page 13. 17. 2; 2; 2.4. 18. 110; 110; 931. 19. 1814; 1814; 4688. 20. 4860; 4860; 2904. 21. 1205; 1205; 1120. 22. $\frac{1}{8}$; 9. 23. $\frac{3}{8}$; 240. 24. 140; 2800. 25. 32; 3224. 26. 1586; 1586; 2834.

Page 14. 10. 0.1; 10%. 11. 0.001; $\frac{1}{1000}$. 12. 8. 13. 0.005. 14. 1.5. 15. 0.00005. 16. 0.375. 17. $0.66\frac{2}{3}$. 18. 0.0167. 19. 0.0375.

Page 15. 6. 90%. 7. 620%. 8. 30%. 9. 300%. 10. 59%. 11. 72%. 12. $32\frac{1}{2}$%. 13. 65%. 14. $666\frac{2}{3}$%. 15. 0.375%.

Page 16. 10. $8\frac{1}{3}$%; $2\frac{7}{9}$%. 11. $6\frac{1}{4}$%; $1\frac{9}{16}$%. 12. 50%; $62\frac{1}{2}$%, 100%. 13. $6\frac{1}{4}$%; $23\frac{3}{4}$%; 100%. 14. $6\frac{2}{3}\frac{2}{3}$%; $18\frac{2}{11}$%; 100%; 200%. 15. $11\frac{1}{9}$%; $\frac{1}{8}$.

Page 17. 4. Four hundred seventy-five per cent. 5. Four and seventy-five hundredths per cent. 6. Three and nine tenths per cent. 7. Thirty-nine per cent. 8. Thirty-nine hundredths per cent. 9. Three hundred ninety per cent. 10. Five and twenty-five hundredths (or five hundred twenty-five per cent). 11. Seven and thirty-five hundredths (or seven hundred thirty-five per cent). 12. Six tenths per cent. 13. Six hundredths

per cent. 14. 2.25. 15. 0.0025. 16. 3. 17. 0.00375. 18. 0.004. 19. $\frac{38}{60}$. 20. $\frac{1}{16}$. 21. $3\frac{1}{4}$. 22. $4\frac{3}{4}$. 23. $\frac{1}{3}$. 24. 60%. 25. $33\frac{1}{3}$%. 26. $37\frac{1}{2}$%. 27. 80%. 28. 250%. 29. 375%. 30. 0.28. 31. 0.34. 32. 0.006. 33. $0.66\frac{2}{3}$. 34. 6. 35. 3.55. 36. 2.5. 37. 2.75. 38. 3.25. 39. 50. 40. 0.00006. 41. 45.5. 42. $\frac{3}{8}$; 64%; $0.6\frac{3}{8}$; $\frac{18}{25}$.

Page 18. 1. Fifty per cent; fifty hundredths; 2; 2; 2. 2. 8; $\frac{1}{2}$; 50; 50%. 3. Twenty-five per cent; twenty-five hundredths; 4; 4; 4. 4. $\frac{1}{8}$; $12\frac{1}{2}$; $12\frac{1}{2}$%. 5. 8; 8. 6. 1. 7. 3. 8. 6; 12. 9. 16; 48. 10. 5; 25. 11. 14; 24. 12. 36; 80. 13. $\frac{3}{20}$; $\frac{3}{20}$; $\frac{3}{20}$. 14. 0.0075; $\frac{3}{400}$. 15. $6.66\frac{2}{3}$; $\frac{20}{3}$.

Page 20. 10. $14\frac{2}{7}$%, $28\frac{4}{7}$%, $42\frac{6}{7}$%, $57\frac{1}{7}$%, $71\frac{3}{7}$%, $85\frac{5}{7}$%, 100%. 11. $11\frac{1}{9}$%, $22\frac{2}{9}$%, $33\frac{1}{3}$%, $44\frac{4}{9}$%, $55\frac{5}{9}$%, $66\frac{2}{3}$%, $77\frac{7}{9}$%, $88\frac{8}{9}$%, 100%. 12. $8\frac{1}{3}$%, $16\frac{2}{3}$%, 25%, $33\frac{1}{3}$%, $41\frac{2}{3}$%, 50%, $58\frac{1}{3}$%, $66\frac{2}{3}$%, 75%, $83\frac{1}{3}$%, $91\frac{2}{3}$%, 100%. 13. $6\frac{1}{4}$%, $12\frac{1}{2}$%, $18\frac{3}{4}$%, 25%, $31\frac{1}{4}$%, $37\frac{1}{2}$%, $43\frac{3}{4}$%, 50%, $56\frac{1}{4}$%, $62\frac{1}{2}$%, $68\frac{3}{4}$%, 75%, $81\frac{1}{4}$%, $87\frac{1}{2}$%, $93\frac{3}{4}$%, 100%. 14. 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, 90%, 100%. 15. 5%, 10%, 15%, 20%, 25%, 30%, 35%, 40%, 45%, 50%, 55%, 60%, 65%, 70%, 75%, 80%, 85%, 90%, 95%, 100%. 16. $1\frac{1}{4}$. 17. $1\frac{3}{5}$. 18. $\frac{5}{12}$. 19. 10%. 20. 50%. 21. 20%. 22. 4. 23. 25%. 24. $1\frac{3}{5}$. 25. $636.90. 26. $931.16. 27. $641.09. 28. $1461.01. 29. $6663.12. 30. $25,721.23. 31. $53,380.08. 32. $59,256.75. 33. $98,172.54. 34. $132,451.44.

Page 22. 8. 262.5. 9. 1487.5. 10. $20,160. 11. $27,500. 12. $56. 13. 3; 47. 14. $15.40. 15. 1.3. 16. $407. 17. $20,368,800. 18. $74.52. 19. 178; 1246. 20. $2375. 21. $593.75. 22. $6; $34.

Page 24. 7. 16.58 lb. 8. 15.96 lb.; $4.15. 9. Second; 52¢. 10. $1458.

Page 27. 1. 5%; $3\frac{1}{3}$%. 2. 3%; 20%. 3. 4%; 5%. 4. $6\frac{1}{4}$%; $16\frac{2}{3}$%. 5. 25%; 1%. 6. 1650%; 550%. 7. $266\frac{2}{3}$%; 150%. 8. $33\frac{1}{3}$%; 20%; 10%. 9. 50%; $33\frac{1}{3}$%; 25%. 10. 50%; 25%; $16\frac{2}{3}$%. 11. $14\frac{2}{7}$%. 12. $166\frac{2}{3}$%. 13. $12\frac{1}{2}$%. 14. 9%. 15. 5.3%. 16. 8.6%. 17. $7\frac{1}{7}$%. 18. 25%. 19. 23%. 20. 75%. 21. 18%. 22. 48%. 23. 50%; 100%; 40%; 20%; 200%; 500%. 24. 75%; $58\frac{1}{3}$%. 25. $66\frac{2}{3}$%; 50%. 26. $6\frac{1}{4}$%; $12\frac{1}{2}$%. 27. 17.1%; 82.9%; 100%.

Page 29. 1. 62,000. 2. 20,000. 3. 87. 4. 288. 5. $76.50. 6. 400. 7. 1500. 8. 400. 9. 1400. 10. 1100. 11. 296. 12. 1470. 13. 159. 14. 496. 15. 520. 16. 194.7. 17. 340. 18. 20.2. 19. 62. 20. 11,320. 21. $31; $23.25. 22. $49; $78.40. 23. 113 ft. 4 in.; 249 ft. 4 in. 24. 362 lb. 8 oz. 25. $200. 26. 440. 27. 120%; $300. 28. $7550. 29. 120; 96. 30. 672. 31. 700.

Page 31. 13. 50%. 14. $33\frac{1}{3}$%. 15. $33\frac{1}{3}$%. 16. 20%. 17. 20%. 18. $16\frac{2}{3}$%; $12\frac{1}{2}$%. 19. 9%. 20. 8%. 21. $11\frac{1}{9}$%. 22. 200%. 23. $66\frac{2}{3}$%. 24. 100%. 25. 25%. 26. 20%. 27. 7%. 28. 7%. 29. 14%. 30. 5%. 31. 13%. 32. 20%. 33. 9%. 34. 11%. 35. 18%. 36. 16%.

Page 33. 1. $12\frac{1}{2}\%$. 2. 25%; 20%; 25%. 3. 8%; 92%. 4. 132. 5. 371. 6. 19.2 mi. per hour. 7. $2520. 8. 45. 9. 0.8%. 10. $180. 11. 25%. 12. $33\frac{1}{3}\%$. 13. $6\frac{1}{4}\%$. 14. 300%. 15. 78.8%. 16. 60.3%. 17. 23.3%; 26.7%. 18. 0.7%. 19. $18; $207.

Page 36. 1. $70; $770. 2. $112; $912. 3. $118.75; $1068.75. 4. $94.25; $819.25. 5. $300; $2300. 6. $400; $2900. 7. $525; $3525. 8. $720; $4720. 9. $336; $4536. 10. $297; $3897. 11. $468; $5268. 12. $1125; $8625. 13. $819.25; $4720; $8625.

Page 37. 1. $40. 2. 6%. 3. $200. 4. $100. 5. 8%. 6. $333.33. 7. $1200. 8. 20%. 9. $750. 10. $6\frac{1}{2}\%$.

Page 39. 1. 30%; 40%. 2. 11.7%. 3. 8.9%. 4. 15%. 5. $30.15. 6. $6440. 7. $4760. 8. 5%. 9. $4000; $28,000. 10. $60. 11. $9800; $11,760.

Page 41. 1. $400. 2. $1000. 3. $1500. 4. $1400. 5. $2000. 6. $2000. 7. $10,000.

Page 42. 1. $1.80. 2. 15%. 3. $2.60; $1.40; 10.8%. 4. $6.80. 5. 12%. 6. $19. 7. $7.10; $10.70; $19.40; $27.90. 8. $5.15. 9. 25%. 10. $2.50.

Page 45. 1. $49.50; $1050.50. 2. $1; $49. 3. $1.50; $73.50. 4. $2.25; $72.75. 5. $3.40; $81.60. 6. $2.70; $57.30. 7. $2.60; $77.40. 8. $3.80; $76.20. 9. $4; $196. 10. $6.25; $243.75. 11. $12; $308. 12. $200; $4800. 13. $240; $4560. 14. $341; $5859. 15. $342; $6858. 16. $37.50; $1212.50. 17. $39.38; $1710.62. 18. $75; $1800. 19. $78.75; $2171.25. 20. $135; $3240. 21. $147.15; $3122.85. 22. $450; $14,550. 23. $400; $19,600. 24. $750; $24,250. 25. $750; $29,250.

Page 47. 1. $60. 2. $264; $4536. 3. $1459.20. 4. $60; $690. 5. $136; $6936. 6. $420. 7. $7.65. 8. $107.18. 9. $114. 10. $750. 11. $14.

Page 49. 1. $30. 2. 6%. 3. $700. 4. $742. 5. $335.20. 6. $6.69. 7. $76.80. 8. 8%. 9. Second.

Page 50. 1. $14. 2. 5%. 3. $340. 4. 4%. 5. $45. 6. $30. 7. 2%. 8. $22.50. 9. $2000. 10. 8%. 11. $33. 12. $240. 13. 20%. 14. $42. 15. 5.4%. 16. $420. 17. $60. 18. 1.6%. 19. $147.50. 20. $112.50. 21. $12; 6%. 22. $15; 5%. 23. $20; 4%. 24. $36; 6%. 25. $26; 4%. 26. $15; 2%. 27. $22.50; 3%. 28. $24.75; 3%. 29. $51; 4%. 30. $12; 6%. 31. $16; 8%. 32. $15; 5%. 33. $50; 10%. 34. $56; 8%. 35. $27; 3%. 36. $22.50; 3%. 37. $9.50; 1%. 38. $24.25; 1.5%.

Page 52. 19. $23 60. 20. $31.00. 21. $34.80. 22. $64.95. 23. $63.30. 24. $52.60. 25. $47.80. 26. $41.25. 27. $71.40. 28. $75.60. 29. $29.92. 30. $52.65. 31. $35.80. 32. $52.20. 33. $34.74.

Page 55. 1. $196. 2. $336. 3. $696. 4. $543.20. 5. $121.60. 6. $1960. 7. $2425. 8. $2944. 9. $4512. 10. $6900. 11. $20. 12. $40. 13. $50.

14. $600. 15. $525. 16. $340. 17. $387. 18. $558. 19. $672. 20. $750. 21. $880. 22. $1400. 23. $1032. 24. $1760.

Page 57. 1. $162. 2. $576. 3. $256. 4. $427.50. 5. $472.50. 6. $249.90. 7. $6.25. 8. No; no difference; no difference. 9. $17.20; 17.2%. 10. 19%. 11. 20.8%. 12. 20.2%. 13. 19.04%. 14. 26.4%. 15. 28%. 16. $1043.10; $1043.10. 17. $38.25. 18. First; $325.50. 19. $837.90; $837.90; $837.90.

Page 58. 1. $167.33. 2. $3472.87. 3. $499.61. 4. $415.97. 5. $2009.93. 6. $2485.65.

Page 60. 1. $21.36. 2. $37.77; $4.71. 3. $14.45. 4. $22.34. 5. $14.33. 6. $1.25. 7. 30%.

Page 63. 1. $2200. 2. $3200. 3. $13,750. 4. $7000. 5. $8100. 6. $9200. 7. $9400. 8. $7500. 9. $9000. 10. $12,200. 11. $8700. 12. $8200. 13. $9800. 14. $4900. 15. $9800. 16. $19,600. 17. $14,500. 18. $7000. 19. $4500. 20. $5500. 21. $6500. 22. 12,500. 23. $12,500. 24. $14,000. 25. $29,000. 26. $22,500. 27. $20,300.

Page 64. 1. 90¢. 2. 70¢. 3. 80¢. 4. 85¢. 5. 70¢. 6. 90¢. 7. $1. 8. $1.10. 9. $1. 10. $1.25. 11. $1.50. 12. $1. 13. $1.25. 14. $1.25. 15. $1.20. 16. $1.10. 17. $1.10. 18. $1.20. 19. $1.30. 20. $1.30. 21. $1.30. 22. Second.

Page 65. 1. $39.20. 2. $72. 3. $100. 4. $120. 5. $112.50. 6. $7000. 7. $3240. 8. $304.50. 9. $1.65. 10. 66¢. 11. $1.20; 48¢. 12. $21,000; $33,600. 13. $9072. 14. $62.50; $3000.

Page 66. 1. $54.20. 2. $76.92. 3. $43.60. 4. $129.80. 5. $56.16. 6. $143.20. 7. $189.90. 8. $218.40. 9. $426. 10. $555. 11. $967.75. 12. $1567.80. 13. $10,000. 14. $4000. 15. $7000. 16. $30.12. 17. $24.09. 18. $31.65.

Page 68. 1. $248.40; $7452. 2. $5448. 3. $419.30; $8386. 4. $147.42. 5. $295.30; $8859; $1141. 6. $4076. 7. $2541; $1310. 8. $4230; $3482.50. 9. $1259.91. 10. $2683.20. 11. $1400. 12. $500. 13. $25.

Page 70. 1. $8.40; until her death or remarriage. 2. $145.60. 3. $20.40; $408.

Page 74. 1. $174.40. 2. $124.10. 3. $189. 4. $260.70. 5. $440. 6. $24.60. 7. $196. 8. $550. 9. $203.75. 10. $97.20. 11. $140.40. 12. $172.80. 13. $99.36. 14. $151.20. 15. $97.20. 16. $187.92. 17. $209.52. 18. $145.80. 19. $183.60. 20. $259.20. 21. $540. 22. $702. 23. $831.60. 24. $972. 25. $1620. 26. $257.50. 27. $15,000; $18,000.

Page 75. 1. $52.80. 2. $61.65. 3. $78. 4. $84.70. 5. $123.20. 6. $183.20. 7. $196.35. 8. $52.11. 9. $59.25. 10. $108. 11. $208.80. 12. $235.60. 13. $224.40. 14. $362.50. 15. $367.50. 16. $127.26.

17. $207.56. 18. $493.28. 19. $707.30. 20. $312.41. 21. $383.67. 22. $593.78. 23. $803.88.

Page 77. 1. $106.80; $124.60; $133.50; $153.08; $174.44. 2. $178; $213.60; $284.80; $293.70; $455.68. 3. $138.84. 4. $405.84. 5. $165.54. 6. $119.26. 7. $142.40. 8. $160.20. 9. $151.30. 10. $267. 11. $400.50. 12. $445. 13. $578.50. 14. $756.50. 15. $979. 16. $1335. 17. $1343.90. 18. $1424. 19. $1557.50. 20. $2225. 21. $12,015. 22. $4895. 23. $6230. 24. $15,575. 25. $52.87. 26. $2139.40.

Page 79. 1. $36.80; $10.50. 2. Gained; $539.50. 3. $400 (macadam); $18.18 (concrete). 4. 2100.2%.

Page 81. 11. $3575; ad valorem. 12. $700; specific. 13. $660; $3000; $1200. 14. $28; $100; $400. 15. $1312.50; $1425; $1875. 16. $60. 17. $45. 18. $380. 19. $37.50. 20. $36. 21. $3062.50. 22. $1356. 23. $2000. 24. $2887.50.

Page 84. 1. $52.13. 2. $41.27. 3. $46,785.63. 4. $975. 5. $2750. 6. $15,600. 7. $358,605. 8. 30.3%. 9. 1011.1%.

Page 87. 13. 40¢. 14. 75¢. 15. 11¢. 16. 147.

Page 88. 1. 58.3%. 2. $22.40. 3. 17.9%. 4. $1300. 5. Walter. 6. $5.43. 7. $64.80. 8. 90.3 lb. 9. 48 lb. 10. 9.2%.

Page 89. 1. Four hundred thousand seventy-five; forty and seventy-five ten-thousandths; four thousand and seventy-five hundredths; four hundred seventy-five thousandths; four hundred seventy-five hundred-thousandths. 2. $57.98. 3. 31 ft. 1 in. 4. $\frac{13}{16}$. 5. $500.64. 6. $132.02. 7. $376,929. 8. 39.14. 9. 1.02. 10. $24.75. 11. $12. 12. $54. 13. $168.75. 14. $17.88. 15. $44.33. 16. $5.50. 17. $15.75. 18. $6.60. 19. $13.30. 20. $400. 21. $522.50. 22. $73.50. 23. $9180. 24. $6600. 25. $18,843.75. 26. $23,000.

Page 93. 1. $2.58. 2. Not profitable; $1.16, loss. 3. $2.44, loss. 4. 33¢, gain; yes; $2.77. 5. 40 lb. nitrogen, 160 lb. phosphoric acid, 40 lb. potash. 6. 12 lb. nitrogen, 24 lb. phosphoric acid, 12 lb. potash. 7. Phosphoric acid; 12 lb. nitrogen, 12 lb. potash; 80 lb. nitrogen, 80 lb. potash. 8. Potash; nitrogen; 12 lb. nitrogen, 24 lb. phosphoric acid; 24 lb. phosphoric acid, 12 lb. potash; 80 lb. nitrogen, 160 lb. phosphoric acid; 160 lb. phosphoric acid, 80 lb. potash.

Page 95. 2. $41.60. 3. John, $7.80; William, Clarence, and Roger, $5.20 each; Mary, Susan, Ella, and Ann, $3.25 each; Henry and Frank, $2.60 each. 4. $67 60. 5. $75.45. 6. $58.91. 7. $42.30. 8. $35.85. 9. $33.90. 10. $314.01. 12. $46.42. 13. $43.60. 14. $44.85. 15. $48.10. 16. $182.97. 17. 1st, Ann, $1.92; 2d, William, $1.89; 3d, Mary, $1.86;

4th, Ella and Henry, $1.79; 5th, Susan, $1.74; 6th, Frank, $1.70; 7th, Clarence, $1.47; 8th, John, $1.13; 9th, Roger, $1.06. **18.** $496.98; $248.49. **19.** $49.70; $25.75; $15.80.

Page **98.** I. 1. 50%; 200%. 2. $33\frac{1}{3}$%; 50%. **3.** $15.75; $157.50; $1575. 4. $5250; $60. 5. $6300; $5684.

II. 1. 6; 6; 6. 2. $87\frac{1}{2}$%. **3.** 2.8%. **4.** $\frac{1}{3}$; $\frac{2}{3}$; $\frac{5}{6}$. **5.** 75%; $87\frac{1}{2}$%; $6\frac{1}{4}$%; $62\frac{1}{2}$%.

III. 1. $8\frac{1}{3}$%; $0.8\frac{1}{3}$%; $0.08\frac{1}{3}$%. 2. 90%; 7.5%; 2.5%. **3.** 50%; $12\frac{1}{4}$%; $2\frac{1}{12}$%. **4.** 1200 ft.; 2400 ft. **5.** 96 ft. 8 in.

IV. 1. $13. 2. $240. 3. $76. 4. $66.80. 5. $8400.

V. 1. $94.88. 2. $282.50. 3. $196.20. 4. $25. 5. $75.

Page **100.** 10. $2120; $4240; $7950. **11.** $22.50. **12.** $35. **13.** $23.75. **14.** $25.50. **15.** $3.38. **16.** $2.98. **17.** $4.28. **18.** $1.63. **19.** $18.78. **20.** $22.01. **21.** $50.12. **22.** $55.51.

Page **101.** 1. $30. 2. $37.50. 3. $57.60. 4. $12. 5. $28. 6. $102. 7. $132. 8. $192. 9. $264. 10. $30. 11. $41.25. 12. $64.80. 13. $24.30. 14. $42.90. 15. $156.75. 16. $13.50. 17. $135. 18. $1237.50. 19. $200. 20. $540. 21. $1152. 22. $675. 23. $1402.50. 24. $3000. 25. $1800. 26. $3375.

Page **102.** 1. $3. 2. $5.50. 3. $7. 4. $22.50. 5. $7.13. 6. $3.13. 7. $10.50. 8. $12.38. 9. $10.43. 10. $14.47. 11. $17.27. 12. $23.77. 13. $5. 14. $4.75. 15. $8.50. 16. $17.50. 17. $54. 18. $50. 19. $120. 20. $128.80.

Page **103.** 1. $52.32; $26.16; $17.44; $34.88; $43.60. 2. $36; $18; $30; $3; $27. 3. $43.20; $3.60; $1.80; 60¢; 12¢. 4. $9.02; $4.51; $3.01; 75¢; 25¢.

Page **104.** 1. $26.25. 2. $30. 3. $62.50. 4. $101.50. 5. $89.25. 6. $1200. 7. $107.25. 8. $152. 9. $32.03. 10. $42. 11. $98.44. 12. $118.13. 13. $59.06. 14. $78.75. 15. $108.28.

Page **105.** 1. $87.50; $105; $70; $96.25. 2. $19,228. 3. $86.28. 4. $3695.63. 5. $2278.13. 6. $11.60. 7. $2756.25. 8. $3402. 9. $1084.58. 10. $625.

Page **108.** 1. $35.50. 2. $44.38. 3. $53.25. 4. $39.94. 5. $62.13. 6. $26.36. 7. $33.73. 8. $39.36. 9. $47.44. 10. $66.76. 11. $185.68. 12. $206.31. 13. $345.91. 14. $276.55. 15. $3611.84. 16. $66.22. 17. $31; $38.75. 18. $92.52; $106.40. 19. $73.70; $78.31. 20. $36.43; $38.35. 21. $623.19; $509.89. 22. $3036.56. 23. $4373.75. 24. $6864.67.

Page **109.** 6. $11.25. 7. $31.25. 8. $50.75. 9. $83.70. 10. $18.40. 11. $4.35. 12. $2.82. 13. $3.50. 14. $41.25. 15. $90.00. 16. $60.50. 17. $100.43.

Page 111. 1. $2.33. 2. $2. 3. 8¢. 4. $10. 5. $4.50. 6. 48¢. 7. $9.31. 8. $12.75. 9. $9.25. 10. $19.38. 11. $6.60. 12. $19.75. 13. $11.15. 14. $8.70. 15. $13.14. 16. $41.11. 17. $175. 18. $39.25. 19. $33. 20. $23.08. 21. $3.75; $3.75. 22. $8.14; $8.15. 23. $11.06; $11.08. 24. $11.13; $11.11. 25. $6.75; $6.75; $6.75. 26. $7.20, $7.20, $7.20; $13.13, $13.13, $13.13; $22.40, $22.40, $22.40; $30.45, $30.45, $30.45.

Page 112. 1. $2. 2. $1.66. 3. $3.32. 4. $10. 5. $5. 6. $9.38. 7. $6.75. 8. $58.38. 9. $4.50. 10. $3.74. 11. $8.25. 12. $2.68. 13. $5.44. 14. $1.50; $3. 15. $4.15; $2.08. 16. $2.50; $2.50. 17. $30; $60. 18. Doubles; doubles; doubles; 4; 8.

Page 113. 1. First. 2. $5540. 3. $15,800. 4. First; $6.70. 5. $51. 6. $4.25.

Page 116. 1. $279.13. 2. $120. 3. $759.38. 4. 147; 114; $2082.93. 5. $1392.06.

Page 120. 1. House, $300; food, $375; clothing, $300; advancement, $300; other expenses, $150; savings, $75. 2. House, $500; food, $625; clothing, $375; advancement, $625; other expenses, $250; savings, $125. 3. House: $560, $640, $800. Food: $700, $800, $1000. Clothing: $420, $480, $600. Advancement: $700, $800, $1000. Other expenses: $280, $320, $400. Savings: $140, $160, $200. 4. House, $228; food, $516; clothing, $132; other expenses, $264; savings, $60. 5. House, $810; food, $750; clothing, $450; advancement, $300; health, $210; miscellaneous, $210; savings, $270.

Page 122. 1. $450. 2. $1155. 3. $535.90. 4. $20,377.50. 5. $642. 6. $1870; $399.

Page 125. 1. $3000; $3038.76. 2. $4720; $4764.06. 3. $3920; $3937.02. 4. $2520; $2532.40. 5. $1933.75; $1940.25. 6. $713.07; $717.77. 7. $12,400; $12,653.19. 8. $502.09; $507.42. 9. $225.28. 10. $731.59. 11. $765.41. 12. $541.21. 13. $1093.08. 14. $2143.72. 15. $10; $8.75; $8.13; $9.38. 16. $9.50. 17. $111.50. 18. 4½% simple interest; $9.48. 19. $164.84. 20. $488.84. 21. $487.22. 22. $531.62. 23. $1380.85; $1408.45.

Page 128. 17. $31. 18. 40¢. 19. $120; Feb. 1, 1929; 20¢; $2.60. 20. $3.80. 21. $6.25.

Page 130. 11. $509.00. 12. $1243.85. 13. $523.73. 14. $2461.90. 15. $1668.75. 16. $392.64. 17. $920.75.

Page 132. 1. $587.75. 2. $167.57. 3. $180.92. 4. $1199.10. 5. $934.62. 6. $1534.05. 7. $2399.67.

Page 134. 1. $15,938.03. 2. $15,938.03. 3. $18,806.97. 4. $2706.65.

Page 138. 1. $1.20. 2. $4.05. 3. $2.63. 4. $38.63. 5. $54.38. 6. $21.25. 7. $38.50. 8. $40. 9. $112.50. 10. $1.76. 11. $5.40. 12. $9.06. 13. $11.33. 14. $4.13. 15. $10.63. 16. $41.67. 17. $131.25. 18. $146.67.

Page 139. 1. $3.25. 2. $7.00. 3. 72¢. 4. 33¢. 5. $15. 6. $19.50. 7. 80¢. 8. $4.20. 9. 46¢. 10. 24¢. 11. $18. 12. $24.36. 13. $3.30. 14. $8.80. 15. 56¢. 16. 45¢. 17. $77. 18. $55.66. 19. 60¢. 20. $6.90. 21. 41¢. 22. 24¢. 23. $67.50. 24. $97.29.

Page 140. 1. $74.25. 2. $89.55. 3. $49.25. 4. $246.25. 5. $270.87. 6. $420.75. 7. $1485. 8. $1781.10. 9. $4430.25. 10. 17¢. 11. 58¢. 12. $1.00. 13. $1.60. 14. $2.71. 15. $4.50. 16. $5.21. 17. $18.96. 18. $59.38. 19. $25.83. 20. $14.83. 21. $37.92. 22. $96.56. 23. $68.75. 24. $82.19.

Page 141. 2. $671.63. 3. $1231.25. 4. $1500. 5. $1477.50. 6. $750; $6.55 more. 7. $870.62. 8. $962.81. 9. $422.87. 10. $2532.94. 11. $3205.31. 12. $4234.06.

Page 143. 1. $6.25; $743.75. 2. $1.88; $223.12. 3. $5; $495. 4. $3.75; $446.25. 5. $2.81; $222.19. 6. $83.63; $5491.37. 7. $31.25; $7468.75. 8. $23.75; $4726.25. 9. 21¢; $42.29. 10. $1.26; $124.24. 11. $2.75; $272.50. 12. $5.63; $369.87. 13. $1.91; $455.59. 14. $1.24; $295.26. 15. $3.60; $284.00. 16. $4.69; $370.71. 17. $30.94; $2719.06. 18. $36.84; $3238.16. 19. $1250; $6500. 20. $1747.07. 21. $300.25. 22. $799.95. 23. $552.91. 24. $325.31. 25. $999.97. 26. $1258.95.

Page 144. 1. $17.50. 2. $10.13; $7.13. 3. $49. 4. $153. 5. Same. 6. More; 50¢. 7. $158.59; $12,528.91. 8. $27.75.

Page 148. 17. 0.22%. 18. $345.84. 19. 0.24%.

Page 151. 1. 6 ft. 4 in. 2. 10 ft. 7 in. 3. 6 lb. 4. 10 lb. 3 oz. 5. 5 yd. 16 in. 6. 11 yd. 22 in. 7. 23 ft. 8. 8 qt. 9. 10 qt. 1¼ pt. 10. 7 bu. 2 pk. 11. 9 bu. 3¼ pk. 12. 7 pk. 2 qt. 13. 12 pk. 5¾ qt. 14. 20 qt. 15. 11 T. 1200 lb. 16. 22 T. 17. 11 mi. 1440 yd. 18. 22 mi. 480 yd. 19. 10 mi. 3200 ft. 20. 23 mi. 520 ft. 21. 17 T. 900 lb.

Page 152. 1. 3 ft. 7 in. 2. 3 yd. 31 in. 3. 3 rd. 193 in. 4. 3 mi. 5275 ft. 5. 3 lb. 11 oz. 6. 3 bu. 27 qt. 7. 3 gal. 3 pt. 8. 2 rd. 4½ yd. 9. 2 ft. 4 in. 10. 6 in. 11. 2 bu. 27 qt. 12. 2 yd. 31 in. 13. 2 rd. 193 in. 14. 2 mi. 315 rd. 15. 2 mi. 5275 ft. 16. 2 T. 1995 lb. 17. 2 hr. 55 min. 18. 2 mo. 25 da. 19. 7 yd. 27 in. 20. 2 mi. 5235 ft. 21. 2 yr. 330 da. 22. 2 mi. 1730 yd. 23. 2 mi. 290 rd. 24. 2 T. 1970 lb. 25. 3 T. 30 lb. 26. 2 mi. 5250 ft. 27. 3 mi. 30 ft. 28. 2 sq. yd. 1266 sq. in. 29. 1 sq. ft. 44 sq. in. 30. 1 sq. yd. 1196 sq. in.

Page 153. 1. 5 ft. 6 in. 2. 4 yd. 18 in. 3. 4 rd. 18 in. 4. 5 lb. 2 oz. 5. 81 ft. 8 in. 6. 81 lb. 4 oz. 7. 85 gal. 8. 85 bu. 9. 112 ft. 6 in. 10. 195 lb. 11. 76 ft. 12. 75 lb. 13. 73 yd. 12 in. 14. 84 bu. 15. 348 ft. 16. 372 gal. 17. 340 yd. 18. 354 gal. 19. 462 ft. 6 in. 20. 93 ft. 21. 7 mi. 120 ft. 22. 8 T. 1400 lb. 23. 34 hr. 48 min. 24. 30 mi. 288 rd. 25. 30 T. 288 lb. 26. 34 min. 48 sec. 27. 35 yr. 28 wk. 28. 31 wk. 120 hr. 29. 30 hr. 288 sec. 30. 30 da. 288 min.

Page 154. 1. 9 ft. 3 in. 2. 9 lb. 3 oz. 3. 4 ft. 11 in. 4. 4 lb. 14 oz. 5. 4 yd. $23\frac{3}{8}$ in. 6. 4 ft. 9 in. 7. 2 bu. $1\frac{5}{8}$ pk. 8. 2 ft. $4\frac{1}{2}$ in. 9. 2 lb. $5\frac{5}{8}$ oz. 10. 2 yd. $12\frac{1}{2}$ in. 11. 2 gal. $1\frac{5}{6}$ qt. 12. 2 gal. $3\frac{1}{8}$ pt. 13. 2 lb. $14\frac{1}{4}$ oz. 14. 2 yd. $2\frac{7}{8}$ ft. 15. 2 yd. $31\frac{3}{4}$ in. 16. 2 bu. $3\frac{3}{4}$ pk. 17. 2 ft. $10\frac{3}{4}$ in. 18. 2 rd. $14\frac{11}{16}$ ft. 19. 2 rd. $173\frac{1}{2}$ in. 20. 2 gal. $3\frac{3}{4}$ qt. 21. 2 gal. $7\frac{1}{4}$ pt. 22. 2 mi. $1540\frac{1}{4}$ yd. 23. 2 mi. $280\frac{1}{4}$ rd. 24. 2 mi. $4620\frac{1}{4}$ ft. 25. 4 ft. 26. 4. 27. 4 ft. 1 in. 28. $4\frac{1}{17}$. 29. 49. 30. $4\frac{1}{16}$. 31. 65. 32. $4\frac{1}{36}$. 33. 145. 34. $4\frac{1}{9}$. 35. 33. 36. $2\frac{25}{36}$. 37. $36\frac{1}{8}$. 38. $2\frac{19}{72}$. 39. $27\frac{1}{8}$. 40. $2\frac{11}{36}$. 41. $6\frac{11}{12}$. 42. $95\frac{1}{3}$. 43. 572. 44. 13,728. 45. 101. 46. 1616. 47. $20\frac{4}{15}$. 48. $243\frac{1}{8}$.

Page 156. 1. 6 sq. in. 2. $7\frac{1}{2}$ in.; $3\frac{1}{8}$ sq. in. 3. 3 sq. in. 4. $\frac{1}{2}''$; $\frac{1}{10}''$; 2″. 5. First two; third. 6. Third; $24\frac{1}{2}$ sq. in., 25 sq. in., 26 sq. in. 7. Neither; $36\frac{1}{8}$ sq. ft., $36\frac{1}{8}$ sq. ft. 8. 1 sq. in.; 2 in., 4 sq. in.; 3 in., 9 sq. in. 28. Second; 152 sq. ft. 29. 608 sq. ft. 30. 208. 31. 2272; 252.4. 32. 71 ft. 33. 12 sq. ft.; 12 sq. ft.; 12 sq. ft.; 12 sq. ft.; same.

Page 161. 1. 88 in. 2. 22 in. 3. 132 in. 4. 110 in. 5. 198 in. 6. 330 ft. 7. 374 yd. 8. 352 ft. 9. 418 yd. 10. 37.7 in. 11. 47.1 ft. 12. 194.9 in. 13. 160.3 ft. 14. 11 in. 15. 15.4 in. 16. 19.8 in. 17. 8.5 in. 18. 11 ft. 19. 12.83 ft. 20. 23.83 ft. 21. 25.67 ft. 22. 7 in.; 22 in. 23. 44 in. 24. 88 in. 25. 176 in. 26. 264 in. 27. 37.7 in. 28. 50.3 in. 29. 66 in. 30. 30.8 in. 31. 13.10 ft. 32. 66 ft. 33. 7.07 ft.; 35,350 ft.

Page 162. 1. 23.9 ft.; 12.0 ft. 2. 7 in. 3. 14 in. 4. 28 in. 5. 35 ft. 6. 49 ft. 7. 63 ft. 8. 9.5 in. 9. 25.5 in. 10. 47.7 ft. 11. 0.7 in. 12. 2.8 in. 13. 4.2 in. 14. 4.5 in. 15. 8.0 in. 16. 10.2 in. 17. $10\frac{3}{5}$ in. 18. 21 ft. 19. 0.7 in. 20. $26\frac{1}{4}$ ft. 21. 43.0 ft. 22. 28 ft. 23. 1.6 in. 24. 1.8 in. 25. 14 in. 26. $10\frac{1}{2}$ in.

Page 163. 1. $16\frac{1}{2}$ ft. 2. $24\frac{1}{2}$ in. 3. 1.3 in. 4. 1342 ft. 5. 214.3 in. 6. 16.06 ft. 7. 20.02 in. 8. 14.74 ft. 9. 18.92 in. 10. 202.4 in. 11. 33 ft. 12. 42 ft. 13. 56 ft. 14. 21.7 in. 15. 63 in. 16. 308 in. 17. 572 in. 18. 61.6 in. 19. 968 ft. 20. 53.4 in. 21. 19.7 in. 22. 10.7 in. 23. 19 ft. 4.4 in. 24. 11 in. 25. 94.3 in.

Page 165. 1. 154 sq. ft. 2. 154 sq. in. 3. 616 sq. in. 4. 3850 sq. in. 5. $38\frac{1}{2}$ sq. in. 6. 124.7 sq. in. 7. $346\frac{1}{2}$ sq. in. 8. 12.6 sq. in. 9. 78.6 sq. in. 10. 254.6 sq. ft. 11. 180.74 sq. ft. 12. 95.07 sq. ft. 13. 14.75 sq. yd.

14. 2 ft. 3 in.; **15.** 91 sq. ft. **15.** 1386 sq. in. **16.** 616 sq. in. **17.** 2464 sq. in. **18.** 154 sq. ft. **19.** 24.6 sq. in. **20.** 75.5 sq. in. **21.** 0.79 sq. yd. **22.** 3.14 sq. yd. **23.** 7.1 sq. ft. **24.** 17.11 sq. ft. **25.** 9.63 sq. ft. **26.** 86.63 sq. ft.

Page **166.** **1.** $\frac{1}{4}$. **2.** 4. **3.** 1386 sq. ft. **4.** $962\frac{1}{2}$. **5.** 7.7 in.; 3.9 in.; 46.6 sq. in. **6.** 78.5 sq. in. **7.** 128.9. **8.** 37. **9.** 975 lb.

Page **168.** **1.** 224; 288; 112; 216; 1032. **2.** 2632. **3.** 3600. **4.** 8314 bd. ft.; $565.35. **5.** $429.76. **6.** $2872.50. **7.** $4997.61. **8.** 1024. **9.** 576. **10.** 1680. **11.** 960. **12.** 3456.

Page **170.** **1.** 93. **2.** $88.20. **3.** 9; $5.85. **4.** 10; $8.50. **5.** $5.40.

Page **171.** **1.** 132. **2.** 23. **3.** 107. **4.** $55. **5.** 64%. **6.** 8. **7.** No; yes. **8.** $\frac{1}{6}$ yd.; $41\frac{5}{6}$ yd. **9.** Yes; $2\frac{1}{6}$.

Page **172.** **1.** 5. **2.** 26 in.; 110 in. **3.** 15. **4.** 3. **5.** 13. **6.** $1.24.

Page **174.** **2.** 11 A.M. **3.** 9 A.M.; 9 A.M.; 8 A.M.; 9 A.M. **4.** 12 M. **5.** 10 A.M. **6.** 6 A.M.

Page **175.** **2.** 1 hr. 48 min. 17 sec. **3.** 5 hr. 2 min. 2 sec. **4.** 9 P.M. **5.** 4 A.M. **6.** 8 hr. 22 min. 42 sec. A.M.; 8 hr. 2 min. 42 sec. A.M.

Page **176.** **2.** 30° 47′ 30″. **3.** 75° W. **4.** 90° W. **5.** 22° 45′ W.

Page **177.** **2.** 999,999. **3.** 3.70. **4.** 24.64. **5.** $8.36. **6.** $16.77. **7.** 19 ft. 4 in. **8.** $7\frac{1}{32}$. **9.** 3.59; 1.21. **10.** 45.84; 3991.47. **11.** 1.54 sq. in. **12.** 275; 27.5; 2750. **13.** $6.20; $43.40. **14.** $13.06. **15.** $288. **16.** First; $3.81.

Page **180.** **2.** 395. **3.** 0.5. **4.** 540. **5.** 2.4¢. **6.** 107.

Page **181.** **1.** $7.50. **2.** $12.33; $21.75. **3.** $90. **4.** $67.20.

Page **182.** **1.** $1200. **2.** $736.05. **3.** $352. **4.** $60. **5.** 21¢. **6.** $2065. **7.** 10.9%. **8.** 1980. **9.** 4600.

Page **183.** **1.** 15,068 bd. ft.; $1024.62; $200.20. **2.** $62.30. **3.** $138.75. **4.** $471.75. **5.** $835.20. **6.** $2917.82.

Page **184.** I. **1.** 1. **2.** $166\frac{2}{3}$. **3.** $62\frac{1}{2}$. **4.** 33. **5.** 4. **6.** 2. **7.** $25. **8.** $210. **9.** $500. **10.** $45.36.

II. **1.** $2.25; $2.25; $2.25. **2.** $41.25. **3.** 5%. **4.** 50.3 in. **5.** 201.1 sq. in.

III. **1.** Multiplies it by 4. **2.** 3.8 in. **3.** 1.9 in.; 1134.6 sq. in. **4.** $3\frac{3}{4}$ sq. ft. **5.** 0.2%.

IV. **1.** $18.75. **2.** $3450. **3.** $6.07. **4.** $6.19. **5.** $187.31.

Page **187.** **1.** 362. **2.** $50.58.

Page **189.** **1.** $9.86. **2.** $24.00. **3.** $16.41. **4.** $27.43. **5.** $16.22. **6.** $27.05. **7.** $15.89. **8.** $43.87. **9.** $10.75. **10.** $47.94. **11.** $25,074.29. **12.** $22,103.23. **13.** 24,462.4. **14.** 3662.22.

Page 191. 9. 1889. 10. 1778. 11. 2001. 12. 3148. 13. 1214. 14. 1062. 15. 2327. 16. 3327. 17. 1739. 18. 2071. 19. 3364. 20. 2272. 21. 34,055. 22. 8187. 23. 42,894. 24. 35,180. 25. 5.8874. 26. 251.494. 27. 3254.79. 28. 2709.24. 29. $124,600. 30. $1,983,750.

Page 192. 6. $1276. 7. $3809. 8. $3355. 9. $4266. 10. $4462. 11. $1710.01. 12. $3549.25. 13. $1333.84. 14. $1233.85. 15. $1564.24. 16. $3303.88. 17. $4244.57. 18. $1216.19. 19. $2616.31. 20. $4729.92. 21. $12,083.70. 22. $6282.29. 23. $12,599.23. 24. $35,472.39. 25. $74,031.61. 26. $689.60. 27. $104.81. 28. $1,341,374.25.

Page 193. 1. 773,664. 2. 984,768. 3. 1,665,296. 4. 2,391,592. 5. 2,195,949. 6. 2,167,056. 7. 6,706,656. 8. 2,158,560. 9. 3,505,016. 10. 1,613,792. 11. $3.50. 12. $72,510. 13. $287.50. 14. $250. 15. $3,211,200. 16. 1,469,264. 17. 1,389,789. 18. 3,071,880. 19. 4,015,920. 20. 5,485,172. 21. 3,403,944. 22. 6,195,441. 23. 39,930,966. 24. 37,295,449. 25. 16,032,016. 26. $4000. 27. $16,000. 28. $1881.25. 29. $80,000. 30. $80,000. 31. $13,654.75. 32. $10,174.50. 33. $13,639.92. 34. $32,181.32. 35. 178.7. 36. 134. 37. 1.34. 38. 0.134. 39. 432. 40. 432. 41. 2200. 42. 1600. 43. $10,600. 44. 238,320. 45. $60,000. 46. 4400. 47. 5452.4. 48. 97. 49. 74.2. 50. 742. 51. 74,200. 52. 742. 53. 248.2. 54. 189,000. 55. 18.9. 56. 57.27.

Page 194. 1. 842; 345; 657; 752; 557. 2. 2593; 2775; 494; 933; 938. 3. 2308; 1131; 692; 741; 881. 4. $1712\frac{1}{2}$; $1392\frac{1}{3}$; $1570\frac{3}{4}$; $1458\frac{1}{5}$; $556\frac{1}{6}$. 5. $920\frac{1}{7}$; $1206\frac{1}{8}$; $385\frac{5}{9}$; $1215\frac{2}{7}$; $1928\frac{3}{5}$. 6. $740\frac{5}{6}$; $915\frac{7}{8}$; $617\frac{2}{3}$; $143\frac{3}{7}$; $429\frac{3}{4}$. 7. $96\frac{2}{5}$. 8. $126\frac{3}{4}$. 9. 122. 10. 270. 11. 53. 12. $371\frac{1}{2}$. 13. $115\frac{2}{3}$. 14. $86\frac{5}{7}$. 15. $69\frac{3}{8}$. 16. 78. 17. 84. 18. 21. 19. 30, remainder 381. 20. 121. 21. 270. 22. 248. 23. 363. 24. 475. 25. 4380. 26. 1717. 27. $3.05. 28. $3.30. 29. $3.85. 30. $2.03. 31. $3.21. 32. $3761.69. 33. $4606.83. 34. $8426.93. 35. $7104.91. 36. $171.33. 37. 34.8.

Page 195. 1. 19.37. 2. 6.78. 3. 30.72. 4. 6.70. 5. 27.94. 6. 20.1. 7. 1.7. 8. 0.17. 9. 17. 10. 1.7. 11. 2.3. 12. 23. 13. 2.3. 14. 2.1. 15. 21. 16. 2.1. 17. 1000. 18. 10,000. 19. 10,000. 20. 2090. 21. 20.9. 22. 17. 23. 8.2. 24. 8.2. 25. 9100. 26. 51. 27. 6. 28. 980. 29. 78. 30. 7.1. 31. 81. 32. 65. 33. 91. 34. 0.7. 35. 21. 36. 1.2. 37. 513. 38. 0.51. 39. 0.51. 40. 5.10. 41. 1.1. 42. 11. 43. 1.85. 44. 1076. 45. 10.76. 46. 2000. 47. 2.03. 48. 2.03. 49. 24,500. 50. 800. 51. 174. 52. 10.76. 53. 51.53. 54. 911. 55. 2110.1. 56. 0.81. 57. 0.02. 58. 0.01. 59. 1.97. 60. 20.10.

Page 196. 1. 22,849. 2. 29,864. 3. $107.64. 4. $229.75. 5. $745.47. 6. 2342. 7. 2946. 8. 785. 9. 4468. 10. $10.68. 11. $16.89. 12. $70.65. 13. $45.60. 14. $498.58. 15. $835.43. 16. 63,000; 0.09. 17. 24,000;

38.4. 18. 120,000; 0.13. 19. 110,000; 7.04. 20. 275,000; 1.42. 21. 400,000; 0.98. 22. 240,000; 1.71. 23. 180,000; 1.28. 24. 500,000; 1.28. 25. 1728; 12. 26. 34,560; 1.67. 27. 6912; 12. 28. 13,824; 6. 29. 20,736; 144. 30. 82,944; 36. 31. 37,500; 60. 32. 32,000; 0.02. 33. 93,000; 16.53. 34. 144,846; 95.23. 35. 204,838; 38.44. 36. 495,198; 32.73. 37. 639,088; 11.47. 38. 1,835,704; 30.09. 39. 4,992,728; 7.94. 40. 399,350; 3.76. 41. 1,541,080; 6.92. 42. 1,092,570; 7.00.

Page 198. 1. $\frac{5}{8}$. 2. $\frac{7}{16}$. 3. $\frac{9}{16}$. 4. $\frac{13}{16}$. 5. $1\frac{9}{16}$. 6. $\frac{5}{16}$. 7. $\frac{13}{16}$. 8. $1\frac{5}{16}$. 9. $1\frac{9}{16}$. 10. $1\frac{11}{16}$. 11. $\frac{23}{32}$. 12. $\frac{31}{32}$. 13. $1\frac{1}{32}$. 14. $1\frac{23}{32}$. 15. $\frac{17}{32}$. 16. $1\frac{1}{24}$. 17. $1\frac{11}{24}$. 18. $\frac{5}{6}$. 19. $1\frac{5}{6}$. 20. $\frac{21}{64}$. 21. $3\frac{9}{16}$. 22. 4. 23. $14\frac{1}{12}$. 24. $15\frac{5}{12}$. 25. $9\frac{11}{24}$. 26. $11\frac{17}{24}$. 27. $18\frac{7}{24}$. 28. $16\frac{13}{24}$. 29. $92\frac{13}{30}$. 30. $138\frac{13}{16}$. 31. $167\frac{3}{5}$. 32. $142\frac{2}{15}$. 33. $128\frac{3}{4}$. 34. $122\frac{48}{80}$. 35. $16\frac{3}{8}$. 36. $16\frac{5}{16}$. 37. $12\frac{28}{30}$. 38. 14. 39. $14\frac{27}{32}$. 40. $12\frac{13}{24}$. 41. $\frac{5}{8}$. 42. $\frac{3}{8}$. 43. $\frac{7}{16}$. 44. $\frac{9}{16}$. 45. $\frac{11}{32}$. 46. $\frac{23}{32}$. 47. $\frac{17}{32}$. 48. $\frac{3}{16}$. 49. $\frac{1}{32}$. 50. $\frac{13}{32}$. 51. $\frac{5}{32}$. 52. $\frac{27}{64}$. 53. $\frac{3}{16}$. 54. $\frac{13}{64}$. 55. $27\frac{1}{8}$. 56. $57\frac{1}{4}$. 57. $63\frac{1}{10}$. 58. $44\frac{25}{32}$. 59. $61\frac{13}{16}$. 60. $64\frac{19}{24}$. 61. $70\frac{43}{48}$. 62. $2\frac{1}{16}$. 63. $5\frac{3}{16}$. 64. $2\frac{1}{16}$. 65. $4\frac{1}{16}$. 66. $3\frac{7}{32}$. 67. $3\frac{17}{32}$. 68. $14\frac{15}{16}$.

Page 199. 1. $\frac{3}{4}$. 2. $1\frac{1}{4}$. 3. $\frac{5}{8}$. 4. $\frac{7}{8}$. 5. $1\frac{1}{8}$. 6. $1\frac{3}{8}$. 7. $\frac{3}{8}$. 8. $1\frac{1}{8}$. 9. $1\frac{1}{8}$. 10. $1\frac{3}{8}$. 11. $\frac{1}{4}$. 12. $\frac{1}{4}$. 13. $\frac{3}{8}$. 14. $\frac{1}{8}$. 15. $\frac{3}{8}$. 16. $\frac{1}{8}$. 17. $\frac{1}{8}$. 18. $\frac{5}{8}$. 19. $\frac{3}{8}$. 20. $\frac{1}{8}$. 21. $\frac{1}{8}$. 22. $\frac{3}{8}$. 23. $\frac{1}{16}$. 24. $\frac{3}{16}$. 25. $\frac{5}{16}$. 26. $\frac{7}{16}$. 27. $\frac{1}{32}$. 28. $\frac{21}{32}$. 29. $\frac{9}{32}$. 30. $\frac{15}{32}$. 31. 2. 32. $\frac{1}{2}$. 33. $1\frac{1}{8}$. 34. $\frac{5}{8}$. 35. $\frac{6}{7}$. 36. $1\frac{1}{8}$. 37. $\frac{8}{9}$. 38. $1\frac{1}{8}$. 39. $\frac{16}{21}$. 40. $\frac{9}{10}$. 41. 119. 42. 1116. 43. 2170. 44. 1524. 45. 2280. 46. 1904. 47. 1062. 48. 1740. 49. $1321\frac{7}{8}$. 50. $13\frac{3}{4}$. 51. $27\frac{1}{8}$. 52. $64\frac{3}{16}$. 53. $19\frac{19}{32}$. 54. $37\frac{31}{32}$. 55. $25\frac{7}{8}$. 56. $3\frac{1}{10}$. 57. $2\frac{19}{30}$. 58. $2\frac{2}{3}$. 59. $\frac{3}{8}$. 60. 2. 61. $\frac{1}{2}$. 62. 3. 63. $\frac{1}{3}$.

Page 200. 1. $1\frac{7}{8}$. 2. $7\frac{7}{8}$. 3. $15\frac{5}{8}$. 4. $17\frac{1}{8}$. 5. $26\frac{7}{16}$. 6. $19\frac{5}{16}$. 7. $\frac{1}{8}$. 8. $\frac{3}{8}$. 9. $5\frac{3}{4}$. 10. $4\frac{3}{16}$. 11. $5\frac{1}{3}$. 12. $4\frac{11}{15}$. 13. $1\frac{1}{8}$. 14. $1\frac{7}{8}$. 15. $14\frac{11}{16}$. 16. $16\frac{7}{8}$. 17. $26\frac{1}{4}$. 18. $21\frac{7}{9}$. 19. $2\frac{2}{3}$. 20. $\frac{3}{8}$. 21. $\frac{4}{7}$. 22. $1\frac{3}{4}$. 23. $1\frac{1}{9}$. 24. $\frac{9}{10}$. 25. $\frac{23}{30}$. 26. $\frac{17}{20}$. 27. $\frac{17}{30}$. 28. $9\frac{11}{12}$. 29. $6\frac{9}{16}$. 30. $18\frac{1}{4}$. 31. $2\frac{1}{2}$. 32. $69\frac{43}{64}$. 33. 2. 34. $7\frac{1}{4}$. 35. 2; 3. 36. $1\frac{5}{8}$; $1\frac{27}{32}$.

Page 201. 1. 35.37. 2. 14.739. 3. 129.55. 4. 1561.5. 5. 505.1. 6. 114.34. 7. 14.717. 8. 14.641. 9. 27.05. 10. 29.33. 11. 77.37. 12. 12.27. 13. 404.79. 14. 462.51. 15. 8.922. 16. 3.433. 17. 65.5248. 18. 10,935.42. 19. 1286.2616. 20. 104.7591. 21. 35,241.5049. 22. 1583.914365. 23. 14.556528. 24. 17.653408. 25. 1065.86. 26. 97.48. 27. 0.91. 28. 93.82. 29. 1.46. 30. 127.49. 31. 11.15. 32. 15.32. 33. 1.24. 34. 18.78. 35. 656.52. 36. 2252.86.

Page 202. I. 1. 847. 2. 121. 3. 175,692. 4. $1\frac{1}{3}$. 5. 57.6. 6. 19.2. 7. 737.28. 8. 2. 9. $6\frac{5}{8}$. 10. $1\frac{1}{8}$. 11. $10\frac{21}{32}$. 12. $1\frac{9}{22}$. 13. $\frac{22}{31}$. 14. 9. 15. $2\frac{1}{4}$. 16. $5\frac{2}{3}$. 17. 816. 18. 2640. 19. 3960. 20. 4620.

II. 1. 79.2. 2. 39.6. 3. 1176.12. 4. 11.7612. 5. 117.612. 6. 3. 7. 3. 8. 300. 9. 0.03. 10. $8\frac{3}{5}$. 11. $\frac{5}{8}$. 12. $17\frac{7}{16}$. 13. $1\frac{5}{31}$. 14. $\frac{31}{36}$. 15. 135. 16. $61\frac{7}{8}$. 17. 45. 18. 10. 19. 17. 20. 34.

III. 1. $1\frac{3}{8}$. 2. $\frac{7}{8}$. 3. $1\frac{3}{8}$. 4. $2\frac{1}{8}$. 5. $\frac{3}{8}$. 6. $1\frac{1}{8}$. 7. $\frac{21}{64}$. 8. $25\frac{25}{32}$. 9. $3\frac{1}{8}$. 10. $1\frac{5}{8}$. 11. $\frac{6}{11}$. 12. 5.04. 13. 0.075. 14. 0.0075. 15. 3. 16. $5\frac{5}{8}$ yd. 17. 5. 18. $15\frac{3}{4}$. 19. $3\frac{5}{72}$. 20. $\frac{1}{4}$.

Page 205. 21. 125. 22. 600. 23. 128. 24. 50. 25. 72. 26. 80. 27. 480. 28. 240. 29. 200. 30. 200. 31. 125. 32. 12,500. 33. 60,000. 34. 12,800. 35. 5000. 36. 7200. 37. 8000. 38. 48,000. 39. 24,000. 40. 45,000.

Page 207. 1. 180. 2. 360. 3. 1630. 4. 3620. 5. 1580. 6. 1270. 7. 5610. 8. 5480. 9. 9480. 10. 8760. 11. 1700. 12. 1400. 13. 42,100. 14. 39,400. 15. 56,100. 16. 61,900. 17. 88,100. 18. 92,400. 19. 91,200. 20. 71,300. 21. 4000. 22. 4500. 23. 7000. 24. 7750. 25. 18,000. 26. 22,500. 27. 23,500. 28. 21,500. 29. 13,500. 30. 17,000. 31. 9000. 32. 5500. 33. 4250. 34. 2750. 35. 8250. 36. 8000. 37. 1750. 38. 17,500. 39. 13,250. 40. 16,000. 41. 6000; 24,000. 42. 8000; 32,000. 43. 4000; 16,000. 44. 90,000; 360,000. 45. 92,000; 368,000. 46. 29,000; 116,000. 47. 16,250; 65,000. 48. 16,625; 66,500. 49. 79,625; 318,500. 50. 80,250; 321,000. 51. 120,125; 480,500. 52. 20,625; 82,500. 53. 496,500; 1,986,000. 54. 904,250; 3,617,000. 55. 576,250; 2,305,000. 56. 7.6; 1.52; 0.304. 57. 9.4; 1.88; 0.376. 58. 19.6; 3.92; 0.784. 59. 32; 6.4; 1.28. 60. 44; 8.8; 1.76. 61. 188; 37.6; 7.52. 62. 177.6; 35.52; 7.104. 63. 54; 10.8; 2.16. 64. 77; 15.4; 3.08. 65. 72.4; 14.48; 2.896. 66. 117.6; 23.52; 4.704. 67. 186.8; 37.36; 7.472. 68. 550; 110; 22. 69. 1127; 225.4; 45.08. 70. 1999.8; 399.96; 79.992.

Page 208. 16. 40. 17. 40. 18. 49. 19. 64. 20. 15. 21. 15. 22. 30. 23. 30. 24. 22.5. 25. 29. 26. 42.5. 27. 58.5. 28. 120. 29. 133.5. 30. 299.6. 31. 300. 32. 100. 33. 120. 34. 50. 35. 160. 36. 11. 37. 8.4. 38. 8.4. 39. 36. 40. 77. 41. 45. 42. 100. 43. 275. 44. 275. 45. 500. 46. 480. 47. 120. 48. 827. 49. \$5. 50. \$1.07. 51. \$9.75. 52. \$4.80. 53. \$27.50. 54. \$65.71.

Page 209. 1. $33\frac{3}{8}$. 2. \$36.75. 3. \$3.57. 4. 74. 5. 75; 75. 6. 42. 7. Crescents'; 1; Stars'. 8. 30. 9. \$17.50. 10. \$441.60.

Page 210. 1. \$54.00. 2. \$52.00. 3. \$41.25. 4. \$52.50. 5. \$40.00. 6. \$46.88. 7. \$35.25. 8. \$33.75. 9. \$31.50. 10. \$30.00. 11. \$31.88. 12. \$29.75. 13. \$27.63. 14. \$25.50. 15. 40%. 16. 39%. 17. 38%. 18. 40%. 19. 34%. 20. 41%. 21. 12%. 22. 15%.

Page 211. 1. \$13; \$637. 2. \$39; \$1261. 3. \$66; \$1584. 4. \$94.75; \$1800.25. 5. \$112; \$3088. 6. \$158.40; \$3361.60. 7. \$134; \$1541.

8. $191.25; $2058.75. 9. $297.50; $8202.50. 10. $510; $11,490. 11. $406.25; $12,093.75. 12. $620.63; $15,929.37. 13. $1017.50; $17,482.50. 14. $1546.88; $20,953.12. 15. $977.81; $17,647.19. 16. $985.63; $19,764.37. 17. $6.44; $74.06. 18. $4.85; $64.50. 19. $5.13; $46.14. 20. $4.42; $93.83. 21. $8.40; $58.80. 22. $12.96; $73.44. 23. $13.47; $98.78. 24. $25.60; $125.00. 25. $50; 10%. 26. $24; 4%. 27. $35; 5%. 28. $26; 4%. 29. $17.60; 5½%. 30. $38.40; 8%. 31. $41.60; 6½%. 32. $32.85; 4½%. 33. $350; 3½%. 34. $625; 5%. 35. $569; 4%. 36. $1300; 8%.

Page 212. 17. $100; $600. 18. $50; $200. 19. $150; $450. 20. $80; $880. 21. $25; $50. 22. $126; $1134. 23. $30; $130. 24. 15¢; $1.33. 25. $25; $50. 26. $100; $150. 27. $75; $100. 28. $100; $120. 29. $80; $100. 30. $60; $80. 31. $75; $80. 32. $150; $225. 33. $75; $200. 34. $68.06; $82. 35. $548.85; $762.29. 36. $816.75; $1485. 37. 70%; 233⅓%. 38. 1½¢; 1¢. 39. 10¢; 6⅔¢; 50%. 40. 10¢, 6¼¢, 60%; 10¢, 2½¢, 300%; 50¢, 33⅓¢, 50%.

Page 214. 1. 8⅓%. 2. 30%. 3. 45.8%. 4. 15.5%. 5. 45%. 7. 111.5%; 63.9%; 97.7%; 585.2%. 8. 67.9%; 517.3%. 9. 318.6%. 10. John, $55.34; Peter, $51.06; Mary, $38.48; Robert, $35.15; Alice, $30.16; Helen, $19.95; Ruth, $7.36. 11. June, 16.3%; July, 11.2%; Aug., 9.2%; Sept., 11.8; Oct., 15.1%; Nov., 16.7%; Dec., 19.7%.

Page 216. 1. 91.1 lb. 2. 713.0%; 105.3%. 3. 12.3%; 31.6%. 4. 78.1%; 62.6%; 152.7%. 5. 3.7%. 6. 63.2%.

Page 217. 1. 120.1%; 174.6%. 2. 40.5%; 35.7%. 3. 12.8%; 38.5%; 12.5%. 4. 26.2%; 39.6%; 60%. 5. 24.7%; 19.8%. 6. 37.1%; 58.9%.

Page 218. 1. 20.0¢, 80.0%; 6.0¢, 14.3%; 3.5¢, 53.8%; 0.5¢, 5.3%; 9.75¢, 156.0%; 0.4¢, 4.0%; 10.0¢, 50.0%; 1.5¢, 20.0%; 8.0¢, 36.4%; 10.0¢, 66.7%. 2. 18.70¢; 26.51¢; 7.80¢; 47.7%.

Page 219. 1. 43.0%; 20.5%; 16.0%; 5.4%; 4.7%; 3.9%; 3.8%; 2.8%. 2. 17.9%. 3. 59.5%. 4. 43.9%. 5. 72.5%. 6. 71.4%. 7. 21.7%.

Page 220. 1. 593.4 lb., butter fat; 100.1 lb., water; 18.1 lb., salt; 8.5 lb., curd. 2. $5.30; $4.85; $4.80; $4.55; $4.50; $4.45; $4.40. 3. 20.5%. 4. 1785.7; 3500.

Page 221. 1. 12.7%. 2. 93.4%. 3. 41⅔%. 4. 74%; 26%. 5. Second. 6. 54.2%. 7. 3.5%. 8. 40%. 9. 89%.

Page 222. 1. $157.76. 2. $394.40. 3. $631.04. 4. $3718.50. 5. $3032. 6. $25. 7. $101.44. 8. $3124.39. 9. $1,317,753.56. 10. $1.43. 11. $1700.79. 12. $4553.60. 13. $9513. 14. $525. 15. $159,160.

Page 224. 1. $8.16. 2. $3.40. 3. $5.10. 4. $11.42. 5. $17. 6. $108.80. 7. $125.12. 8. $79.56. 9. $85. 10. $113.90. 11. $136. 12. $163.20. 13. $183.60. 14. $221. 15. $278.80. 16. $748. 17. $1026.80. 18. $1700. 19. $3060. 20. $9180. 21. $5.15. 22. $11.47. 23. $17.93. 24. $136.71. 25. $111.13. 26. $125.32. 27. $183.75. 28. $238.88. 29. $227.85. 30. $882. 31. $1065.75. 32. $1888.95. 33. $24.60. 34. $16.07. 35. $75.01. 36. 45.9%. 37. 8.7%.

Page 226. 33. Multiplies it by 4. 34. $3\frac{3}{8}$ sq. ft. 35. 22.22 sq. ft.

Page 227. 1. 240 sq. rd. 2. $5450\frac{1}{2}$ sq. ft. 3. 167.06 sq. in. 4. 15.6 sq. in. 5. $16\frac{7}{8}$ sq. ft. 6. 336 sq. rd. 7. 4692.3 sq. ft. 8. 6850 sq. ft.; 6000.6 sq. ft.; 10,144.9 sq. ft.; 5204.4 sq. ft. 9. 9633.8 sq. ft.; 9983.9 sq. ft. 10. 18,031.2 sq. ft. 11. 24,940.7 sq. ft. 12. 19,315.7 sq. ft. 13. 19,315.7 sq. ft.

Page 229. 1. 300 cu. yd. 2. 216,000. 3. 4320 cu. in. 4. 54. 5. 3. 6. 3456. 7. 864. 8. 31,104. 9. $94\frac{1}{2}$.

Page 230. 1. 6; 4. 2. 12. 3. 8. 4. 1; 6. 5. 4; 24. 6. 1; 8. 7. 10; 10. 9. 24 in.; 24 sq. in. 10. 36 in.; 9 sq. in.; 27 cu. in. 11. 5 in.; 25 sq. in.; 150 sq. in. 12. 7 in.; 49 sq. in.; 294 sq. in.; 343 cu. in. 13. 6 in.; 72 in.; 216 sq. in.; 216 cu. in. 14. 12 in.; 144 in.; 864 sq. in.; 1728 cu. in.

Page 232. 1. 133; $1130\frac{1}{2}$. 2. 272; 1904. 3. 4896; $181\frac{1}{3}$. 4. $331.02. 5. 540. 6. $106.67; $133.33. 7. $12\frac{3}{4}$. 8. 391.1. 9. 169,200 cu. ft. 10. 12,800. 11. $400.50. 12. 30. 13. 15 cu. ft. 14. 240 cu. ft. 15. 8. 16. 4; 4. 17. 8. 18. 8. 19. 4. 20. 8. 21. $7\frac{13}{16}$. 22. 1520. 23. 21. 24. 9072. 25. 60 cu. ft.; $62\frac{1}{2}$ cu. ft.; 65 cu. ft. 26. $54\frac{1}{6}$. 27. 800. 28. 13,858. 29. 219.1; $328.65. 30. 200-pound and 25-pound pieces. 31. $26,400.

Page 235. 1. 8 in.; 4 sq. in. 2. 5 in.; 25 sq. in. 3. 3 in.; 12 in. 4. 15 in.; 225 sq. in. 5. 4 in.; 16 in. 6. $5\frac{1}{2}$ in.; $30\frac{1}{4}$ sq. in. 7. 9.6 in.; 5.76 sq. in. 8. 4.1 in.; 16.81 sq. in. 9. 8 ft. 4 in.; 600 sq. in. 10. 12 ft. 2 in.; 1260 sq. in. 11. 16 ft.; 2300 sq. in. 12. 6 ft. 8 in.; 396 sq. in. 13. 5 cu. ft. 14. $18\frac{3}{4}$ cu. ft. 15. 21 cu. ft. 16. $31\frac{1}{2}$ cu. ft. 17. $33\frac{3}{4}$ cu. in. 18. 102 cu. in. 19. $14\frac{1}{16}$ cu. in.

Page 236. 1. $2.40. 2. $2.97. 3. $4.29. 4. $5; $4.46. 5. $4.32. 6. 49¢. 7. 11¢. 8. 800; 88¢; 4¢; 84¢. 9. 2800; $2.52; 13¢; $2.39. 10. 6100; $7.32; 37¢; $6.95. 11. 4600; $4.60; 23¢; $4.37. 12. 3300; $4.62; 23¢; $4.39. 13. 3900; $3.32; 17¢; $3.15. 14. 3600; $4.86; 24¢; $4.62. 15. 4400; $4.62; 23¢; $4.39. 16. $172.80. 17. $84.15. 18. $26.40.

Page 238. 1. 1526 K. W. H. 2. 318 K. W. H. 3. $12.18. 4. $187.13. 5. $9.35. 6. $9.50.

Page 241. 1. 153.6 bu.; 129.6 bu. 2. 7.2 bu. 3. 900 gal. 4. $437\frac{1}{2}$ gal. 5. 40. 6. $56\frac{1}{4}$. 7. 921.6. 8. 1814.4. 9. 3072. 10. 1176. 11. 990.

12. $770.40. **13.** $9\frac{1}{2}$; $16\frac{1}{2}$. **14.** 54. **15.** 52. **16.** 17.7. **17.** 5.5. **18.** 7.4; 14.9. **19.** 8.1. **20.** 8.9. **21.** 41. **22.** 11,000 cu. ft. **23.** $28\frac{1}{2}$. **24.** 7.3. **25.** $3.60. **26.** $2112\frac{1}{2}$. **27.** $8\frac{3}{4}$ M.

Page 243. **1.** 4752. **2.** 159.6 cu. in., larger. **3.** 49.6 cu. in. **4.** 489.6 cu. in., larger. **5.** First; 370.2 cu. in. **6.** $1235\frac{1}{4}$ cu. in. **7.** First; 50.9 cu. in. **8.** $869\frac{1}{2}$ cu. in.; 428.0 cu. in.; 1847.7 cu. in.; 1059.7 cu. in.

Page 245. **1.** 12 sq. ft. **2.** 6 ft. **3.** 15 in. **4.** $112\frac{1}{2}$ sq. ft. **5.** $43\frac{1}{3}$ ft. **6.** $12\frac{1}{2}$ ft. **7.** 27.5 ft. **8.** 39 in. **9.** 7.2 in. **10.** 11 ft. **11.** 12 ft. **12.** 12 ft.; 12 ft.; $9\frac{1}{2}$ ft.

Page 246. **1.** 364.5. **2.** $40.50. **3.** 720 gal. **4.** 7.4. **5.** 60. **6.** 160; yes. **7.** $117\frac{1}{3}$ bu. **8.** 68 bu. **9.** $1036.80.

Page 247. **1.** $33\frac{1}{3}$. **2.** 35.1. **3.** 35.7. **4.** $62\frac{1}{2}$. **5.** $41\frac{2}{3}$. **6.** $41\frac{2}{3}$. **7.** $33\frac{1}{3}$. **8.** $33\frac{1}{3}$. **9.** 37.0. **10.** $1\frac{3}{4}$. **11.** $\frac{9}{10}$. **12.** $\frac{6}{7}$. **13.** 500. **14.** 30 T.; 16 T.; 28 T.; 28 T.

Page 249. **1.** 988 cu. in. **2.** 279.3 lb. **3.** 10,875 cu. in. **4.** 144 cu. in. **5.** 186 cu. ft. **6.** $\frac{1}{2}$ cu. ft. **7.** 516 cu. in. **8.** 194.7 cu. in. **9.** 8.4 in. **10.** 8 in. **11.** 9 in. **12.** 9.3 in. **13.** $955\frac{1}{2}$ cu. in. **14.** 105 cu. in. **15.** 117 cu. ft. **16.** 2301.3 cu. in. **17.** 15,169.0 cu. in. **18.** $3906\frac{1}{4}$ cu. in. **19.** 10 in.; 10 in.

Page 250. **1.** 2100 cu. ft. **2.** 20 ft.; 50,285.7 cu. ft. **3.** 46.2. **4.** 169,714.3. **5.** $144\frac{3}{8}$ cu. ft. **6.** 2079. **7.** 14 ft.; 3080 cu. ft.; 23,100 gal. **8.** 36 cu. in. **9.** 72 cu. ft. **10.** 351 cu. in. **11.** $19\frac{1}{2}$ cu. in. **12.** $17\frac{1}{2}$ cu. in. **13.** 1320 cu. in. **14.** $57\frac{3}{4}$ cu. in. **15.** 42 cu. ft. **16.** $2085\frac{1}{2}$ cu. in. **17.** 485.2 cu. in. **18.** 3 ft.; 6.5 ft. **19.** 18 sq. ft.; 15.3 sq. ft. **20.** 1357.7 cu. in.; 6436.6 cu. in.; 377.1 cu. in.

Page 252. **13.** 13.6 sq. ft. **14.** 13.6 sq. ft. **15.** 29.3.

Page 253. **2.** 630. **3.** 301.7 sq. in. **4.** 69.9. **5.** 5104. **6.** 235.7 sq. ft. **7.** $244\frac{3}{4}$. **8.** 110 sq. ft. **9.** 27,456 sq. ft. **10.** 1100.

Page 254. **1.** 1964.3. **2.** No; 308 cu. ft. **3.** 34.7 ft. **4.** 2828.6. **5.** 558. **6.** 1848. **7.** $220.10; $396.18. **8.** $73.15. **9.** $4812\frac{1}{2}$. **10.** $137.28. **11.** 175.2. **12.** 120.3. **13.** 233.3. **14.** 280. **15.** 60.

Page 256. **1.** 84 cu. in. **2.** $623\frac{1}{3}$ cu. ft. **3.** $538\frac{2}{3}$ cu. ft. **4.** 382.3 cu. in. **5.** 4000 cu. in. **6.** 159.5 cu. in. **7.** 3932.9 cu. in. **8.** 1095.1 cu. in. **9.** 94,765,056 cu. ft.; 7,960,264.7 T.

Page 258. **1.** 192 sq. in. **2.** $229\frac{1}{2}$ sq. in. **3.** $2\frac{3}{8}$ sq. ft. **4.** 37.4 sq. in. **5.** 15 sq. ft. **6.** 2.2 sq. ft. **7.** 1490.7 cu. in. **8.** 623.7 cu. in. **9.** 33,264 cu. in. **10.** $1437\frac{1}{3}$. **11.** 960 sq. ft. **12.** $183\frac{1}{3}$. **13.** 80 in. **14.** 2040 sq. ft. **15.** $115\frac{1}{2}$ sq. ft. **16.** 22.4. **17.** 47.1 sq. in. **18.** 40 sq. in. **19.** 64 cu. in. **20.** 440 sq. in. **21.** $42\frac{2}{3}$ cu. in. **22.** 2464 cu. in. **23.** 9856 cu. in. **24.** 946,176 sq. ft.

Page 261. 1. 616 sq. in. 2. 3773 sq. ft. 3. 2,359,727,071,000 sq. mi. 4. 86.4 lb. 5. 19.4. 6. 76.9 lb. 7. 12 oz. 8. 268,190,476,000 cu. mi. 9. 14,663,000 sq. mi.; 5,278,793,000 cu. mi. 10. 102.9. 11. 905.1 cu. in.; 1357.7 cu. in.; 452.6 cu. in.; 2; 3.

Page 262. 1. $74\frac{1}{4}$ cu. in. 2. 16.0 cu. in. 3. 157.1 cu. ft. 4. $179\frac{2}{3}$ cu. in. 5. 36 sq. in. 6. 48 cu. in. 7. 15 sq. in.; 96 sq. in.

Page 263. 15. 343 cu. in. 16. $1437\frac{1}{3}$ cu. in.

Page 264. 1. 5 ft. 8 in.; $5\frac{2}{3}$ ft.; 1 yd. 32 in. 2. 116 gal. 1 qt.; $116\frac{1}{4}$ gal. 3. 42 qt. 1 pt.; $42\frac{1}{2}$ qt.; 10 gal. 5 pt.; $10\frac{5}{8}$ gal. 4. 630. 5. $78\frac{1}{8}$ lb. 6. 400. 7. 6 yd. 8. 108,060 lb. 9. 216; 72. 10. 252. 11. 7. 12. 1988; 2000; 2004. 13. 1,486,200 cu. yd. 14. 2220.2 lb. 15. Prism 3 times as large as the cone. 16. 118,800,000. 17. 6500. 18. 1320. 19. 70 lb. 20. 71.00 in.; 71.03 in.; 71 in.; second; 0.03 in. 21. 150 in.; 149.94 in.; 150.00 in. 22. Multiplies it by 8.

Page 267. 1. 1850.39 in. 2. 1358.27 in. 3. 2047.24 in. 4. 2657.48 in. 5. 127.56 in. 6. 1.11 in. 7. 184.25 in. 8. 118.11 in. 9. 2.95 in. 10. 666.01 ft. 11. 123.03 ft. 12. 51.18 in. 13. 46.8 mi. 14. 72 mi. 15. 27.9 mi. per hour. 16. 198; 100,000. 17. 1152; 1000.

Page 268. 1. 7,500,000,000,000 $cm.^2$ 2. 370,000 $cm.^2$ 3. 42.96 $cm.^2$ 4. 62,500 $cm.^2$ 5. 19.75 $m.^3$ 6. 0.82642765384 $m.^3$ 7. 0.007 $m.^3$ 8. 3.5 $m.^3$ 9. 0.000937 $m.^3$ 10. 0.486 $m.^3$ 11. 432; 30,000; second. 12. 512; 1000; second.

Page 269. 1. 170 kg. 2. 194.55 kg. 3. 92.73 kg. 4. 10 kg. 5. 22 kg. 6. 272,727.27 kg. 7. 3.27 kg. 8. 7.29 kg. 9. 79 kg. 10. 7.38 kg. 11. 4.9 kg. 12. 6.35 kg. 13. 4.4 lb. 14. 44 lb. 15. 660 lb. 16. 6600 lb. 17. 750 l. 18. 4 l. 19. 225 l. 20. 50 l. 21. 25 l. 22. 2000 l. 23. 1500 l. 24. 683 l. 25. 180.4. 26. 50.

Page 270. 1. 1090.9. 2. 125.4 mi. 3. $19\frac{1}{4}$. 4. 1 t.; 2205 lb. 5. 17,640 lb. 6. 1049.75 l. 7. 1 kg.; 1049.75 kg. 8. 277.7 gal.; 2304.9 lb. 10. 12.5.

Page 273. 24. $\frac{8}{42}$; $\frac{3}{18}$ and $\frac{5}{30}$. 26. $\frac{15}{24}$, $\frac{30}{48}$, $\frac{40}{64}$. 27. $\frac{14}{18}$, $\frac{21}{27}$, $\frac{63}{81}$. 28. 6; 6. 29. 10. 30. 21. 31. $\frac{1}{2}$; 2. 32. 3; 3. 33. $\frac{1}{3}$; $\frac{1}{3}$. 34. 16; $\frac{1}{2}$; 16; 128 lb. 35. 4%. 36. 25%. 37. 4%. 38. $37\frac{1}{2}$%. 39. 10. 40. 12. 41. 2. 42. 6. 43. 4 ft. 44. $\frac{3}{4}$; $\frac{1}{2}$; $\frac{1}{2}$; $\frac{1}{6}$.

Page 275. 1. $\frac{1}{2}$; $\frac{1}{2}$; 1. 2. $\frac{2}{5}$; 40%. 3. $\frac{1}{5}$; 20%. 4. $\frac{1}{4}$. 5. $\frac{1}{4}$. 6. 8; 24. 7. $\frac{1}{4}$; 2; $\frac{1}{3}$; 1; 4; 2; water; $\frac{1}{4}$; 2; $\frac{1}{3}$. 8. 10; 3; 12; $\frac{1}{4}$.

Page 276. 7. 18 clear days; 12 cloudy days. 8. $9; $6; $7; $10.50.

Page 278. 1. $7\frac{1}{2}$ min. 2. 4 hr. 3. 21.9 sec. 4. 12 min. 40 sec. 5. $2\frac{1}{2}$ in. 6. 576 sq. mi. 7. 30.0 min. 8. 1034.8. 9. 214.2 in. 10. A shorter distance; 75 mi. 11. $6. 12. 5. 13. 36. 14. 18. 15. 82. 16. 175. 17. 22.7. 18. 4.1. 19. 15.8. 20. 0.24. 21. 3.5.

Page 281. 1. 29 ft. 2. 43 ft. 2 in. 3. 55 ft. 4. 35 ft. 5. 36 ft. 6 in. 6. 39 ft. 9 in. 7. 34 ft. 7 in. 8. 57 ft. 8 in. 9. 51 ft.

Page 282. 1. 45.8 ft. 2. 35 ft. 9 in. 3. 80 ft. 4. 37 ft. 5. 50 ft. 6. 27 ft.

Page 283. 1. 3 in. 2. $12\frac{1}{2}$ ft. 3. Yes. 4. 30 lb. 5. 80 lb.

Page 284. 1. $1.60. 2. $9.66. 3. $14.85. 4. $25.20. 5. 7 yd. 6. 5 books. 7. $27. 8. 7 pencils.

Page 285. 1. $625.88; $228.76. 2. $1619.57; $244.45. 3. $1462.91; $277.55. 4. $948.76; $851.26. 5. $275.46. 6. $424.62. 7. $77,467.46. 8. 355 ft. 9. 54; 27; 1080. 10. 27; 0.27. 11. 7 ft. 4 in.; 14 ft. 8 in. 12. 7 lb. 2 oz.; 5 lb. $5\frac{1}{2}$ oz. 13. 0.5; 50%. 14. 0.75; 75%. 15. 0.625; $62\frac{1}{2}$%. 16. 0.66$\frac{2}{3}$; 66$\frac{2}{3}$%. 17. 0.83$\frac{1}{3}$; 83$\frac{1}{3}$%. 18. 0.41$\frac{2}{3}$; 41$\frac{2}{3}$%. 19. 0.91$\frac{2}{3}$; 91$\frac{2}{3}$%. 20. $2984.80. 21. 7056 cu. in. 22. 775.8 lb. 23. 75.5 sq. in. 24. 5.0 sq. ft.; 1.2 cu. ft. 25. 12. 26. 19. 27. 0.28. 28. 0.4. 29. 3. 30. 125.

Page 288. 1. 280 lb. 2. $12. 3. $1.50. 4. $8.80. 5. 20 da. 6. $176,030. 7. $157.70. 8. 20. 9. $317.25. 10. 36. 11. $13.60. 12. $392.

Page 290. 1. 201,450. 2. $14,101.50. 3. 3.1%. 4. 355. 5. 216. 6. $3.60. 7. $4. 8. $3.30.

Page 291. 1. $1,504,800,000. 2. $2.28. 3. $22,800,000. 4. 78,000,000. 5. 1,790,800 bu. 6. 111,925. 7. 102$\frac{3}{4}$ lb. 8. $3,279,780,000. 9. 547,500,000; $153,300,000.

Page 292. I. 1. $2111.25. 2. $127.48. 3. $9020. 4. $2479.29. 5. $23,640. 6. $1\frac{1}{16}$. 7. $\frac{13}{16}$. 8. $\frac{7}{8}$. 9. $1\frac{23}{31}$. 10. $\frac{31}{64}$.

II. 1. $210. 2. $250. 3. $652. 4. $59.38. 5. 8. 6. $583.33. 7. $2400. 8. 33$\frac{1}{3}$%. 9. 12$\frac{1}{2}$%. 10. 15%.

III. 1. $300.93. 2. 42%. 3. $55.25. 4. $29.23. 5. $194.54.

IV. 1. 468.6 sq. in. 2. 4752 cu. in. 3. 200 bu. 4. 86.6 sq. in.

V. 1. 250, 375. 2. 1008, 720. 3. 14. 4. 35. 5. $2\frac{13}{16}$.

Page 296. 1. $7.50. 2. $21.60. 3. $9.15. 4. $45.00. 5. $5.07. 6. $25.20. 7. $56.80. 8. $26.55. 9. $8.40. 10. $18.50. 11. $43.40. 12. $12.80. 13. $8.25. 14. $11.13. 15. $22.20. 16. $20.40. 17. $11.50. 18. $9. 19. $4.10. 20. $37.75. 21. $7.30. 22. $45.60. 23. $100.50. 24. $25.80. 25. $40.50. 26. $21.60. 27. $25.76. 28. $9.94. 29. $7.00. 30. $12.50. 31. $14.50. 32. $56.40. 33. $20. 34. $15. 35. $3.76. 36. $2.25. 37. $2.90. 38. $6.00. 39. $12.30. 40. $1.38. 41. $6.20. 42. $1.80. 43. $4.50. 44. $6.50. 45. 95¢. 46. $1.40. 47. 60¢. 48. $8.50. 49. $9.33. 50. $7.00. 51. $10.95. 52. $2.40. 53. $3.60. 54. $5.30. 55. $6.63. 56. $1.40. 57. $3.13. 58. $11.25. 59. $8.25. 60. $4.25. 61. $2. 62. $11.25. 63. $9.75. 64. $2.10. 65. $5.92. 66. $2.88.

Page 297. 1. \$82.88. 2. \$688.84. 3. \$226.07. 5. \$567.71. 6. \$580.50. 7. 88¢. 8. \$3.50. 10. \$772.50. 11. \$35.98. 12. \$2287.76; \$12.76. 13. \$50; \$40; \$7210.

Page 301. 1. \$149.70. 2. 0.3%. 3. 0.1%. 4. \$249.75. 5. \$498.33. 6. \$971.10.

Page 302. 5. \$66.18. 7. \$1.50.

Page 306. 1. \$500. 2. \$270. 3. \$1,250,000. 4. \$2,750,000. 5. 2%. 6. 5%.

Page 308. 7. \$8727.50. 8. \$41,010. 9. \$21,810. 10. \$48,025. 11. \$7141.25. 12. \$12,961.88. 13. \$12,237.50. 14. \$1200.25. 15. \$317.88. 16. \$8832. 17. \$5013.75. 18. \$18,487.50. 19. \$57,735. 20. \$4986.25. 21. \$25,410. 22. \$632.50. 23. \$841.50. 24. \$192,840. 25. \$5177. 26. \$265.25. 27. \$5238.50. 28. \$8421.76.

Page 312. 1. \$350. 2. 5.8%. 3. \$802.50, gain. 4. \$446.25, loss. 5. \$32.25, loss. 6. \$14.13, gain. 7. \$2830, gain. 8. \$2066.25, loss. 9. 3.9%. 10. 4.2%. 11. 5.0%. 12. \$65.

Page 313. 1. \$225,000. 2. \$1,700,000. 3. \$255,000,000. 4. \$1.88. 5. A little more than \$1,000,000,000; \$70,000,000. 6. \$152,350,646. 7. \$177,501,966.13. 8. \$297,034,497.50. 9. \$191,250,000. 10. \$106.25. 11. \$18.75. 12. \$180.63. 13. \$480. 14. \$620.

Page 316. 1. \$1.70. 2. \$163.90; savings bank. 3. Yes; \$315. 4. \$750, \$300; the one at 6%. 5. \$225; \$540; the one at 5%. 6. \$30. 7. \$28.75. 8. \$35. 9. \$34.38. 10. \$34.97. 11. \$58. 12. \$585. 13. \$373.50. 14. \$105.

Page 317. 1. 6.9%. 2. \$575. 3. 8.0%. 4. \$741.50. 5. \$410; \$190. 6. $6\frac{1}{4}$%.

Page 318. 1. \$1020; \$1082.42; \$1126.14; \$1171.62. 2. \$180. 6. \$25.38; \$50.75; \$101.50; \$253.75.

Page 326. 4. 9. 5. 4.

Page 331. 3. \$60,000; yes. 5. 100 baskets. 6. 45.

Page 332. 2. Lower. 5. 98.6°, 100°; 98.6°, 99.8°; 98.8°, 98.8°; 98.6°, 98.6°, 98.6°. 6. − 10°, 15°, − 8°, 10°; 28°.

Page 337. 1. Yes. 2. $\frac{1}{6}$; 1000 lb. 3. 71. 4. \$1030.80; \$438.75. 5. \$592.05. 6. 93¢. 7. 59¢; \$1.76 less.

Page 340. 25. 5.29. 26. 22.09. 27. $6\frac{1}{4}$. 28. $11\frac{1}{9}$. 29. 5.76. 30. 12.96. 31. 0.49. 32. 0.6561. 33. 1,440,000. 34. 12,974,404. 35. 14. 36. 22. 37. 25. 38. 18. 39. 27. 40. 24. 41. 36. 42. 33. 43. 3.5. 44. 6.4. 45. 2.2 in. 46. 1.4 ft. 47. 2.1 in. 48. 66 in. 49. 77 yd. 50. 3.3 yd. 51. 324 ft. 52. 440 in. 53. 560 ft. 54. 48.4 in. 55. 512 in. 56. 520 ft. 57. 320 ft.; 960 ft. 58. 540 ft.; 1080 ft.; 2. 59. 80 ft.; 50 ft. 60. 680 ft.; 2040 ft.; 3; $\frac{1}{3}$.

Page 343. 1. 12. 2. 13. 3. 25. 4. 27. 5. 30. 6. 60. 7. 57. 8. 61. 9. 63. 10. 71. 11. 82 ft. 12. 87 ft. 13. 95 ft. 14. 97 ft. 15. 89 ft. 16. 83 yd. 17. $\frac{12}{13}$. 18. $\frac{13}{18}$. 19. $\frac{31}{32}$. 20. $\frac{33}{34}$. 21. $\frac{44}{71}$.

Page 345. 1. 25.6 ft. 2. 84 in. 3. 24.0 in. 4. 1.7 in.; 2.4 in. 5. 12.73 in.; 25.46 in. 6. 85 ft. 7. 2.83 ft. 8. 36.06 in. 9. 95 in. 10. 125 ft. 11. 22 ft. 6 in. 12. 16 ft. 6 in. 13. 42.52 ft. 14. 85 ft. 15. 535 ft. 16. 76.5 in. 17. 153 ft. 18. 91.92 in. 19. 137.18 ft. 20. 42.9 ft. 21. 15 ft.

Page 347. 1. 5. 2. $\frac{1}{6}$. 3. 9. 4. $\frac{1}{9}$. 5. 8. 6. $\frac{1}{8}$. 7. 12. 8. $\frac{1}{12}$. 9. 16. 10. 18. 11. 2.4 in. 12. 7 in. 13. 10 in. 14. 14 in. 15. 15 ft. 16. 25 ft. 17. 28 ft.

Page 350. 1. 5. 2. 7. 3. 8. 4. 9. 5. 2.646. 6. 3.606. 7. 5.385. 8. 6.856. 9. 8.185. 10. 9.539. 11. 2. 12. 3. 13. 4. 14. 2.351. 15. 2.571. 16. 2.668. 17. 3.037. 18. 3.583. 19. 3.708. 20. 4.626. 21. 0.9849. 22. 0.7483. 23. 31. 24. 46. 25. 72. 26. 83. 27. 88. 28. 93. 29. 95. 30. 98. 31. 18. 32. 21. 33. 27. 34. 35. 35. 44. 36. 46. 37. 81. 38. 93. 39. 13 in. 40. 3.39 ft. 41. 688.90 cu. in. 42. 14.69 cu. in. 43. 42; 43; 15. 44. 22; 29; 82; 93.

Page 354. 1. 10 ft. 2. $10a$. 3. $10n$. 4. $10x$. 5. $10w$. 6. $12a$; 36; 72; 108. 7. 20 in. 8. 20 yd. 9. $20x$. 10. $20y$. 11. $22n$. 12. $26a$. 13. 5 in. 14. $5x$. 15. $5a$. 16. $7n$. 17. $20y$. 18. $12x$; 12; 36; 6; 3. 19. 12 in. 20. $12a$. 21. $12b$. 22. $12n$. 23. $12x$. 24. $19x$. 25. 17 ft. 26. $17x$. 27. $28a$. 28. $159y$. 29. $10a$; 30; 50; 10. 30. 6 in.; $6a$; $6x$; $6n$.

Page 356. 1. $a + b$. 2. $x - y$. 3. $y - x$. 4. ab. 5. $\frac{m}{x}$. 6. $\frac{x}{m}$. 7. $a + b + c$. 8. $2a + 3b$. 9. $5a - 3b$. 10. 34; 0; 0. 11. 35; 2; 0; 65.4. 12. 4; $8\frac{1}{2}$; 0; 0.685; $\frac{1}{4}$. 13. 5; 11. 14. $\frac{5}{6}$; 0.

Page 358. 1. $3a$. 2. b^2; 9; $\frac{1}{4}$; 1. 3. $9n$; 54; 9; $4\frac{1}{2}$; 0. 5. 5; 3. 6. 8; $\frac{1}{2}$. 7. 8; e^3. 8. 24 in.; $12e$. 9. $3a^2b$. 10. 84; 3. 11. 12; 18; 1. 12. 6; 1. 13. 15; 3.

Page 359. 1. $9x$. 2. $6m$. 3. $14a$. 4. $11x$. 5. $16x$. 6. $20x$. 7. $21x$. 8. $4y$. 9. $17m$. 10. $12b$. 11. $13b$. 12. $4c$. 13. $29x$. 14. $9a$. 15. $25a$. 16. a. 17. $\frac{1}{2}a$. 18. $\frac{3}{8}x$. 19. $1\frac{1}{2}x$. 20. $\frac{5}{8}y$. 21. $7.5x$. 22. $2.4y$. 23. $8x$. 24. $4x$. 25. $19a$. 26. $13a$. 27. $17a$. 28. $13.7a$. 29. $36n$. 30. $9.1n$. 31. $22n$. 32. $14n$. 33. $12w$. 34. $20w$. 35. 0. 36. $55b$. 37. $16b$. 38. $4.8b$. 39. 0. 40. $7x$. 41. $4.4m$. 42. $7.72a^2$. 43. $9.54m^2$. 44. $4.16n^3$.

Page 361. 1. a^2. 2. b^2. 3. x^2. 4. $2x^2$. 5. x^3. 6. a^3. 7. b^3. 8. $3a^2$. 9. $4b^2$. 10. ab^2. 11. a^2b. 12. ac^2. 13. a^2c. 14. c^3. 15. x^2y. 16. $3a^2b$. 17. $3ab^2$. 18. $4x^2y$. 19. $4xy^2$. 20. $4x^2y^2$. 21. 2; 2; 2; x. 22. x. 23. 4. 24. a^2. 25. 5. 26. ab. 27. $7a$. 28. $9x$. 29. $9y$. 30. 10; 25. 31. 6. 32. 2.

33. 4. **34.** 9. **35.** 3. **36.** 9. **37.** 12. **38.** 8. **39.** 3. **40.** 24. **41.** 9. **42.** 9. **43.** $3xy$. **44.** $4x^2y$. **45.** $7ab^2c$. **46.** $3a^4b^2c^2$. **47.** $4a^2bx$. **48.** $10a^2b^2x^2$. **49.** 0. **50.** $36a^3b^2x$.

Page 362. **1.** 10. **2.** 0. **3.** 6. **4.** 13. **5.** 12. **6.** 2. **7.** 34. **8.** 30. **9.** 125. **10.** 375. **11.** 12. **12.** 0. **13.** 11. **14.** 1. **15.** 9. **16.** 2. **17.** 4. **18.** 180. **19.** 900. **20.** 200. **21.** 38. **22.** 8. **23.** 30. **24.** 50. **25.** 165. **26.** 175. **27.** 725. **28.** 5. **29.** 40. **30.** 20. **31.** 42. **32.** 42. **33.** 252. **34.** 294. **35.** 26,460. **36.** 84. **37.** 140. **38.** 588. **39.** 5040. **40.** 7560. **41.** 251. **42.** 1765. **43.** 1470. **44.** 36. **45.** 12. **46.** 84. **47.** 14. **48.** 356. **49.** 216. **50.** 8575. **51.** 1.

Page 365. **1.** 43. **2.** 66. **3.** 87. **4.** 47. **5.** 100. **6.** $\frac{1}{8}$. **7.** 1. **8.** $\frac{3}{4}$. **9.** $\frac{1}{2}$. **10.** $1\frac{1}{4}$. **11.** 2.5. **12.** 5.5. **13.** 3. **14.** 2. **15.** 9.5. **16.** 40. **17.** 40. **18.** 47. **19.** 61. **20.** 71.

Page 366. **1.** 40. **2.** 46. **3.** 37. **4.** 50. **5.** 42. **6.** 10. **7.** 21. **8.** 14. **9.** 80. **10.** 48. **11.** 10. **12.** 0. **13.** 1. **14.** 11. **15.** 3. **16.** x; x; 3; **6.** **17.** 9. **18.** 7. **19.** 11. **20.** 3.5. **21.** 2.8. **22.** $\frac{1}{2}$. **23.** 10. **24.** 1. **25.** 9. **26.** 7. **27.** 9. **28.** 1.

Page 367. **1.** 14. **2.** 10. **3.** 12. **4.** 27. **5.** 240. **6.** 28. **7.** 36. **8.** 40. **9.** 66. **10.** 800. **11.** 70. **12.** 90. **13.** 160. **14.** 270. **15.** 1400. **16.** 20. **17.** 21. **18.** 3. **19.** 240. **20.** 4000. **21.** $\frac{1}{2}x = 13$; **26.** **22.** 30. **23.** 52. **24.** 50. **25.** 88. **26.** 138. **27.** 400. **28.** 2. **29.** 30. **30.** 76.

Page 368. **1.** 2. **2.** 4. **3.** 8. **4.** 17. **5.** 30. **6.** 20. **7.** 4. **8.** 5. **9.** 9. **10.** 2. **11.** 2. **12.** 20. **13.** 2. **14.** 5. **15.** 3. **16.** 4. **17.** $\frac{1}{4}$. **18.** 16. **19.** 7. **20.** 60. **21.** 900. **22.** 27. **23.** 8. **24.** 5. **25.** 5. **26.** 8. **27.** 3. **28.** $\frac{5}{8}$. **29.** $3\frac{1}{3}$. **30.** $\frac{1}{8}$. **31.** $\frac{1}{8}$. **32.** 3.

Page 370. **1.** 2. **2.** 9. **3.** 3. **4.** 21. **5.** 2. **6.** 6. **7.** 1. **8.** 21. **9.** 3. **10.** 21. **11.** 3. **12.** 7. **13.** 4.2. **14.** 18. **15.** 46. **16.** 46. **17.** 3. **18.** 15. **19.** 14.4. **20.** 206.4. **21.** 10. **22.** 19. **23.** 19. **24.** 10. **25.** 7. **26.** 2.4. **27.** 4. **28.** 4. **29.** 1. **30.** 3. **31.** 3. **32.** 2. **33.** 2. **34.** 5. **35.** 3. **36.** 4.2. **37.** 28. **38.** 54. **39.** 80. **40.** 96.

Page 372. **1.** 12. **2.** 9. **3.** 7. **4.** 10. **5.** 13. **6.** 15. **7.** 24. **8.** 13. **9.** 3. **10.** 11. **11.** 4. **12.** 14. **13.** 5. **14.** 11. **15.** 4. **16.** 11. **17.** 1. **18.** 2. **19.** $5\frac{1}{3}$. **20.** $1\frac{3}{4}$. **21.** 4.9. **22.** 4.1. **23.** 20. **24.** 1.2. **25.** $3\frac{1}{6}$. **26.** $1\frac{1}{2}\frac{1}{2}$. **27.** 1. **28.** 35. **29.** 1. **30.** 213. **31.** 5. **32.** 7. **33.** 7. **34.** 5. **35.** $\frac{1}{8}$. **36.** 10. **37.** 19.8. **38.** 1. **39.** $4\frac{3}{8}$. **40.** $2\frac{7}{9}$.

Page 374. **1.** 39. **2.** 14. **3.** 9. **4.** 72. **5.** 9. **6.** 2. **7.** 18. **8.** 13. **9.** 56. **10.** 16. **11.** 84. **12.** 7. **13.** 36. **14.** $3\frac{1}{2}$. **15.** 24. **16.** 23. **17.** 24. **18.** $5.40. **19.** $720. **20.** 4. **21.** 2600 lb. **22.** 40 ft. **23.** 20 ft. **24.** 4. **25.** 7. **26.** 3. **27.** 24. **28.** 12, 84. **29.** 30, 60. **30.** 25, 75. **31.** 3, 12, 6.

Page 377. 1. 64. 2. 48. 3. 36. 4. 54. 5. 72. 6. 82. 7. 62. 8. 220,221. 9. 240,241. 10. 9. 11. 16. 12. 132. 13. 48. 14. \$1250. 15. \$96. 16. \$65. 17. 12. 18. $22\frac{1}{2}$ ft.; $67\frac{1}{2}$ ft. 19. \$360. 20. \$2. 21. 15 yr. 22. 13 yr. 23. 20 ft., 12 ft. 24. 78 ft., 36 ft. 25. 15 yr. 26. \$35,000.

Page 380. 1. 32. 2. 8. 3. 21. 4. 45. 5. 10. 6. 15. 7. 30. 8. 90. 9. 132. 10. 195. 11. 5. 12. 3. 13. 14. 14. 15. 15. 14. 16. $6\frac{3}{4}$. 17. 23. 18. 54. 19. 54. 20. 79. 21. 12.6; 12.6. 22. 44. 23. 44. 24. 88. 25. 88. 26. 22. 27. 22. 28. 13.2. 29. 13.2. 30. 150 sq. in. 31. 27 cu. in.

Page 383. 1. $r = \frac{C}{2\pi}$. 2. 3.5 in. 3. 0.7 in. 4. 0.8 in. 5. 3.9 in. 6. 15.9 in. 7. $r = \frac{S}{2\pi h}$. 8. 1.8 in. 9. 1.4 in. 10. 2.0 in. 11. 3.2 in. 12. $h = \frac{V}{\pi r^2}$. 13. 0.8 in. 14. 0.6 in. 15. 0.6 in. 16. 1.3 in. 17. $r = \sqrt{\frac{V}{\pi h}}$. 18. 2.3 in. 19. 1.6 in. 20. 2.1 in. 21. 1.8 in. 22. $r = \frac{i}{pt}$; $t = \frac{i}{pr}$.

Page 384. 1. \$20. 2. $c = rs$. 3. \$36. 4. \$650. 5. $p = rc$. 6. $r = \frac{p}{c}$. 7. $p = \frac{ri}{100}$. 8. $r = \frac{100\,p}{i}$. 9. \$125. 10. $t = \frac{ar}{1000}$. 11. $r = \frac{1000\,t}{a}$. 12. $a = \frac{1000\,t}{r}$. 13. $b = \frac{A}{h}$; $h = \frac{A}{b}$. 15. $h = \frac{2\,A}{b + b'}$. 16. 10 in. 17. 60 mi.; $D = rt$. 18. $r = \frac{D}{t}$; $t = \frac{D}{r}$. 20. \$6; $C = np$. 21. $n = \frac{C}{p}$; $p = \frac{C}{n}$. 22. $r = \sqrt{\frac{S}{4\pi}}$. 23. 38,808 cu. in. 24. $163\frac{1}{3}$. 25. $1306\frac{2}{3}$. 26. 10.

Page 386. 1. 528. 2. 572. 3. 616. 4. 660. 5. 3696. 6. 30,888. 7. 924. 8. 226.3. 9. 22. 10. 154. 11. 616. 12. 12,873.1. 13. 640. 14. \$83.33. 15. $A = p + prt$. 16. \$5500. 20. $h = \frac{T}{2\pi r} - r$. 21. $d = hm$; $87\frac{1}{2}$.

Page 387. 1. $6\frac{3}{16}$; $1\frac{9}{16}$. 2. $8\frac{1}{8}$; $1\frac{5}{8}$. 3. $10\frac{9}{16}$; $4\frac{11}{16}$. 4. $12\frac{11}{16}$; $5\frac{7}{16}$. 5. $10\frac{25}{32}$; $5\frac{9}{32}$. 6. $10\frac{1}{16}$; $2\frac{13}{16}$. 7. $\frac{6}{16}$; $\frac{5}{6}$. 8. $\frac{7}{256}$; $1\frac{3}{4}$. 9. $\frac{3}{8}$; $1\frac{7}{10}$. 10. $84\frac{3}{8}$ ft.; $16\frac{2}{3}$ ft. 11. \$13.63; 87¢. 12. \$9.08; \$7.56. 13. 24.3 mi.; 8.5 mi. 14. 36.08 in.; 16.56 in. 15. 248.832; 120. 16. 2949.7804; 76. 17. 238.45095; 9.5. 18. \$5.40. 19. \$9.75. 20. \$27.24. 21. \$78.24. 22. 5. 23. 25. 24. 2 mi. 25. 827.14. 26. 98.47. 27. 11.75. 28. 4.09. 29. 15. 30. 14. 31. 19. 32. 20. 33. 9. 34. 4. 35. 2. 36. 8. 37. 2. 38. 33.

Page 394. 1. 12; 3. 2. 40. 3. 240; 480. 4. 40. 5. 45; 90; 4500. 6. 32; 16. 7. 96; 6. 8. 53. 9. 8; 120; 9. 10. 16. 11. July 2. 12. \$58.80. 13. 0.2, sulphur; 0.18, lime; 0.02, arsenate of lead; 80, sulphur;

72, lime; 8, arsenate of lead. **14.** $21.28. **15.** $28; $19.95; $34.30. **16.** $384.98; 12 20-dollar bills, 9 10-dollar bills, 3 5-dollar bills, 18 2-dollar bills, 1 1-dollar bill, 3 50-cent pieces, 1 quarter, 12 dimes, 3 pennies. **17.** $3.68.

Page 396. **1.** 20%. **2.** 20%, below. **3.** 25%. **4.** 25%. **5.** 11.2 gal. **6.** 60%. **7.** 12½%. **8.** $1. **9.** 1242. **10.** 9.1%. **11.** $80.50. **12.** $821.10. **13.** $207.90. **14.** $11.25. **15.** $21.60. **16.** $3. **17.** $300; 60¢. **18.** 38¢. **19.** $705.60. **20.** $10.90.

Page 398. **1.** 3¾ lb. sugar, 1¼ cups vinegar, 2½ cups water. **2.** 2 cups sugar, ¼ cup milk, 1½ teaspoonfuls butter, ⅜ cup walnuts; 6 cups sugar, ¾ cup milk, 4½ teaspoonfuls butter, 1⅛ cups walnuts; 10 cups sugar, 1¼ cups milk, 7½ teaspoonfuls butter, 1⅞ cups walnuts. **3.** $1.40. **5.** 8. **6.** 52. **8.** 36¢. **9.** 10¢. **10.** 52¢. **11.** $1.14. **12.** 25¢. **13.** $1.60; 40%. **14.** 55¢; 19.3%. **15.** 12%. **16.** 67. **17.** 6250; 2,281,250. **18.** $264,625. **19.** $172,006.25. **20.** $2.34. **21.** $585,000. **22.** $2.63. **23.** $13.50. **24.** 44¢.

Page 401. **1.** 1176. **2.** $1096.48. **3.** 23,733. **4.** $391.60. **5.** Second; $1.26. **6.** 40.8 lb.

Page 402. **1.** 3⅛ lb., copper sulphate; 3⅛ lb., lime; 243¾ lb., water. **2.** 47¼ lb., copper sulphate; 47¼ lb., lime. **3.** 357. **4.** $43.65. **5.** $22.55; $51.45. **6.** 0.4%. **7.** 0.5%.

Page 403. **1.** $2474.00; $1379.52. **2.** 77.1%; 79.9%; 79.2%; 84.7%; 110.3%. **3.** $5.47, $76.75, $73.02, $48.15, $2.67; $109.76. **4.** $1632.15, $647.43; $984.72.

Page 404. **1.** 2427.8. **2.** 162.6 cu. ft. **3.** 75%; 42.9%. **4.** 64,800 lb. **5.** 5 ft. 6⅔ in. **6.** 51,397.5. **7.** 2420 lb. **8.** 15; 3¾.

Page 405. **1.** $85,312.50. **2.** $12. **3.** $586.20. **4.** $120. **5.** First; 4.5%. **6.** $4728. **7.** 66,000 gal. **8.** $8007.

Page 406. **1.** $66. **2.** $35. **3.** 14.3%; 10%. **4.** 27.5%. **5.** 10%. **6.** $50. **7.** $2.50. **8.** $2.25. **9.** $171.60.

Page 407. **1.** 53,865; 1995. **2.** 5712; $102. **3.** $3459.50. **4.** 13.0%. **5.** $1200. **6.** $8030. **7.** 6.6 sq. ft.; 10½ ft. **8.** 88 sq. ft. **9.** 5 ft. **10.** 14.3 ft. **11.** 84; $36.65. **12.** $233.75. **13.** 975 bbl., cement; 325 cu. yd., sand; 487½ cu. yd., crushed rock. **14.** 90 bbl., cement; 36 cu. yd., sand; 42 cu. yd., crushed rock. **15.** ⅔; 18; 36. **16.** 16⅔; 10 cu. yd.; 11.7 cu. yd. **17.** 10½ bbl., cement; 3½ cu. yd., sand; 5¼ cu. yd., crushed rock. **18.** $49.50.

Page 410. **1.** $180; $90; $100; $275; $240; $240; $106.25; $112.50; $1343.75. **2.** $6. **3.** $20. **4.** $6. **5.** $30. **6.** $20. **7.** $9.60. **8.** $1080. **9.** $187.50. **10.** $41.25. **11.** $300. **12.** $937.50. **13.** $540. **14.** $5.38. **15.** The mortgage. **16.** 5.4%; 5.4%; 5.5%; the bond. **20.** $53. **21.** $407.

22. $32, decrease. **23.** $6900; $414. **24.** $46.89; $3.11 less. **25.** $600.40, loss. **26.** $839.60, gain.

Page 413. **1.** 999,999. **2.** 111; 222; 333; 444; 555; 666; 777. **5.** 20 lb. **6.** 14 lb. **7.** 49. **8.** 24 ft. **9.** 5¢. **10.** 39,700 lb. **11.** 30 da.

Page 414. I. **1.** $6.56. **2.** $507.50. **3.** $2.50 per $100. **4.** $129.38. **5.** $4\frac{1}{2}\%$.

II. **1.** 31.2 sq. in. **2.** 1200. **4.** 40.31; 9.42; 9.63. **5.** 35 ft.

III. **1.** 8. **2.** 2. **3.** 3. **4.** 6. **5.** 2. **6.** 5. **7.** 1. **8.** 3.

IV. **1.** 24. **2.** $31\frac{1}{2}$. **3.** 113.5. **4.** 209.4. **5.** 105.2. **6.** 10,483.7. **7.** 242.32. **8.** 6447.50.

V. **1.** $h = \frac{A}{b}$. **2.** $l = \frac{V}{wh}$. **3.** $r = \frac{C}{2\pi}$. **4.** $b = \frac{2A}{h}$. **5.** $h = \frac{V}{\pi r^2}$. **6.** $r = \frac{1}{2}\sqrt{\frac{S}{\pi}}$. **7.** $p = \frac{i}{rt}$. **8.** $r = \sqrt{\frac{3V}{\pi h}}$.

Page 415. **1.** 506 mi. **2.** 9148 mi. **3.** 194. **4.** 853. **5.** $8.68. **6.** $4.21.

Page 416. **1.** 2225 mi. **2.** 3440 mi. **3.** 1020 mi. **4.** 4460 mi. **5.** 406 mi. **6.** Lindbergh's route; 1585 mi. **7.** 2673 mi. **8.** 735 mi. **9.** 2385 mi.

Page 418. **1.** 9.1. **2.** 35.9. **3.** 15.5. **4.** 19.8. **5.** 29.6. **6.** 46.5 da. **7.** 20.5 da. **8.** 89; 178.4.

Page 419. **1.** $63. **2.** $41.90. **3.** $52.50. **4.** $24.50. **5.** $637. **6.** $88.50.

Page 420. **1.** 4 00 P.M.; 6 45 P.M.; 2 hr. 45 min. **2.** 2 hr. 40 min.; 5 min. **3.** 7237 ft.; 5280 ft.; 5989 ft. **4.** 1957; 709. **5.** 52; 38. **6.** 34. **7.** 33. **8.** 1 hr. 45 min. **9.** 1 hr. 40 min. **10.** 25.7; 27.3. **11.** 27.4; 25.8.

Page 422. **1.** $4.28; $13.50. **2.** $10.62. **3.** $3.35; $4.14; $6.84; $16.70; $21.06. **4.** $3.24; $6.08; $6.48; $11.92. **5.** $48.24. **6.** 555 mi.; Green River. **7.** Salt Lake City; 9 00 P.M. **8.** Colorado Springs; 12 05 P.M. **9.** $48.62. **10.** $79.

Page 423. **1.** 75¢. **2.** $2.10. **3.** 75 ft. **4.** $3.05. **5.** $13.00. **6.** 80¢. **7.** $3.29. **8.** $57.20. **9.** $2\frac{1}{2}$¢. **10.** $1718.50.

Page 424. **1.** 17,622. **2.** $74,153.34; $90,467.07; $118,511.86. **3.** 408. **4.** 970. **5.** 1380. **6.** 81.1 mi. **7.** 55.8. **8.** 124. **9.** 85. **10.** 3164. **11.** $8,187,470.98. **12.** 183. **13.** 75%; 18,590 bu. barley, 37,180 bu. oats, 167,310 bu. wheat. **14.** $1,344,776.35. **15.** 345.

Page 426. **1.** 558. **2.** 90.9%. **3.** 19.5%; 21.2%; 26.7%; 32.6%. **4.** *At $1,* 66.7%; *at 75¢,* 96.9%; *at 50¢,* 92.7%; *at 25¢,* 100%. **5.** *At $1,* $80; *at 75¢,* $94.50; *at 50¢,* $76; *at 25¢,* $50; *total,* $300.50. **6.** *At $1,* 26.6%; *at 75¢,* 31.4%; *at 50¢,* 25.3%; *at 25¢,* 16.6%. **7.** $105.18. **8.** $75.13. **9.** $120.19.

Page 427. 1. 52%. 2. 44%. 3. 53%. 4. 52%. 5. 52%. 6. 65%. 7. 61%. 8. 55%. 9. 46%. 10. 59%. 11. 43%. 12. 61%. 13. 46%. 14. 54%. 15. 47%. 16. 51%. 17. 52%. 18. 52%.

Page 428. 1. $153.92; 12.9%. 2. $143.69; 12.0%. 3. $146.01; 11.9%. 4. $149.50; 12.5%. 5. $153.45; 12.0%. 6. $156.71; 12.1%. 7. $158.10; 11.5%. 8. $182.28; 12.2%. 9. $182.75; 11.6%. 10. $178.33; 12.2%. 11. $189.95; 9.5%. 12. $185.07; 9.6%. 13. $169.96; 11.4%. 14. $180.89; 10.2%. 15. $173.68; 11.4%. 16. $182.51; 9.9%.

Page 429. 1. 12½%. 2. 29.4%. 3. 30.9%. 4. $7920. 5. $1250. 6. 10%. 7. 88.9%. 8. $1200; $900. 9. $8.50. 10. 14.9% loss.

Page 430. 1. $330. 2. $255. 3. $200. 4. $295. 5. $340. 6. $310. 7. $630. 8. $515. 9. $1270. 10. $1395. 11. $370. 12. $2280. 13. $1840. 14. $1250. 15. $660. 16. $735. 17. $440. 18. $550. 19. $500. 20. $485. 21. $1590. 22. $1435. 23. $810. 24. $410. 25. $2425. 26. $2000. 27. $2870. 28. $2205. 29. $2720. 30. $1105. 31. $1810. 32. $955. 33. $2910. 34. $3310. 35. $1345. 36. $2315. 37. $8090. 38. $1155.

Page 431. 1. $180. 2. $231.87. 3. $2601. 4. $1375. 5. $15,000. 6. 5%. 7. $2150; $107.50. 8. 2%.

Page 432. 1. $1135.25. 2. $1163.75. 3. $1173. 4. $1147.50. 5. $1230. 6. $1596. 7. $1480. 8. $1143.75. 9. $1046.50. 10. $1575. 11. $5800. 12. $8850. 13. $10,500. 14. $12.75. 15. $27. 16. $32.73. 17. $20.70.

Page 433. 1. $5. 2. $30; $43.33; $63.75. 3. $4; $8; $10; $12; $16. 4. 50%; 33.3%; 40%. 5. 55.6%; 38.5%; 36.4%; 51.3%. 6. 33.3%; 37.5%; 52.9%; 41.7%. 7. $186.24. 8. $3138.75. 9. $19.21. 10. $43.94. 11. $11.14. 12. $66.73.

Page 434. 1. $72. 2. $64. 3. $25.05. 4. $351. 5. $221. 6. $126.50. 7. $222.50. 8. $56. 9. $420. 10. $27.

Page 435. 1. $9.60. 2. $13.50. 3. $13.50; $702. 4. $4.20; $1092. 5. $21.60; $302.40. 6. $4650.75. 7. $698.33.

Page 436. 1. $660; $55. 2. 26%. 3. 17½%. 4. 25%. 5. $4907. 6. 9%. 7. 5.3%. 8. $81.59. 9. $2188.

Page 437. 1. $210; $1260. 2. $2340. 3. $240. 4. $1350. 5. $750. 6. $2600. 7. $4974.55.

Page 438. 1. $10. 2. 6 mo. 3. $21.25. 4. 6%. 5. $12. 6. 1½ yr. 7. 2¼%. 8. $41.67. 9. 8 mo. 10. $5.73. 11. $21.88. 12. 6 mo. 13. $17.50; $26.25; $4.38; $2.92; $13.13; $35; $52.50; $8.75. 14. $76,875. 15. $550.

Page 439. **1.** 13.3%; $15 less. **2.** Rent, heat, light, 26.9%; food, 33.1%; clothing, 15.4%; advancement, 8.6%; savings, 7.1%; miscellaneous, 8.9%. **3.** Rent, $262.50; groceries, $437.50; meats, $210; clothing, $227.50; 28.6%. **4.** $1825; $27,375. **5.** 20%; $10.80.

Page 440. **1.** $1689.20; $9.20 more. **2.** $7.50. **3.** $6.25. **4.** $33.75. **5.** $1506. **6.** $6309.74. **7.** $49.50. **8.** $855.56. **9.** Gains; $10.59.

Page 441. **1.** 10 gal. 1 qt. **2.** 26 gal. 2 qt. **3.** 688 gal. **4.** 62 lb. 14 oz. **5.** 72 ft. 10 in. **6.** 35 T. 1440 lb. **7.** 6 hr. 20 min. **8.** 28 sq. yd. 8 sq. ft.; 7 sq. yd. $7\frac{1}{2}$ sq. ft.

Page 442. **1.** 689; 1033. **2.** 3000. **3.** 35. **4.** 3168; 380,161. **5.** 105,600. **6.** 5100. **7.** 2; 1; 2. **8.** 1999. **9.** 47. **10.** $7.25. **11.** 10. **12.** 9601. **13.** 18. **14.** 10; $6\frac{1}{4}$ in. **15.** $71\frac{1}{4}$; $5.70. **16.** $45.10. **17.** $3.37. **18.** 48. **19.** $5\frac{7}{8}$. **20.** To buy a gross; $1.20. **21.** 2500. **22.** 8.1 hr. **23.** $5\frac{1}{4}$.

Page 444. **1.** $74\frac{3}{8}$. **2.** 1280. **3.** 48. **4.** 22; $39.60. **5.** $158.40. **6.** 1980 sq. ft.; 220 sq. yd. **7.** $\frac{5}{16}$. **8.** 6.9. **9.** $990.

Page 445. **1.** 1.1 in. **2.** No; 7 in. **3.** 84 ft. **4.** 63 in. **5.** 14 in. **6.** No; $\frac{1}{4}$-mile track. **7.** 18 in. by 18 in. by 13 in. **8.** 75.

Page 446. **1.** 144. **2.** 144; 1. **3.** 144; 1. **4.** 144; 1. **5.** 144; 1. **6.** 576; 4. **7.** 12 in.; 1; 1; 12. **8.** 1; 12. **9.** 2; 24. **10.** 2; 24. **11.** 12. **12.** 2; 4; 8. **13.** 8; 8; 16. **14.** 18; 16; 30. **15.** 36. **16.** 192. **17.** $9.60. **18.** $116.64. **19.** $9.73. **20.** $11.78. **21.** $199.68. **22.** $913.92. **23.** $25.09. **24.** $66.96. **25.** $179.20.

Page 448. **1.** 11,304. **2.** 2.1; 1333. **3.** 3520. **4.** 5.6. **5.** 150; 30. **6.** 36.9%. **7.** $602.18. **8.** 96,800.

Page 449. **1.** $308.15. **2.** $429.15. **3.** $285.41. **4.** $1774.23.

Page 450. **1.** $5\frac{3}{4}$. **2.** $7\frac{3}{8}$ in. **3.** 34. **4.** 32. **5.** $9.72. **6.** 45¢. **7.** $1.82. **8.** 9. **9.** $29\frac{1}{2}$ yd. **10.** 6. **11.** First; 32¢.

Page 451. **1.** $18. **2.** 40%. **3.** $12\frac{1}{2}$%. **4.** 20%. **5.** $3480. **6.** Food, $1050; rent, $700; clothing, $630; advancement, $420; other expenses, $350; investment, $350. **7.** 468. **8.** $715.38, 47.7%; $834.62, 47.7%. **9.** $1800; $2100; $900. **10.** 110.16 lb. **11.** 500. **12.** 714.3. **13.** 2080 gal. **14.** $1071. **15.** $2475. **16.** An increase of 20%; $64. **17.** Sand, 108; soda, 21; lime, 18; alumina, 3. **18.** Copper, 962.5; tin, 262.5; zinc, 25. **19.** First.

Page 453. **1.** $12\frac{1}{2}$. **2.** $\frac{1}{4}$; 0.25; 25. **3.** 25¢; 0.50; 50. **4.** 1; $\frac{1}{8}$; 0.125. **5.** $\frac{1}{10}$; 10; 10. **6.** $\frac{1}{16}$; 0.0625; $6\frac{1}{4}$. **7.** $\frac{8}{1}$; 8.00; 800. **8.** 0.02; 2; 1. **9.** 17.2 yr. **10.** 4.2. **11.** 1.5; 14.5. **12.** $101.62. **13.** $49,407.60.

Page 454. 1. 8; $276. 2. 20; $30. 3. $3.84; $44.16. 4. $9; $66. 5. 10; $21.60. 6. $150; $300. 7. $1.80. 8. $38.25. 9. $17. 10. Second; $49.02. 11. $300. 12. Second; 3.9%.

Page 455. 1. $22.50. 2. 5%. 3. $4900. 4. $47.50. 5. $261.79. 6. $355.50. 7. $246.45. 8. $260. 9. $412.50. 10. $40. 11. $435.00; $1.87 more.

Page 456. 1. $\frac{3}{4}$; $\frac{1}{2}$; $1\frac{1}{2}$ lb.; 3 lb.; 2 lb.; 4 lb. 2. 3.8¢; $1.84. 3. $1230. 4. $145. 5. $100. 6. 15%. 7. $1437.50. 8. $8000. 9. 25%. 10. $6240; 12%.

Page 457. 1. 3.2 in. 2. 3.9%. 3. 4.1%. 4. $4220. 5. 27.9 T. 6. 45.5%. 7. 120%; $300. 8. 14.6. 9. 1100; 8250.

Page 458. 1. $514,300. 2. $194.60. 3. $223.79. 4. $26 per $1000; $2.60 per $100; 26 mills per $1. 5. $21.34. 6. $35,000. 7. $26,520. 8. $121.50. 9. $7372.50.

Page 459. 1. 63.3. 2. 7.8 A. 3. 33 sq. in. 4. 3.3 sq. ft. 5. 33 sq. ft; 10. 6. 7 sq. mi. 8. 6.9 sq. in. 9. 104 sq. ft. 10. 37.3 sq. ft.

Page 461. 1. 47.9. 2. $2\frac{3}{4}$ in. 3. $49\frac{1}{2}$ cu. ft. 4. 121.6. 5. 63.9 lb. 6. 71.5 lb. 7. $133\frac{1}{3}$. 8. 16,200 gal. 9. 540 lb. 10. 9200 cu. yd.

Page 462. 1. 107.22 cu. in. 2. 4019. 3. 14.9. 4. $13\frac{1}{3}$. 5. 46.4 gal. 6. 706.5 lb. 7. 10-gallon can. 8. $5\frac{1}{2}$.

Page 463. 1. 26.8 cu. in. 2. $263.90. 3. 201,143,000 sq. mi. 4. 116.3 sq. in. 5. 462. 6. 66 cu. in. 7. 6084.6 sq. in. 8. 9.4. 9. 1458.0. 10. 452.6 sq. in. 11. 6.7 in. 12. 1925. 13. 28.3. 14. 46 lb. 11 oz. 15. 110.9 sq. in. 16. 178.2. 17. 10,183,381,000 cu. mi. 18. 83.2 sq. in.; 83.2 sq. in. 19. 6 ft. 6 in. 20. 5.3. 21. $160.16. 22. 6 in.

Page 465. 1. 12.3 sq. in. 2. 8.94 sq. in. 3. 7.2 sq. in. 4. 34.3 sq. in. 5. 14.2 sq. in. 6. 75.5 sq. in. 7. 274.6 cu. in. 8. 5.94 cu. in. 9. 48.5 cu. in. 10. 381.9 cu. in. 11. 2.21 cu. in. 12. 7.5 cu. in. 13. 10 lb.; 50 lb. 14. 201.6. 15. 23.2. 16. $1546\frac{7}{8}$. 17. 7.5 A. 18. 2.4 sq. ft. 19. 18,669. 20. 68.5. 21. 2940 sq. ft. 22. 1.4 ft. 23. 740.7. 24. $176.40.

Page 467. 4. Unchanged; unchanged; increased; decreased. 5. $\frac{3}{10}$; 30%. 6. 5 ft.; 15 ft. 7. 37.5 lb. 8. 80. 9. Length, 60 ft.; height, 54 ft. 10. $3.44; $5.16.

Page 468. 1. 30 ft. 2. $\frac{5}{4}$; 45 ft. 3. 64 ft. 4. 40 ft. 5. 27 in. 6. 50.7 ft. 7. 37.5 ft. 8. 22.4 ft. 9. 67.2 ft.

Page 469. 1. 7.4%. 2. 5.7%. 3. 18¢. 4. $33.80. 5. First; $1.74. 6. $6\frac{1}{4}$%. 7. $8\frac{1}{3}$%. 8. 9.1%. 9. $2739.75.

Page 470. 1. 22,500. 2. $157,500. 3. $48.75. 4. $3\frac{1}{2}$%. 5. 2.6%. 6. $325. 7. 7%. 8. $87.50. 9. $500.

Page 471. 1. $4106. 2. $11,881.88. 3. $4951.25. 4. $3224.25. 5. $13,815.63. 6. $10,799.25. 7. $4749.75; $4737.75. 8. $25,686.63. 9. $11,895. 10. $7201.00. 11. $8030.50.

Page 472. 1. $1215. 2. $99.25; 6.0% annually. 3. 7.3% annually. 4. $1275.00. 5. $78.75. 6. 0.07% less. 7. $773, gain. 8. $6125.

Page 473. 1. $431.25. 2. 5.54%. 3. $1184.75. 4. $6.35 less. 5. $737.62. 6. Bonds; $30.00. 7. $1505.

Page 474. 1. $1\frac{1}{2}$ in. 3. 395 lb.; 18°. 7. 7.1 in. 8. 2 ft.

Page 475. 1. 39 ft. 2. 88 ft. 3. 3.7 in. 4. 8.9 in. 5. 325 ft. 6. 40.7 yd. 7. 10.0 in. 8. 17.8 in. 9. 21.6 ft. 10. 5.5 in. 11. 3.0 in. 12. 4.4 ft. 13. 29.4 cm. 14. 53.0 cm. 15. 4.7 m. 16. 156.2 ft. 17. 59 mm. 18. 30 ft. 19. 127.3 ft. 20. 25.6 ft. 21. 2.9 in.; 2.7 ft. 22. 24 rd.; 96. 23. 36 ft. by 36 ft.

Page 476. 1. 2. 2. 31.7 lb. 3. 62,304.7 lb. 4. 54,308.6 lb. 5. 144.4. 6. 5646.7; 1344.5. 7. 2 hr. 15 min.; 3 hr. $22\frac{1}{2}$ min. 8. 1 yd. 31 in. 9. 18.1; $217\frac{1}{2}$. 10. 51.2 mi. 11. 35 ft. 12. 288. 13. $19.46. 14. $186.30. 15. $525. 16. $2\frac{1}{2}$%. 17. $190.62. 18. $16.72. 19. Copper, 673.8; zinc, 17.5; tin, 183.7. 20. $490. 21. 4%. 22. $271.80. 23. $195.98. 24. $155.25. 25. 40% gain. 26. 3%. 27. $357.20; 24%. 28. $116.37. 29. 4.2.

Page 479. *Exercise 1.* 1. 114.8; 116.0; 123.1; 129.4; 132.6; 134.5; 143.4; 135.9. 2. 155.6; 154.4; 147.3; 141.0; 137.8; 135.9; 127.0; 134.5. 3. 50.41; 68.89; 237.16; 470.89; 620.01; 718.24; 1274.49; 795.24. 4. 10.92; 12.77; 23.69; 33.38; 38.31; 41.23; 54.92; 43.38. 5. 2.37; 2.77; 5.13; 7.23; 8.3; 8.93; 11.9; 9.4.

Exercise 2. 1. 125.9; 130.9; 128.4; 134.3; 137.3; 138.6; 136.3; 136.9. 2. 78.3; 73.3; 75.8; 69.9; 66.9; 65.6; 67.9; 67.3. 3. 1393.92; 1971.92; 1670.42; 2422.08; 2857.68; 3057.62; 2708.48; 2797.52. 4. 371.83; 442.25; 407.04; 490.14; 532.39; 550.70; 518.31; 526.76. 5. (1) 6.6; 7.85; 7.23; 8.7; 9.45; 9.78; 9.2; 9.35. (2) 33; 39.25; 36.13; 43.5; 47.25; 48.88; 46; 46.75. (3) 35.2; 41.87; 38.53; 46.4; 50.4; 52.13; 49.07; 49.87.

Page 480. *Exercise 3.* 1. 375.1; 392.7; 360.8; 425.7; 403.7; 437.8; 422.4; 413.6. 2. 970.6; 969.0; 971.9; 966.0; 968.0; 964.9; 966.3; 967.1. 3. 119.35; 124.95; 114.8; 135.45; 128.45; 139.3; 134.4; 131.6. 4. (1) 2; 2; 2; 2; 2; 2; 2; 2. (2) $\frac{1}{2}$; $\frac{1}{2}$; $\frac{1}{2}$; $\frac{1}{2}$; $\frac{1}{2}$; $\frac{1}{2}$; $\frac{1}{2}$; $\frac{1}{2}$. (3) 48.03; 50.28; 46.20; 54.51; 51.69; 56.06; 54.08; 52.96. 5. 250.1; 251.7; 248.8; 254.7; 252.7; 255.8; 254.4; 253.6.

Exercise 4. 1. 4120; 4230; 4340; 4450; 4560; 4670; 4540; 4730. 2. 41,200; 42,300; 43,400; 44,500; 45,600; 46,700; 45,400; 47,300.

3. 5150; 5287.5; 5425; 5562.5; 5700; 5837.5; 5675; 5912.5. **4.** 47.6; 47.6; 47.6; 47.6; 47.6; 47.6; 47.6; 47.6. **5.** (1) 82.4; 84.6; 86.8; 89; 91.2; 93.4; 90.8; 94.6. (2) 20.6; 21.15; 21.7; 22.25; 22.8; 23.35; 22.7; 23.65. (3) 0.0412; 0.0423; 0.0434; 0.0445; 0.0456; 0.0467; 0.0454; 0.0473.

Exercise 5. **1.** 94.3; 94.3; 94.3; 94.3; 94.3; 94.3; 94.3; 94.3. **2.** 1295; 1317.5; 1340; 1362.5; 1385; 1407.5; 1330; 1372.5. **3.** 4.32; 4.39; 4.47; 4.54; 4.62; 4.69; 4.43; 4.58. **4.** 60.43; 61.48; 62.53; 63.58; 64.63; 65.68; 62.07; 64.05. **5.** (1) 41.44; 42.16; 42.88; 43.6; 44.32; 45.04; 42.56; 43.92. (2) 64.75; 65.88; 67; 68.13; 69.25; 70.38; 66.5; 68.63.

Page 481. *Exercise 6.* **1.** 161.7; 162.6; 163.5; 164.4; 165.3; 166.2; 163.7; 164.9. **2.** 48.5; 47.6; 46.7; 45.8; 44.9; 44.0; 46.5; 45.3. **3.** 369.93; 375.24; 380.55; 385.86; 391.17; 396.48; 381.73; 388.81. **4.** 7.6; 7.71; 7.82; 7.93; 8.04; 8.15; 7.84; 7.99. **5.** (1) 7.84; 7.95; 8.06; 8.18; 8.29; 8.4; 8.09; 8.24. (2) 0.78; 0.80; 0.81; 0.82; 0.83; 0.84; 0.81; 0.82. (3) 78.38; 79.5; 80.63; 81.75; 82.88; 84; 80.88; 82.38.

Exercise 7. **1.** 262.1; 263.0; 263.9; 264.8; 265.7; 266.6; 264.1; 265.0. **2.** 32.3; 31.4; 30.5; 29.6; 28.7; 27.8; 30.3; 29.4. **3.** (1) 44.94; 45.5; 46.06; 46.63; 47.19; 47.75; 46.19; 46.75. (2) 4493.75; 4550; 4606.25; 4662.5; 4718.75; 4775; 4618.75; 4675. **4.** (1) 115.04; 116.48; 117.92; 119.36; 120.8; 122.24; 118.24; 119.68. (2) 11.50; 11.65; 11.79; 11.94; 12.08; 12.22; 11.82; 11.97. (3) 1150.4; 1164.8; 1179.2; 1193.6; 1208; 1222.4; 1182.4; 1196.8. **5.** (1) 14.38; 14.56; 14.74; 14.92; 15.1; 15.28; 14.78; 14.96. (2) 28.76; 29.12; 29.48; 29.84; 30.2; 30.56; 29.56; 29.92. (3) 43.14; 43.68; 44.22; 44.76; 45.3; 45.84; 44.34; 44.88.

Exercise 8. **1.** 82,700; 83,600; 84,500; 85,400; 86,300; 87,200; 86,900; 87,800. **2.** 454.85; 459.8; 464.75; 469.7; 474.65; 479.6; 477.95; 482.9. **3.** (1) 2.07; 2.09; 2.11; 2.14; 2.16; 2.18; 2.17; 2.20. (2) 20.68; 20.9; 21.13; 21.35; 21.58; 21.8; 21.73; 21.95. (3) 206.75; 209; 211.25; 213.5; 215.75; 218; 217.25; 219.5. **4.** (1) 3308; 3344; 3380; 3416; 3452; 3488; 3476; 3512. (2) 330.8; 334.4; 338; 341.6; 345.2; 348.8; 347.6; 351.2. (3) 33.08; 33.44; 33.8; 34.16; 34.52; 34.88; 34.76; 35.12. **5.** 196.7; 197.6; 198.5; 199.4; 200.3; 201.2; 200.9; 201.8.

Page 482. *Exercise 9.* **1.** 53 ft. 4 in.; 70 ft.; 106 ft. 8 in.; 123 ft. 4 in.; 156 ft. 8 in.; 166 ft. 8 in.; 171 ft. 8 in.; 190 ft. **2.** 7 ft. 5 in.; 6 ft. 7 in.; 4 ft. 9 in.; 3 ft. 11 in.; 2 ft. 3 in.; 1 ft. 9 in.; 1 ft. 6 in.; 7 in. **3.** (1) 5 ft. 4 in.; 7 ft.; 10 ft. 8 in.; 12 ft. 4 in.; 15 ft. 8 in.; 16 ft. 8 in.;

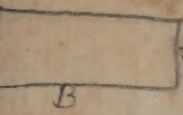
17 ft. 2 in.; 19 ft. (2) 5 ft. 8 in.; 7 ft. $5\frac{1}{4}$ in.; 11 ft. 4 in.; 13 ft. $1\frac{1}{2}$ in.; 16 ft. $7\frac{3}{4}$ in.; 17 ft. $8\frac{1}{2}$ in.; 18 ft. $2\frac{7}{8}$ in.; 20 ft. $2\frac{1}{4}$ in. (3) 56 ft. 8 in.; 74 ft. $4\frac{1}{2}$ in.; 113 ft. 4 in.; 131 ft. $\frac{1}{2}$ in.; 166 ft. $5\frac{1}{2}$ in.; 177 ft. 1 in.; 182 ft. $4\frac{3}{4}$ in.; 201 ft. $10\frac{1}{2}$ in. **4.** (1) 1 ft. 4 in.; 1 ft. 9 in.; 2 ft. 8 in.; 3 ft. 1 in.; 3 ft. 11 in.; 4 ft. 2 in.; 4 ft. $3\frac{1}{2}$ in.; 4 ft. 9 in. (2) 8 in.; $10\frac{1}{2}$ in.; 1 ft. 4 in.; 1 ft. $6\frac{1}{2}$ in.; 1 ft. $11\frac{1}{2}$ in.; 2 ft. 1 in.; 2 ft. $1\frac{3}{4}$ in.; 2 ft. $4\frac{1}{2}$ in. (3) $5\frac{1}{3}$ in.; 7 in.; $10\frac{2}{3}$ in.; 1 ft. $\frac{1}{3}$ in.; 1 ft. $3\frac{2}{3}$ in.; 1 ft. $4\frac{2}{3}$ in.; 1 ft. $5\frac{1}{6}$ in.; 1 ft. 7 in. **5.** (1) $10\frac{2}{3}$; 14; $21\frac{1}{3}$; $24\frac{2}{3}$; $31\frac{1}{3}$; $33\frac{1}{3}$; $34\frac{1}{3}$; 38. (2) $6\frac{2}{5}$; $8\frac{2}{5}$; $12\frac{4}{5}$; $14\frac{4}{5}$; $18\frac{4}{5}$; 20; $20\frac{3}{5}$; $22\frac{4}{5}$. (3) $1\frac{1}{3}$; $1\frac{3}{4}$; $2\frac{2}{3}$; $3\frac{1}{12}$; $3\frac{11}{12}$; $4\frac{1}{6}$; $4\frac{7}{24}$; $4\frac{3}{4}$.

Exercise 10. **1.** $137\frac{7}{8}$; $137\frac{7}{8}$; $137\frac{7}{8}$; $137\frac{7}{8}$; $137\frac{7}{8}$; $137\frac{7}{8}$; $137\frac{7}{8}$; $137\frac{7}{8}$. **2.** 11,587.5; 12,150; 11,675; 11,787.5; 12,325; 12,100; 12,212.5; 12,300. **3.** 2317.5; 2430; 2335; 2357.5; 2465; 2420; 2442.5; 2460. **4.** (1) 12.76; 13.38; 12.86; 12.98; 13.57; 13.33; 13.45; 13.55. (2) 25.52; 26.76; 25.72; 25.96; 27.15; 26.65; 26.90; 27.09. **5.** 3167 ft. 3 in.; 3321 ft.; 3191 ft. 2 in.; 3221 ft. 11 in.; 3368 ft. 10 in.; 3307 ft. 4 in.; 3338 ft. 1 in.; 3362 ft.

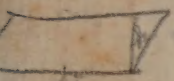
Exercise 11. **1.** 4 yd. 19 in.; 14 ft. 2 in.; 5 yd. 12 in.; 5 yd. 22 in.; 6 yd. 22 in.; 12 ft. 11 in.; 14 ft. 6 in.; 10 ft. 10 in. **2.** 7 yd. 8 in.; 21 ft. 1 in.; 6 yd. 15 in.; 6 yd. 5 in.; 5 yd. 5 in.; 22 ft. 4 in.; 20 ft. 9 in.; 24 ft. 5 in. **3.** (1) 3 yd. $19\frac{1}{2}$ in.; 145 in.; 5 yd. 20 in.; $6\frac{1}{4}$ yd.; $8\frac{3}{4}$ yd.; $107\frac{1}{2}$ in.; 155 in.; $3\frac{3}{4}$ ft. (2) 35 yd. 15 in.; 1450 in.; 55 yd. 20 in.; $62\frac{1}{2}$ yd.; $87\frac{1}{2}$ yd.; 1075 in.; 1550 in.; $37\frac{1}{2}$ ft. (3) $12\frac{3}{4}$ in.; $14\frac{1}{2}$ in; 20 in.; $\frac{5}{8}$ yd.; $\frac{7}{8}$ yd.; $10\frac{3}{4}$ in.; $15\frac{1}{2}$ in.; $\frac{3}{8}$ ft. **4.** (1) 17 in.; $19\frac{1}{3}$ in.; $26\frac{2}{3}$ in.; $\frac{5}{6}$ yd.; $1\frac{1}{6}$ yd.; $14\frac{1}{3}$ in.; $20\frac{2}{3}$ in.; $\frac{1}{2}$ ft. (2) $10\frac{1}{8}$ in.; $11\frac{3}{8}$ in.; 16 in.; $\frac{1}{2}$ yd.; $\frac{7}{10}$ yd.; $8\frac{3}{8}$ in.; $12\frac{2}{5}$ in.; $\frac{3}{10}$ ft. (3) $7\frac{2}{7}$ in.; $8\frac{2}{7}$ in.; $11\frac{3}{7}$ in.; $\frac{5}{14}$ yd.; $\frac{1}{2}$ yd.; $6\frac{1}{7}$ in.; $8\frac{6}{7}$ in.; $\frac{3}{14}$ ft. (4) $4\frac{1}{4}$ in.; $4\frac{5}{6}$ in.; $6\frac{2}{3}$ in.; $\frac{5}{24}$ yd.; $\frac{7}{24}$ yd.; $3\frac{7}{12}$ in.; $5\frac{1}{6}$ in.; $\frac{1}{8}$ ft. **5.** (1) 51; 58; 80; 90; 126; 43; 62; 18. (2) 510; 580; 800; 900; 1260; 430; 620; 180. (3) 204; 232; 320; 360; 504; 172; 248; 72.

September
96 lb.
in.
May
102 lb.
in.

$A = S^2$

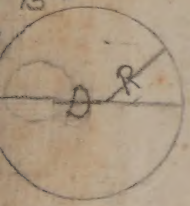


$A = b \times h$



$A = b \times h$

$A = \frac{1}{2} \times B \times h$



$C = \frac{22}{7} \times D$

$R = 2r \times \frac{22}{7}$

$A = \pi R^2$

volume $= \frac{22}{7} \times r^2 \times h$

$\frac{22}{7} \times r$

Obtuse

810
557
1912